Foundations of quantal economy and banking principles.

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Contents

Some words upfront.

What is the value of money? This is an old question and the economist must pay due attention to the virtual monetary value of creation of goods and products. This is the most human endeavour where it becomes important to give a rather silly weight, in terms of natural numbers, to something which may transcend current human society in ways which may be unforseen by the evaluator. Currently, virtual values of houses and non-liquid products are taken into account as collateral for a bank when borrowing money to a demanding party. But how to deal with innovation, or even the potentiality theirof? How do you value the activity of a brilliant new young potential Tesla even when failure is still possible? Should a bank be allowed to take some estimated current value of a estimated future real payoff, maybe not even in monetary units, into account when making up a general ledger? Such an activity is a virtual intermediate one of a quantal nature where the past and future are taken into account in order to give a weight to a current real process even if it just happens once historically. Very often, large amounts of money for the bank go to a waste due to either the incapacity of its employees to value innovaters or visionary entrepeneurs in a humain and correct fashion due to a rather strict Roman book-keeping system or because of rather strict rules placed upon its own inner workings coming from the national banks safegarding their economic stability, so called. We shall argue in this little book or cahier, that this problem is entirely analogous to the problem of defining black-white psychological types resulting in a weighted duality between conservatism and progressiveness, between an elite system and a total democracy, between narrow mindness and a liberitarian world view, between a totally regulated market and an entirely free one, between awareness and impetus where the latter indicates a desire for life or change. In physics, this duality is well known too in thermodynamics, extensive versus intensive variables or order versus temperature. It is indeed no accident that the Maxwell-Boltzmann distribution which expresses total democracy corresponds to the time evolution operator in quantum theory with an imaginary Planck constant. Indeed, impetus in psychology is imaginary wherea bestatigung or narrow mindness is real. When making a choice we narrow our options whereas a floating mind is totally free. This discussion is at this moment held in Europe where the British have their own way of dealing with money, not a totally white or liberitarian one (with a strange queen-"mother" facing on the notes, a notorious lover of horses) but somewhat more attuned to temperature as to order and volume. As we shall see, this is also a black strategy where impetus is made real and evolution becomes state, but a dual one. Indeed, the fact that plenty of bars and restaurants keep on going appears rather strange for a continental person used to merely taking plus and minus of extensive variables. The English have plenty of imagination too but are rigid in chaos and not too evolutionary which reflects in the fact that the state organized by mostly women (see the banknotes). The Italians, in that regard, remain with Roman rigidity, a spear is a spear and Jezus has to bleed (if you want to see this, please go to office 7BF3J8); in Great Britain, a spear may also be a Penis and refer to a whole different kind of punishment ("did you kill your wife with a spear?" refers on the isle to merely having sexual intercourse). Totally black therefore; white banks are therefore Mafia banks where money is born in the Vampire crypts - not necessarily a bad thing (a one Euro account with indefinite payment facilities) but a rather old fahioned Swiss way of dealing with things. In a way, white people need Mafia banks and a mafia government whereas Black people need old fashioned Roman or British commoner banks with a huge amount of paperwork. Having said this (it is no joke), we proceed in this Cahier in the following way: in the first chapter, we introduce the necessary mathematical setting which is nothing but a multi dimensional Heisenberg Lie algebraic one. This has been discussed in my books on psychology in full depth and we shall just copy the argument here almost ad verbatim. In the second chapter, we discuss the quantum dynamics as well as the Maxwell Boltzmann distribution being the state corresponding to the dynamical anti-Hermitian impetus. In the third chapter, the grand canonical quantum ensemble is discussed introducing a chemical (or creative) potential leading to the definition of the "appropriate" white states where the word appropriate refers to metric and variables chosen. In chapter four, we relate the different notions of blackness related to different, being actually conjugate, variables and explain the European conondrum in detail. I proclaim therefore complex (as a mixture between black and white in the several different variables) banks as belonging to the future. These are indeed banks which are intensive-extensive and black-white (in both regards) at the same time. There are several interesting dualities here which reflect in different worldviews and distinct kinds of doing business. I hope this book renunciates both (male and female) view by means of a new form of sexuality called the dickwoman or shemale way of doing business.

Chapter 1

Definitions of black-white psychological profiles.

In this chapter, we define a canonical mapping from behavioural space $\mathcal{B}$ to the black and white complex plane $\mathbb{C}^2$. The notions of black and white depend upon the cultural benchmarks one wishes to uphold and those differ between the sexes too. I suggest the female definition of blackness to coincide with the Wick rotation of the male definition of whiteness; that is, a black female assigns heath to male impetus which is exactly why black females become terribly hot of white men. A white woman is extremely rigid and opposes white males urges for change; this is one kind of duality which holds in nature. The continental European banking system is a black male one, extremely rigid, whereas the British one is hot being extremely chaotic and with an ill defined general ledger. Henceforth, given the different sexual (British woman are found of weak men and hot, but disruptive towards strong men) as well as cultural emphasis, it is impossible to merely join both systems. Sexual and banking revolutions need to take place.

1.1 From behavioral space to black-white.

It is natural to assume that space is described by a convex polygon in N real dimensions with the barycenter as origin of the appropriate coordinate system. In principle, the polygon could extend indefinetly so that we take $\mathbb{R}^N$ mathematically although it does not need to be so in practise. Black people are defined as those with the maximum of rigidity, that is those who have the possibility to live in exactly one profile; mathematically, this reads as

$$|B; \mathbf{b}, \theta\rangle := e^{i\theta}\delta^N(\mathbf{x} - \mathbf{b})$$

where $\mathbf{b}$ is a vector in $\mathbf{R}^N$ and θ is an angle. The corresponding value of the black field is

$$\Psi := ||\mathbf{b}||e^{i\theta}.$$

Now, white people are defined to have constant amplitude on every psychological profile and therefore must correspond to a Fourier wave given that altering the origin cannot change the definition of white. This means that black and white behave as eigenstates of a Heisenberg algebra with respect to one and another where white means momentum or impetus and black indicates position, conservatism or preservation. Mathematically,

$$|W; \mathbf{b}, \theta\rangle := \frac{1}{(2\pi)^{\frac{N}{2}}} \int_{\mathbb{R}^N} d\mathbf{y} e^{i\mathbf{x}.\mathbf{y}} \, e^{i\theta} \delta^N(\mathbf{y} - \mathbf{b}) = \frac{1}{(2\pi)^{\frac{N}{2}}} e^{i\theta} e^{i\mathbf{x}.\mathbf{b}}$$

where again the white field value

$$\Phi := ||\mathbf{b}|| e^{i\theta}.$$

We shall now reason in two different ways of how to arrive at the intermediate states, the state with portion $|\lambda|^2$ of white and $(1 - |\lambda|^2)$ of black is given by a (distributional) eigenstate (in the square integrable sense) of the operators

$$|\lambda|^2 P_r + (1 - |\lambda|^2) X^r$$

where

$$[P_r, X^s] := P_r X^s - X^s P_r = \delta^s_r \, 1.$$

As said before, a linear operator preserves the natural additivity property of the space it acts upon. The space at hand here consists out complex valued functions defined on $\mathbb{R}^N$ and the scalar product is given by

$$\langle f|g\rangle := \int_{\mathbb{R}^N} d\mathbf{x} \overline{f(\mathbf{x})} g(\mathbf{x}).$$

The reader should verify that it satisfies the necessary projective properties, called bilinearity such that it defines a Cartesian distance in an infinite number of dimensions. An eigenstate of an operator X^r is in this case a function f such that

$$X^r(f) = b^r f$$

where b^r is a complex number; moreover we demand that its distance to the zero function or origin is finite or can be conceived as a limit of functions with that property in a suitable way. It must be clear that the black operator X^r is given by multiplication with x^r, that is

$$(X^r(f))(\mathbf{x}) = x^r f(\mathbf{x}).$$

Indeed, such eigenfunction must be zero everywhere except at one point where it is infinitely large; therefore it must be clearly thought of as the limit of proper functions. On the other hand P_r is nothing but the derivative operator with respect to the variable x^r, that is

$$P_r := \frac{\partial}{\partial x^r}$$

and on checks that the Fourier waves are the only proper distributional eigenfunctions as well as that the Heisenberg algebra is satisfied. In particular

$$X^r|B;\mathbf{b},\theta\rangle = b^r|B;\mathbf{b},\theta\rangle$$

so that the eigenvalue b^r is real whereas

$$P_r|W;\mathbf{b},\theta\rangle = ib^r|W;\mathbf{b},\theta\rangle$$

so that the eigenvalue ib^r is pure imaginary. Operators with the former property are called Hermitian whereas the latter are called anti-Hermitian. In standard quantum theory, one chooses $-iP_r$ so that both the momentum and position are hermitian operators; I feel this is unsatisfying as there should be a clear distinction between being and impetus. Usually, one is aware of impetus by successive measurements of being and not by measuring impetus itself. Actually, impetus is awareness as we have argued for above or at least awareness necessitates impetus so that measuring impetus is like awareness of awareness. There is clear distinction between those two concepts and given that quantum theory deals with the lowest form of awareness, and the latter is real, I deem this distinction to be mandatory. Indeed, communication between black and white appears to require a higher form of awareness something we shall argue from the side of the particular mathematical aspects of the dynamical model described above.

Before proceeding with the mathematics, one notices that, by definition, the total amplitude equals the sum of black and white amplitudes; we have dubbed this as *psychic* energy whereas the mechanical process, with its *mechanical energy* changing the fields, and therefore also the separate local amplitudes, require *spatial variations* of the total amplitude or different psychological profiles resulting in the three distinct angles being the black-white ratio as well as two phase factors. Hence, a black person with similar energy as a white counterpart is more rigid and has less chance to adapt in society which results in himor her searching for like minded persona or locking himself up in a small room. The same thing can happen to a person who is too white; too much versatility creates confusion and jealousy so that he needs to isolate himself or look for a very selective public way above the fray. The distinction with an extreme black person of alike magnitude is that the latter is too rigid to be productive and such person becomes totally useless in society. He has to move out in order to survive to a place which makes him less black whereas the white opposite, reflected along the diagonal has to move out or become a superstar.

To return to the quantification process, we have to look for functions f satisfying

$$\left(|\lambda|^2\frac{\partial}{\partial x^r} + (1-|\lambda|^2)x^r\right)f(\mathbf{x}) = \mu^r f(\mathbf{x})$$

solutions which are given by

$$f(\mathbf{x}) := ce^{-a(\mathbf{x}-\mathbf{b})^2}.$$

Here, a satisfies,

$$-|\lambda|^2 2a + (1 - |\lambda|^2) = 0$$

meaning

$$\frac{|\lambda|}{\sqrt{1-|\lambda|^2}} = \frac{1}{\sqrt{2a}} = \tan(\alpha)$$

where α constitutes the angle between the black axis given by $(1, 0)$ and the $(|\Psi|, |\Phi|)$ vector. μ^r is given by $|\lambda|^2 2ab^r = \frac{2a}{2a+1} b^r$ which for large a gives b^r and for small a, $2ab^r$ which necessitates a scaling of b^r such that the modulus of $\frac{2a}{2a+1} b^r$ is independent of a and the latter expression is real for a to infinity and purely complex for a to zero. Rotational invariance between black and white then dictates that

$$b^r = \tilde{b}^r \frac{2a+1}{2a} e^{i\alpha(a)} = \tilde{b}^r \left(\sqrt{\frac{2a+1}{2a}} + i \sqrt{\frac{2a+1}{(2a)^2}} \right)$$

where $\tilde{b^r} \in \mathbb{R}$. The constant c is fixed upon an angle so that the corresponding states are

$$|\lambda, \mathbf{b}, \theta\rangle \to ||\tilde{\mathbf{b}}||(\lambda, \sqrt{1-|\lambda|^2} e^{i\theta}).$$

The number λ provides one also with an angle or perspective, so that mixed profiles offer two perspectives whereas pure black and white ones just one. Extreme democracy or libertarianism versus extreme authorative behaviour.

1.2 Reduction of multiple quibit dynamics to a single individual.

In the remainder of this chapter, we make a different kind of exercise on the same topic. That is, we imagine that the true unitary dynamics on a, say, bosonic Fock space with an arbitrary number of participants gets compactified to an effective black-white theory for one person. In quantum theory, we know very well that this procedure of taking the partial trace leads to an effective non-Hermitian one black-white Hamiltonian due to "radiative noise" from background degrees which have not been properly dynamically implemented. This is an inherent limitation of science, on the one hand, we always have to assume that subsystems are totally isolated when constructing idealized laws (ignoring hereby birth and death) whereas we secretively know that this is approximation belongs to the capable discretion of the experimentator whenn testing individual elements. Indeed, only he can guide us while protecting us maximally from others. The individual black white theory is still infinite dimensional as the "black energy" should be arbitrary and positive and distinct energy states should be statistically independent in your theory. To, nevertheless, proceed in abstracto and say something useful, we take the point of view of Noether who associated free will actions which do not cost any mental energy to dynamical symmetries of your theory. As is well known in physics, these free thoughts ought to correspond to free motoric decisions such as moving forwards, to the left or jumping in the air or to rotate around ones axis. From the stationary point of view, the latter tree rotations are all what is feasable. Moreover, these symmetry operators act linearly on the creative aspects of your theory; for example if a baby is born, then kicking that baby in the ass is going to put in a state between life and death but it is not going to influence other baby's aspects of life unless all of them are sitting in the same car and you want to dump the latter into a lake. Here, the aspect of freeness is important, that each baby is uncoupled to another one. In abstracto, the symmetries we are interested in must act on two operators: the black creation operator of life $a^\dagger$ and the black creation operator of death a where the operation $\dagger$ switches between life and death. A theory where whiteness is associated to killing of black people is rather dangerous so therefore different types of persona are associated to distinct mixtures of $a^\dagger, a$; what we are interested in here is the peculiar type of mixture which should occur in nature based upon understanding in abstracto the linear action on the pair $(a, a^\dagger)$ in terms of a three dimensional group of mental free thoughts or equivalently, free mechanical spin. The algebra of life and death is given by

$$\left[a, a^\dagger\right] = 1$$

which is the usual Heisenberg algebra but then with a real position operator and a complex momentum upholding hereby our interpretations of awareness and impetus. To be aware of its own life or death is of a higher kind and requires the complex algebra. This suggests

$$P = \frac{1}{\sqrt{2}}(a^\dagger - a),\ X = \frac{1}{\sqrt{2}}(a + a^\dagger)$$

where

$$\left[a, a^\dagger\right] = 1$$

as well as

$$[P, X] = 1.$$

The operator a is a so called annihilation operator of the Switchoriem vacuum $|0\rangle$ meaning $a|0\rangle = 0$, whereas $a^\dagger$ is the creation operator of a unit of Switchoriem life. P is the defining operator for white males who treat life and death in an anti-symmetrical way whereas black people treat it symmetrically. Swithchoriems choose either for life or death and have a short or long life (destructive birds or sneaky snakes). The dynamics should choose for life and death equally meaning

$$H = \frac{1}{2}(a^\dagger a + a a^\dagger) = a^\dagger a + \frac{1}{2} = \frac{1}{2}(P^2 + X^2) = \frac{1}{2}(P^\dagger P + X^\dagger X) = \frac{1}{4}((P+X)^2 + (X-P)^2).$$

Indeed, the operator H is invariant with respect to symplectic transformations

$$(X, P) \to (aX + bP, cX + dP)$$

with determinant 1, meaning

$$ad - bc = 1$$

which is required to preserve the Heisenberg algebra, as well as preserving the natural dynamics

$$P^{\dagger}P + X^{\dagger}X = (aX + bP)^{\dagger}(aX + bP) + (cX + dP)^{\dagger}(cX + dP)$$

resulting in

$$ad - bc = 1, |a|^2 + |c|^2 = 1, |b|^2 + |d|^2 = 1, \overline{a}b + \overline{c}d = 0$$

where we will later on multiply by a factor $e^{i\frac{pi}{4}}$. These four conditions result in

$$a = \cos(\theta)e^{i\alpha},\ c = \sin(\theta)e^{i\beta - i\gamma},\ b = -\sin(\theta)e^{-i\beta + i\gamma},\ d = \cos(\theta)e^{-i\alpha}$$

as being the most general solutions. The three rotational parameters are in accordance again with the three dimensional rotation group or $SU(2)$ given that the Hamiltonian determines the time evolution and hence breaks a $U(1)$ symmetry. Our variables are even more specific in the way that we demand a Male-Female duality to exist, given by

$$S : (X, P) \to i(P, X),\ S^2 = -1,\ S^{\dagger} = -S$$

as well as a White-Black symmetry

$$WB : (X, P) \to (-P, X),\ (WB)^2 = -1,\ (WB)^{\dagger} = -WB.$$

They are anti-commuting processes and moreover

$$S(WB) = Z,\ Z : (X, P) \to i(X, -P), Z^{\dagger} = -Z$$

a transformation which simply reverses the impetus and Hermiticity properties as well. The Heisenberg algebra as well as the Hamiltonian are perfectly conserved by our $SU(2)$ tranformations at hand; further conditions have to be imposed now regarding the two dualities S, WB. In order to find the correct ones, notice that our variables have to satisfy

$$(X, P)^T WB(X, P) = -1,\ (X, P)^{\dagger}(X, P) = 2H$$

which express the Heisenberg algebra as well as the positivity of the Hamiltonian operator, for the black-white spiritual charges, in a geometrical fashion. This gives a curious mixture between complex and Hermitian geometry mixing $SU(2)$ transformations with *complex* symplectic ones which are defined by means of the Lie-algebra

$$A = (WB) \circ A^T \circ (WB)$$

which is the complexified $su(2)$. Demanding the transformations to be unitary with respect to the symplectic inner product defined by WB as well as the

Hermitian one imposes no further conditions as is just chooses the Hermitian section

$$A^{\dagger} = A,\, A = (WB) \circ A^{T} \circ (WB).$$

This result was obtained previously in the form that *if* the Heisenberg commutation relations hold, then a $SU(2)$ transformation preserves them. Note also that

$$H = \frac{1}{2}(T(X,P)(T \circ \widetilde{S})(X,P)^{\dagger} - (T \circ (WB))(X,P)((T \circ (WB) \circ \widetilde{S})(X,P))^{\dagger})$$

where $T(X,P) = \frac{1}{\sqrt{2}}(X+P), (T(X,P))^{\dagger} = T \circ S(X,P)$ is the defining operator of the Switchoriem which means that the latter is black-white dual and real symplectic. Furthermore, $-iS = \widetilde{S}, \widetilde{S}^2 = 1$ constitutes a real symmetric $SU(2)$ matrix which is not symplectic given that $-(WB)S(WB) = -S = S^{-1}$ a property which does not hold for iS. The hyperbolic character of the Hamiltonian comes from the anti-commutation relations $S \circ (WB) = -(WB) \circ S$ as well as the trivial property $T \circ \widetilde{S} = T$. So, there is a conjugation between $T, T \circ (WB)$ and $1, \widetilde{S}$ which has a dynamical as well as geometrical meaning. The reader verifies that this way of rewriting the Hamiltonian agrees with the usual expression for conjugate variables (X,P) and $a^{\dagger}, a$ and that, trivially, $T \circ \widetilde{S} = T$. Hence, our energy functional poses reveals that the $SU(2)$ transformation

$$\widehat{T} : (X,P) \to (T(X,P), -(T \circ WB)(X,P))$$

is a rather special one; replacing T by $T \circ WB$ gives

$$\widehat{T} \circ WB : (X,P) \to ((T \circ (WB))(X,P), T(X,P))$$

another one. This suggests one to wonder what conditions on

$$\widehat{T} = \begin{pmatrix} \langle v| \\ \langle v|WB \end{pmatrix}$$

need to be placed in order for this to work out. Denoting by $\langle v| = i(a,b)$, a simple calculation yields that

$$\widehat{T}(X,P) = i(aX + bP, -aP + bX)$$

and it is obvious that for the Heisenberg equations as well as the formulae regarding the Hamiltonian to be satisfied, it is mandatory that

$$a^2 + b^2 = 1,\, |a|^2 + |b|^2 = 1,\, \overline{a}b - \overline{b}a = 0$$

resulting in $a = \cos(\theta), b = \sin(\theta)$ which provides one with a complexified rotation

$$\widehat{T}(\theta) = i \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{pmatrix}$$

where the parameter rannge varies between 0 and 2π. The multiplication law is rather peculiar meaning

$$\widehat{T}(\theta)\widehat{T}(\psi) = -\begin{pmatrix} \cos(\theta-\psi) & -\sin(\theta-\psi) \\ \sin(\theta-\psi) & \cos(\theta-\psi) \end{pmatrix} = (WB) \circ \widehat{T}(\theta-\psi) \circ S$$

which is a standard rotation up to a factor -1. So, our $\widehat{T}(\theta)$ transformations do not constitute a group but a strange twisting and inversion of the second angle takes place. To remedy for that sitution, we must find a natural novel multiplication law as well as an associated unity and inner product such that the usual composition law holds. It is clear that we must simply reflect the angle in a delicate way; this has to do with "Hopf algebra's". Indeed, we may write

$$\widehat{T}(v) = -\langle v| \oplus \langle v(WB)|$$

and

$$-\widetilde{S}\widehat{T}(v)(WB)\widetilde{S} = +\widetilde{S}(\langle v\widetilde{S}(WB)|\oplus\langle v\widetilde{S}|) = -\widetilde{S}\langle v\widetilde{S}(WB)|\oplus\langle v\widetilde{S}| = -\langle v\widetilde{S}|\oplus\langle v\widetilde{S}(WB)|$$

which is the mirror image around the diagonal. The product law we are looking for is given by

$$\widehat{T}(\theta) \star \widehat{T}(\psi) := -\widehat{T}(\theta)Z\widehat{T}(\psi) = \widehat{T}(\theta+\psi)$$

using the fact that

$$(\widehat{T}(\psi))^\dagger = -(\widehat{T}(\psi))^T = -\widehat{T}(\psi),\ \widehat{T}(\theta)^2 = -1$$

as well as

$$\widehat{T}(\psi)(WB)(\widehat{T}(\psi)) = (WB)$$

which reduces further, by means of $(\widehat{T}(\psi))^2 = -1$, to

$$\widehat{T}(\psi) = (WB)\widehat{T}(\psi)(WB).$$

This is an associative product with unit given by $iZ, (iZ)^2 = 1$, $Z^\dagger = -Z$ which suggests the use of the scalar product

$$\langle (X,P)|(X,P)\rangle = (X,P)^\dagger(\pm iZ)(X,P)$$

in which the momentum operator is negatively (positively) squared whereas the position operator positively (negatively). The remainder of this section does not depend upon the sign choice; we have used throughout that

$$i(a,b)\widetilde{S} = i(b,a)$$

which is the operator defining the orthogonal reflection around the diagonal axis. This is consistent with the view that a black male is swapped under S duality to a white female at least and vice versa for a black male. Indeed, we shall now define a relationship between black-white duality (given by the *real*

Heisenberg algbra) and the duality $\widetilde{S}$. The reader noticed that the latter duality is implemented throughout

$$\widetilde{S}\widehat{T}(\theta)(-iZ)$$

which suggests it is much better to use the operators

$$\widetilde{T}(\theta) := -\widehat{T}(\theta)Z$$

given that

$$\widetilde{T}(\theta)\widetilde{T}(\psi) = \widehat{T}(\theta)Z\widehat{T}(\psi)Z = -\widehat{T}(\theta+\psi)Z = \widetilde{T}(\theta+\psi)$$

as well as

$$(iZ)\widetilde{T}(\theta)^{\dagger}(iZ) = \widehat{T}(\theta)Z = -\widetilde{T}(\theta)$$

which fully exploits the abelian character of the group of transformations. The transformations $\widetilde{T}(\theta)$ have also unit determinant meaning they preserve the Heisenberg algebra, they are also unitary (in the standard sense) meaning they preserve the Hamiltonian, and they are unitary as well with respect to the black white inner product meaning that

$$\langle \widetilde{T}(\theta)(X,P)|\widetilde{T}(\theta)(X,P)\rangle = \langle (X,P)|(X,P)\rangle.$$

In standard form

$$\widetilde{T}(v) = \langle vZ| \oplus \langle vS|$$

where

$$|v\rangle = (\cos(\theta), \sin(\theta))$$

indicating a logical symmetry between the sex and black-white tranform which is the thing we were looking for. One notices that the quantal aspects, associated to WB, do not enter in the definition of those transformations. The reader should not confuse what we are doing here with our previous heuristic approach on the space of psychological profiles. There the X quantity was supposed to be positive given that all natural units are bounded from below and henceforth determine an origin. The momentum operator there was really an impetus in the sense that it changed an observable quantity; here, on the other hand, X is not bounded from below at all and the situation is perfectly symmetric. The number operator is in a way $H - \frac{1}{2}1$, a quadratic entity in our basic variables whereas the previous X is assumed to be foundational. The creation operators $a^{\dagger}$ really do produce a quantum of blackness and quantities such as $X \sim H$ go as $a^{\dagger}a$; one should not compare apples with lemons. The theory here is much more foundational as there is a natural operator beyond the symplectic and black-white one which one has to associate with a psychological quantity. The matrices $\widehat{T}(\theta)$ satisfy

$$\widehat{T}(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} |v\rangle_{\pm} = e^{\pm i\theta}|v\rangle_{\pm}$$

and

$$|v\rangle_{\pm} = \frac{1}{\sqrt{2}}(1, \mp i).$$

Given that $\overline{|v\rangle_{+}} = |v\rangle_{-}$ one can form two canonical vectors

$$e^{i\theta}|+\rangle \pm e^{-i\theta}\overline{|+\rangle}$$

from which the first one, given by

$$(\cos(\theta), \sin(\theta))$$

is obviously the correct one. As usual, there has to be a duality between symmetries and ones actions (to rotate or to translate); given that all the physics is in iZ, WB due to the general interacting nature of black and white obliging us to give up on the free Hamiltonian constraint H, we conclude that the symmetry tranformations are given by the unitary Bogoliubov group

$$A : A^{\dagger}(iZ)A = iZ, A^{T}(WB)A = (WB)$$

or even less, by the Heisenberg group

$$A^{T}(WB)A = WB$$

which is six dimensional (which is the most generic possibility). All other options impose constraints between the variables and their momenta; indeed, the Bogoliubov group is three dimensional implying for three constraints on momentum or spin. Obviously, the $\widehat{T}(\theta)$ are extremely special as they only produce a θ angle in the vector $c(\cos(\psi)e^{-i\theta}, \sin(\psi)e^{i\theta})$ given that they preserve the modulus squared of all black and white amplitudes. Hence, we must determine the other Bogoliubov rotation in order to fix ψ as well as the boost parameter c. The reality conditions emerge from noting that the operators $A = iS, i(WB), iZ$ all belong to $su(2)$ *and* $SU(2)$. To make the distinction, we propose that the A conjugate of a vector v is defined by means of

$$v^{c} := Av^{\star}$$

which is equivalent to $X^{\dagger} = X$, $P^{\dagger} = -P$ in case $A = -iZ$. This puts away the creation annihilation algebra of pure birth and death; that a white profile should kill a black one. Instead, white and black have equal weights what regards the killing and birth of black; the fact that $P^{\dagger} = -P$ shows that white has an aversion towards killing whereas black likes it. Taking the † operation really means killing someone, killing a dead spirit is bringing it to life whereas killing an alive spirit is making it dead. White people love their current state therefore. We demand, moreover, that our theories are $-iZ$ conjugate, meaning $v^{c} = v$ or $v^{\star} = (-iZ)v$, which imposes an equivalence between $-iS$ and WB. Indeed, for such variables

$$(X, P)^{T}WB(X, P) = (X, P)^{\dagger}(-iS)(X, P)$$

given that

$$(X, P)^{\dagger} = (X, P)^{T}(iZ)$$

implying

$$(X, P)^{T}(iZ)(-iS)(X, P) = (X, P)^{T} W B(X, P).$$

This explains the Heisenberg algebra; indeed the eye conjugation reveals that $X^{\dagger} = X, P^{\dagger} = -P$ and therefore

$$(X, P)^{\dagger}(-iS)(X, P) = XP + P^{\dagger}X = XP - PX = -1.$$

Unfortunately, the new quadratic form completely destroys the dynamical $SU(2)$ group leaving one with no continuous symmetries whatsoever. So, in a way, life and death cannot be symmetrical in nature as we shall argue further on. Now, we implement a *dynamical aspect* to the theory and that is by insisting that the sex conjugate of a black male is a white female meaning a white female observes states where black people *undergo* processes. They see change whereas black males only see subsequent images. Therefore, a white person should correspond to an anti-Hermitian operator if black people are described by a Hermitian one. It is just a matter of language in a way given a duality between processes and states; it is necessary for some people to see change whereas others see pictures otherwise comprehension of one's actions would be impossible. So, in this vein, C should be given by $\pm iS$, the spiritual belly operator, given that iS preserves the reality condition as well as the Heisenberg algebra but reverses the quadratic form

$$(X, P)^{\dagger} - iZ(X, P)$$

we shall see that such scenario may be troublesome from the dynamical point of view. We will show that marriages with a belly conjugate partner should cause the joint spirit to have an exactly vanishing belly chacra wheres the expectation values of the heart and eye chacra are zero as well in case the couple moves in a thermal bath. However in case the temperature of the heat bath is too large, the effective expected radiated energy by means of the eye and heart will be too large. This was my personal problem with Belgium, that my personality is outside the bandwith of the country which is too narrow in my opinion causing for many hot or cold blooded Switchoriems, on the national benchmark, to poison the atmosphere (hence the poisonous food). In a way, that system thrives upon the old Greek dualities of black and white and level zero and tries to reconsile both by means of suitable marriages at level one which weaken the heart spiritually and likewise so for the eye. Such societies experience jaleousy and behave completely irrational when put under too much pressure. On the other hand, if the pressure or heat is small enough, but not too small, it is joyeous being there.

Humanity does not always follow level-one nature however and wishes to proceed to a higher level by means of neutral conduct at level zero and one as much as possible and henceforth enjoy different marital *contracts* which are of the Belgian nature but then at a higher plane: the highest one at level one is given by the *old* Frech contract where couples switch all chacra's off but this leads to sexual betrayal, voyeurism, cold blooded killing, hypocrecy and jaleousy at higher levels and briefly put, the people as a whole are not ready for it except for some resurrected Egyptian pharaos. Amongt the commoners, the contract enforcing the highest level of *rationality*, which is currently within reach, is given by the shutting down the spiritual heart (Roman system) given that the heart is the source of life, disabling the belly chacra (Belgian system) is in that respect the lowest lowest whereas the hot eye (Polish system) is intermediate, it plays with sensual beauty. In case of a mixture of the belly and heart this gives marital contract MC of the form

$$MC = \begin{pmatrix} 0 & e^{i\theta} \\ e^{-i\theta} & 0 \end{pmatrix}$$

which agree for $\theta = \frac{\pi}{2}$ with $-i(WB)$ whereas $\theta = 0$ with $-iS$, the belly chacra, which expresses a possibly disgusting peace due to too large discrepancies in the spiritual causing jealousy and abnormally high sexuality. One would expect such theta angle to be a dynamical variable varying from culture to culture. Where I come from we have that

$$[MC, WB] = \pm 2iZ$$

which fixes $C = \pm iS$ being the belly operator, the natural contract at level-one as to speak. Another law would be

$$[MC, WB] = \pm 2iS$$

which provides for $C = \mp iZ$ being the eye operator itself, a contract made upon phsycial (level zero) and mental beauty (level two and higher). These contracts are all of a quantal nature, to better understand the details, the reader is referred to [4].

We have already largely discussed a duality in the white-black benchmark related to the different sexes; that is iP_j is the black female operator corresponding to the male impetus P_j. As is well known in quantum theory, this defines a notion of temperature or heat and therefore black woman are hot and energized in the presence of white males. They proclaim chaos whereas the men a democracy which causes democracies governed by a woman to be kind of chaotic in a totalitarian way. On the other hand, the white female benchmark corresponds to iX_j and therefore their temperature is completely arbitrary and variable. These are the hot-warm types who make the black man, who is extremely rigid, completely nuts. When those woman aspire order, they become completely uncertain in gestures, changing between a blush and a totally face, resulting in a lack of comprehension from the male side. The British Queen and PM are white as hell in this sense and play silly games all the time when they want order and law whereas white men just kill to get it. Black woman can easily be distinguished facially from those white ones in the presence of white men as the latter cocky playgirls possess the art of confounding chaos with devilish "unease" to progress in a definite direction. In this vein, there is a clear psychological and astral confusion between males and females: white males and females are democratic - rigid in a certain - uneasy way and they fortify one and another leading to violent discussions and sex. Henceforth, some gap in strength and polarization between males and females in a marriage is needed in order to feel on one hands fortified psychically whereas on the other, conversational aspects need to remain within boundaries. So, banking, as stands up till now, is a black business at the retail level and becomes more white when doing commercial and private banking, with different benchmarks from the European and British side. But what about poor people needing white banking?

Summary.

Our theory is initially just defined by means of a creation - annihilation pair $a, a^\dagger$ where the involution $\dagger$ indicates creation of life; as said before creation of life in a life person means going over to a higer level and death at this one. Also, the vacuum state $|0\rangle$ belongs to the theory. Geometrically, this is expressed by means of
$$(X, P)^T (WB)(X, P) = -1$$
where everything remains at the complex level. The symmetry group of this geometry is the two dimensional Symplectic group which is equivalent to a complexification of $SU(2)$ which is the same as $GL(2, \mathbb{C})$ or the universal cover of the Lorentz group in four real dimensions. To realize the appropriate complexification we have to make one direction imaginary as the associated symplectic form is of the ultrahyperbolic reality $(- - ++)$ with two time and space directions. Now, our observations break this symmetry and the appropriate reduction therefore constitutes in imposing Hermiticity conditions but complexifying in one direction. More in particular, we have introduced the quadratic forms

$$(X, P)^\dagger S(X, P),\ (X, P)^\dagger (-iZ)(X, P), (X, P)^T (-i(WB))(X, P), (X, P)^\dagger (X, P)$$

and imposed the reality condition

$$(X, P)^c = (iZ)(X, P)^\dagger$$

hereby fixing the z-axis in space assuring that all quadratic forms are complex as well as real in nature and that all adjoints vanish effectively. This condition implies that

$$(X, P)^\dagger S(X, P) = (X, P)^T (-i(WB))(X, P) = -1$$

effectively reducing the entire algebra to $su(2)$. To get the appropriate complexified $su(2)$ we either have to opt for the real or complex conventions

$$(X, P)^T (-iS)(X, P), (X, P)^T (-i(WB))(X, P), (X, P)^T (-iZ)(X, P), (X, P)^T (X, P)$$

which generate the quaternion algebra with the charge operator

$$(X, P)^T(X, P)$$

associated to the identity as Hamiltonian. Something very curious happens here *mathematically* given that the identity operator on the Hilbert space is associated to the heart $-i(WB)$. From the philosophical point, this was meant to be as impetus kills awareness or life kills death. Therefore, the other three generators

$$(X, P)^T(-iS)(X, P),\ (X, P)^T(-iZ)(X, P),\ (X, P)^T(X, P)$$

determine a Lie-algebra which *must* be equal to $su(2)$. There are two curious facts here: one is that the heart is associated to *time* and therefore commutes with all operations in space. The second curiosity is that the Hamiltonian of the harmonic oscillator is associated to the identity operator in two dimensions: hence, this one must be associated to the gravitational z-axis given that the height of an object must remain fixed in a gravitational field. The belly and eye operators merely express rotations around the spatial x, y axes which are quite cumbersome given that they require you to move in the z direction. The rotations around this axis still constitute a $U(1)$ symmetry of nature which explains why we can easily rotate on our feet. Some of these aspects were revealed during a telepatic discussion with Prof Norbert Van den Bergh. Insisting upon the Hermitian character of black, by mere choice, imposes that $P^\dagger = -P$, $X^\dagger = X$. Given that this Hermiticity of blackness was arbitrary, we define a sex dual, interchanging black and white by means of $C_1(X, P) = (iP, iX)$ or $C_2(X, P) = (iP, iX)$ which constitute the only two options. Indeed, $-iS$ switches the reality conditions but preserves both expressions of the Heisenberg algebra. Woman, in the same vein, correspond to imaginary momenta in the male psychic Fourier transformation which reflects itself all the time into human behaviour. Males think that females proceed in an imaginary way whereas females feel the same about males namely that the male reality is opposite their own. The reality condition distinguishes awareness from impetus and the fact that $i(WB)$ respects it but destoys $-iS$. One may opt for *different* choices of sex duality but in this one male impetus is related to female awareness with a little tricky imaginary unit creeping in. In a way, this has always been as such as a switchoriem woman makes a switchoriem man's heart tick by means of the belly conjugate, given that impetus really *is* the male heart. This is why black men are interested in black woman from the point of the belly, because they help them to better comprehend themselves which removes beauty and uglyness as well as individualism and keeps them away from mental starvation. The i really makes a huge distinction as the white male, corresponding to 1, is repulsive for the black man at the psychic level. In a way, a black woman has plenty of characteristics of a white male but she is less liberal in the sense that she chooses definetly for small or large measures in her behavioural aspects. She never balances around the Switchoriem; depending upon the particular state in behavioural space of the black male, this provides for some elasticity if the female chooses for the highest weight in her behavioral traits where her husband resides. If this is not so, then opposition occurs and she can kill his heart. White males, then, make black woman crazy as they are oscilatory, a condition dubbed as cyclothemia by black psychiatrists. Indeed, white males have no preference at all whereas black woman do. White females then do not have a preference either but the are not complex cyclic in the psychic way; they are cyclothemic in the real way which results in an infinite positive amplitude at *everything* they do and they can literally do everything. They are extremists which go with full force whereas the white male is much more modest. Indeed, white woman destroy everything which stands in their way whereas white men are much better negotiators. For this very reason, black men often use white females against white dominant (but not complete white) males, by concquering their minds for another goal with money and hereby causing for a spiritual discrepancy breaking the heart. The pure white male cannot be destroyed in this way and literally slices a white female's throat if she were to oppose him because that testifies of evil behaviour. If you cannot stand the biggest liberitarian, then you have malicious intentions regarding the "Wille Zur Macht". There are two conjugations, the first one being $C_1 = -i(WB)$ whereas the other one is $C_2 = i(WB)$ and WB has been shown to correspond to the astral heart.

Banking psychology.

Regarding Europe, it is psychologically divided into three units: those where the belly is largely dominant, those where the eye is, and those of the heart. Some nations profit from taking into account the two highest weights in the definition of the sex conjugate operator

$$MC = i(aS + b(WB) + cZ)$$

where

$$a^2 + b^2 + c^2 = 1$$

due to unitarity and Hermiticity of M

$$(aS + b(WB) + cZ)^\dagger(aS + b(WB) + cZ) = a^2 + b^2 + c^2 = 1.$$

Indeed, it is logical to presume that a retail and commercial banking policy is almost entirely based upon the details of the marriage contract at level-one and possibly level two which shall be discussed further on. Global markets and private banking are situated at a much higher level. Regarding the silent belly contract, Europe has the following nations : Belgium, Czech Republic, Hungaria, Germany, France, Greece, Ireland and possibly some of the Balcan countries. The hot or silent eye nations are Poland, Holland, Czech republic, Slowakia, Denmark, France, England, Ireland, Germany, Greece and Scotland whereas the silent Heart nations are Germany, Scotland, France, England, Spain, Italy, Switzerland, Austria, Holland and Denmark. The European benchmark is henceforth set by Poland, France and Germany whereas Holland, England, Scotland and Denmark may guide the continent towards a better future. On a world scale, I deem the Czech Republic and Ireland to constitute the benchmark and France as well as Germany to be the European countries which should guide the world towards the future. The Vampire board, being the Royals, should be on the level of a French contract at level-one but an Italian at their highest one whereas the devil comes out of Egypt.

Extension towards higher levels.

The attentive reader should have noticed the urge for an intrinsic definition of the levels and why the spiritual level one comes with two to be in peace whereas the physical level zero is not prone to it (or at least way less prone to it). We have to look for dualities in nature to understand this situation better. The first duality is interior-exterior or observation of the self versus observation of others or impetus versus awareness. The exterior impetus has its origin in a spiritual state and vice versa has a spiritual impetus its origin in elemetary thoughts. There has always to be a spark of something and the spark is a state at a higher level. That is one, we noticed in [3, 4] a second correspondance between the motion of the psychic charges (level one) as well as the physical charges (level zero); this is an impetus correspondance coming from level two or higher and being the result of a higher awareness such as elementary thought processes. We shall not go deeper into this issue here but we have suggested already in [3] that at level two the French values of liberté, egalité or fraternité become important and generate the Lorentz algebra in a twistor representation.

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Chapter 2

Maxwell-Boltzmann and intensive-extensive duality.

The idea behind the Maxwell-Boltzmann distribution in physics is associated to the notion of temperature. In banking, this translates as follows, the male system works with identical accounts with a certain amount of Euro's on it (this is amount is a natural number bounded below by zero) implying the "energy gap" between two sucessive values of the accounts is a constant number given by one (a Hamiltonian with equidistant energy levels). So the position operator X in the previous chapter, for one type of account and for one person (all persona are assumed to have equal strategies in the following), is a discrete one associated to the natural numbers n defined by $X|n\rangle = n|n\rangle$ expressing the amount of money on the account. The impetus P or evolution operator for an isolated country with one type of bank account should be given by the identity operator given the constraint that banking affairs must preserve the total amount of money. Therefore, at this level

$$P = 1,\ [P, X] = 0.$$

This is in contrast to the definition of the previous chapter given that there, the momentum was unconstrained. So, a one account theory is completely dull from this perspective given that it is totally void. In the entire psychological setup from the previous chapter, black and white referred to intrinsic behaviour of a person defined with respect to a global benchmark (which is dynamically determined). In that vein, the correct operation to associate to black and whiteness is withdrawing (or depositing) cash money from (on) an account: a white person should be defined as having an arbitrary amount of money on it. Therefore, the constraint is not applicable at this level, but refers to the situation where multiple accounts (possibly belonging to one person) allow for transactions and no cash withdrawals. To describe that situation, we must go over to quantum field theory where several X_a are possible but where the total moneytary constraint is present and implemented by the so called "free Hamiltonian" or energy

H. One could entertain the idea that the canonical momenta P_b should *naively* satisfy

$$[P_a, X_b] = \delta_{a,b}1, \; P_a^\dagger = -P_a, \; [P_a, H] = 0 = [P_a, P_b] = [X_a, X_b]\,.$$

Here, X_b represents a moneytary value of type b whereas P_b is a generic move associated to the same moneytary value whereas H implements the total amount of money present in a fixed currency. Moves on different types must be independent from one and another and hence commute and the moneytary values must be freely generated as well. Such Lie-algebra is certainly possible if money were a continuous unit given that $X_a|x\rangle = x_a|x\rangle$, $P_b = \partial_{x_b}$ and where H *must* equal the elliptic operator $-\sum_b \partial_b^2$. So, therefore, in the setting of cash withdrawals, our definition of whiteness coincides with the psychological one given in the previous chapter even if the account dynamics is unknown. White males indeed don't care at all about the amount of money in a withdrawal and spend at will whereas black woman usually fluctuate around a certain amount. Black men are greedy as hell and don't spend at all.

Studying the situation of pure bank transcriptions we refer to the setting of free quantum field theory, where the identity of the account holders is put into an internal "particle" index and we have to work with smeared creation and annihilation operators as well as several number operators (where $k \geq 0$ expresses the moneytary value). It is useful to introduce the ladder operators

$$L_{a,k}^\dagger = \int dk\, \psi_{a,r}(k) a_r^\dagger(k)$$

creating an account with the amount of $a \in \mathbb{N}$ Pound/Euro on it for the k'th costumer. Obviously

$$[L_{a,r}, L_{b,s}] = \left[L_{a,r}^\dagger, L_{b,s}^\dagger\right], \; \left[L_{a,r}, L_{b,s}^\dagger\right] = \delta_{a,b}\delta_{r,s}1$$

due to the orthogonality property of the functions

$$\int dk\, \psi_{a,r}(k)\overline{\psi_{b,s}(k)} = \delta_{ab}$$

of the respective accounts which are assumed to be identical for each person. Hence, it is natural to construct the operators

$$H := \sum_{a,r} a L_{a,r}^\dagger L_{a,r}, \; P_a := i \sum_{b,c,r,s} L_{b+a,r}^\dagger L_{b,r} L_{c,s}^\dagger L_{c+a,s}$$

$$X_b := \sum_r L_{b,r}^\dagger L_{b,r}$$

where the latter constitute generalized number operators. These constitute the natural operators in our framework, H gives the amount of money in the total circuit whereas X_a portreys the number of accounts of a certain type and finally P_a implements the idea of transferring an amount a of money between two accounts. In case of multiple accounts attached to one person, the definition of whiteness given above could be extended by not merely allowing for cash withdrawals but also implementing transactions between one person's bank accounts preserving hereby the total amount of money (neglecting transferal costs). This still provides one with a Heisenberg operator of the usual kind and we now proceed towards studying higher notions of white and blackness associated to account interchanges. These are not internal to the person anymore but depend upon fine details of previous interactions with other persons as well as debtholders. Given that transcribing money goes in two steps, one would think that a double commutator is more in place as a single one. Indeed, one notices that the relations

$$[H, P_a] = 0$$

are automatically satisfied due to the algebra whereas

$$[X_a, X_b] = 0$$

for the same reason. Likewise, taking into account that

$$\left[L^{\dagger}_{b+a,r} L_{b,r}, L_{c,s}\right] = -\delta_{a+b,c}\delta_{r,s} L_{b,s}, \; \left[L^{\dagger}_{b+a,r} L_{b,r}, L^{\dagger}_{c,s}\right] = \delta_{b,c}\delta_{r,s} L^{\dagger}_{b+a,s}$$

one deduces, using

$$[AB, C] = A\,[B, C] + [A, C]\,B$$

that

$$[P_a, L_{b,r}] = -i \sum_{c;t} L^{\dagger}_{c+a,t} L_{c,t} L_{b+a,r} - i \sum_{c,t} L^{\dagger}_{c,t} L_{b-a,r} L_{c+a,t} - i L_{b,r}$$

where the second and third term on the right hand side vanish if $b - a < 0$. More interesting, with

$$Z_a = \sum_{e,r} L^{\dagger}_{e+a,r} L_{e,r}$$

the reader computes

$$[Z_a, Z_c] = \sum_{e;r} \left(L^{\dagger}_{c+e+a,r} L_{e,r} - L^{\dagger}_{e+c,r} L_{e-a,r} \right) = Z_{c+a} - Z_{a+c} = 0$$

as well as

$$\left[Z_a, Z^{\dagger}_c\right] = \sum_{e;r} \left(-L_{e-a,r} L^{\dagger}_{e+c,r} + L_{e,r} L^{\dagger}_{e+c+a,r} \right) = \sum_{e=M-a+1}^{M} Z^{\dagger}_{c+a}$$

where M is a cutoff in a space. In case $M = \infty$ we deduce that

$$[P_a, P_b] = -\left[Z_a Z^{\dagger}_a, Z_b Z^{\dagger}_b\right] = 0.$$

Finally, we check that

$$[Z_a, X_c] = \sum_r \left(L^\dagger_{c+a,r} L_{c,r} - L^\dagger_{c,r} L_{c-a,r} \right)$$

where, again, the last term requires c to be larger or equal to a. Taking the adjoint results in

$$\left[Z^\dagger_a, X_c\right] = \sum_r \left(L^\dagger_{c-a,r} L_{c,r} - L^\dagger_{c,r} L_{c+a,r} \right)$$

which suggests to study the following expressions

$$[Z_a, [Z_a, X_c]] = \left[Z_a, \sum_r \left(L^\dagger_{c+a,r} L_{c,r} - L^\dagger_{c,r} L_{c-a,r} \right) \right] =$$

$$\sum_r \left(L^\dagger_{c+2a,r} L_{c,r} - 2L^\dagger_{c+a,r} L_{c-a,r} + L^\dagger_{c,r} L_{c-2a,r} \right)$$

whereas

$$\left[Z^\dagger_a, [Z_a, X_c]\right] = X_c - X_{c+a} + (X_c - X_{c-a})\theta(c-a).$$

Also,

$$i\left[Z^\dagger_a Z_a, X_c\right] = iZ^\dagger_a \sum_r \left(L^\dagger_{c+a,r} L_{c,r} - L^\dagger_{c,r} L_{c-a,r} \right) + i \sum_r \left(L^\dagger_{c-a,r} L_{c,r} - L^\dagger_{c,r} L_{c+a,r} \right) Z_a$$

which can be further reduced, means of normal ordering, to

$$i \sum_r \left(L^\dagger_{c,r} L_{c,r} - L^\dagger_{c-a,r} L_{c-a,r} + L^\dagger_{c-a,r} L_{c-a,r} - L^\dagger_{c,r} L_{c,r} \right) +$$

$$i \sum_r \left(L^\dagger_{c+a,r} Z^\dagger_a L_{c,r} - L^\dagger_{c,r} Z^\dagger_a L_{c-a,r} + L^\dagger_{c-a,r} Z_a L_{c,r} - L^\dagger_{c,r} Z_a L_{c+a,r} \right).$$

The first term vanishes so that we arrive at

$$[P_a, X_c] = i \sum_r \left(L^\dagger_{c+a,r} Z^\dagger_a L_{c,r} - L^\dagger_{c,r} Z^\dagger_a L_{c-a,r} + L^\dagger_{c-a,r} Z_a L_{c,r} - L^\dagger_{c,r} Z_a L_{c+a,r} \right).$$

We have that $[Z_0, X_c] = 0$ which follows from $Z_0 = \sum_b X_b$ corresponding to the total number of accounts. For $a = c$ something special happens, we have that

$$[Z_a, X_a] = \sum_r \left(L^\dagger_{2a,r} L_{a,r} - L^\dagger_{a,r} L_{0,r} \right).$$

It means we are on the treshhold were the second term vanishes which happens for $a > c$. Some very curious relationships are given

$$[[Z_a, X_c], X_c] = \sum_r \left(L^\dagger_{c+a,r} L_{c,r} + L^\dagger_{c,r} L_{c-a,r} \right)$$

which is nothing but a sign flip in the second term on the right. Immediate consequences are

$$[[[Z_a, X_c], X_c], X_c] = [Z_a, X_c]$$

as well as

$$[[Z_a, X_c], X_c] + [Z_a, X_c] = 2 \sum_r L^\dagger_{c+a,r} L_{c,r}.$$

Summing this last formula over all c, we obtain that

$$\sum_c [[Z_a, X_c], X_c] + [Z_a, X_c] = 2Z_a.$$

Likewise,

$$\sum_c [[Z_a, X_c], X_c] - [Z_a, X_c] = \sum_{c \geq a} [[Z_a, X_c], X_c] - [Z_a, X_c] = 2Z_a$$

which is a generalized Heisenberg type of relationship. Even more specific, denote by $K = \sum_a Z_a$ the funding operator by an arbitrary amount of money, then we arrive at

$$\sum_c [[K, X_c], X_c] + [K, X_c] = 2K$$

as well as

$$\sum_c [[K, X_c], X_c] - [K, X_c] = 2K$$

which means that the linear operator

$$L := B \to \sum_c [[B, X_c], X_c]$$

has all relevant operators as eigenvectors; indeed, the X_a reside in the kernel whereas the $Z_a, Z^\dagger_a$ have an eigenvalue given by two. It is therefore a Casimir operator on momentum space, but it is not a derivation and therefore does not satisfy the Leibniz rule. We have also the remarkable identity that

$$0 = \sum_n \left[Z_a, X_{(na+b)}\right]$$

where $0 \leq b < a$; under the same conditions

$$[Z_a, [Z_b, X_c]] = \sum_r \left(L^\dagger_{b+c+a,r} L_c - L^\dagger_{b+c,r} L_{c-a} - L^\dagger_{c+a,r} L_{c-b} + L^\dagger_{c,r} L_{c-a-b} \right)$$

implying that

$$[Z_a, [K, X_c]] = 0.$$

On the other hand, using previous results,

$$\left[Z^\dagger_a Z_a, [K, X_c]\right] = \left[Z^\dagger_a, \sum_{b>c,r} L^\dagger_{b,r} L_{c,r} - \sum_{b<c,r} L^\dagger_{c,r} L_{b,r} \right] Z_a$$

whih can be further calculated as

$$\left(\sum_{b>c,r}(L^\dagger_{b-a,r}L_{c,r}-L^\dagger_{b,r}L_{c+a,r})-\sum_{b<c,r}(L^\dagger_{c-a,r}L_{b,r}-L^\dagger_{c,r}L_{b+a,r})\right)Z_a$$

further giving

$$\left(\sum_{b>c-a;r}L^\dagger_{b,r}(L_{c,r}-L_{c+a,r})-\sum_{a\leq b<c,r}(L^\dagger_{c-a,r}-L^\dagger_{c,r})L_{b,r}+\sum_{c\leq b<c+a}L^\dagger_{c,r}L_{b,r})\right)Z_a.$$

Thi

As is well known, when a moneytary cutoff is introduced, the Heisenberg relation

$$[P_a,X_b]=\delta_{a,b}1$$

cannot be properly implemented and has to be approximated as well as possible. More in particular, denote by $|n_{i_1},\dots n_{i_k}\rangle$ a so called *pure* state of k accounts associated to persons $i_1,\dots,i_k$ with amounts of money n_{i_r} on each account; then, the creation operator of one moneytary unit on account j is given by

$$a^\dagger_j|n_{i_1},\dots,n_{i_k}\rangle=\sum_{r=1}^{k}\delta_{j,i_r}\sqrt{n_j+1}|n_{i_1},\dots,n_{i_{r-1}},n_{i_r}+1,n_{i_{r+1}},\dots,n_{i_k}\rangle$$

whereas it adjoint, provided all such states are orthonormal with for different n's and i_j as well as k, is given by

$$a^\dagger_j|n_{i_1},\dots,n_{i_k}\rangle=\sum_{r=1}^{k}\delta_{j,i_r}\sqrt{n_j}|n_{i_1},\dots,n_{i_{r-1}},n_{i_r}-1,n_{i_{r+1}},\dots,n_{i_k}\rangle.$$

This results in the most useful algebra

$$\left[a_j,a^\dagger_k\right]=\delta_{j,k}1$$

supplemented with the condition that no negative n_j can occur. For such a system, the total amount of money, or free energy is given by

$$H=\sum_{r=0}^{\infty}a^\dagger_r a_r.$$

It remains to determine the so-called commutant of H consisting of operators P such that

$$[P,H]=0$$

as well as monetary tastes X_a. It is clear that any moneytary taste X_a should be diagonal in the basis of pure states and that a momentum P_b should map pure states into sums of pure states with the same number of accounts as well as total moneytary value. The general Bosonic observables are

$$X_a|n_{i_1},\ldots,n_{i_k}\rangle = \left(\sum_{r=1}^{k}\delta_{n_{i_r},a}\right)|n_{i_1},\ldots,n_{i_k}\rangle$$

for each natural number $a \in \mathbb{N}$. Those represent the number of accounts with monetary value a on them and does not distinguish between different accounts; they generate all allowed observables algebraically and all commute with one and another

$$[X_a, X_b] = 0.$$

Natural conjugates are generated by operators Z_b which increase the number of accounts with monetary value b by one while changing one of the other *existing* ones democratically by the opposite amount of money. Indeed, for $k \geq 2$

$$Z_a|n_{i_1},\ldots,n_{i_k}\rangle = \sqrt{\frac{1}{k(k-1)}}\sum_{r\neq s:n_{i_r}+n_{i_s}\geq a;n_{i_s}\neq a}|n_{i_1},\ldots,(n_{i_r}+n_{i_s}-a)_{i_r},\ldots,a_{i_s},\ldots\rangle$$

is the obvious candidate operator satifying

$$[Z_e, X_d] \sim \delta_{e,d}L$$

where L is only nontrivial upper triangular in a slight way. This is so on the *subspace* spanned by the pure states for which there exists at least one a, b such that X_d, X_a, X_b are different from zero and $a + b \geq d$. Moreover,

$$[Z_a, Z_b]$$

is only different from zero in a slight way. We are interested in subspaces of states ψ, ϕ of unit norm such that

$$\langle\psi|\,[Z_e, X_d]\,|\phi\rangle \sim \delta_{e,d}\langle\psi|\phi\rangle,\; 0 = \langle\psi|\,[Z_e, Z_d]\,|\phi\rangle.$$

Hence, we may conclude that

$$[X_e, Z_d] \sim \delta_{e,d}1, [Z_e, Z_d] \sim 0$$

on that subspace. The Z_d are however not anti-Hermitian as those should also allow for a decrease in the a number by one; henceforth, we are left with

$$P_a := \frac{1}{2}(Z_a - Z_a^{\dagger}).$$

The space of complex valued functions of interest is given by the Hilbert space spanned by the pure states; the *black* states are by definition the pure ones endowed with a metric of the kind

$$\langle\psi|\phi\rangle_f := \langle\psi|\sum_{a=0}^{\infty}c_a(X_a - b_a)^2|\phi\rangle$$

where $b_a, c_a > 0$ and $c_0 > 0$, $X_0 = 1$. The b_a represent a kind of banking standard over a statistical ensemble of many units; the latter is defined by means of a density matrix diagonal in pure states $|v_i\rangle$. That is,

$$\rho = \sum_i p_i |v_i\rangle\langle v_i|$$

such that $\sum_i p_i = 1$ and $p_i > 0$. More in particular, by definition

$$\mathrm{Tr}(\rho(X_a - b_a)) = \sum_i p_i \langle v_i|(X_a - b_a)v_i\rangle = 0$$

for any $a > 0$. The squares are logical given that a vector ψ must transform for evaluation by means of X_a, the separate squares are logical given that products $X_a X_b$ for $a \neq b$ vanish on the pure states. All coefficients must be positive to garantuee positivity of the evaluation. This scalar product gives rise to a second involution $\star$ and whereas it is obviously so that $X_a^\star = X_a$, the impetus operators should be redefined by $P_a = \frac{1}{2}(Z_a - Z_a^\star)$ whose eigenvalues are purely imaginary. White states then correspond to the eigenvectors of all of P_a on a subspace, spanned by black states, on which all commute with one and another (only a finite one will be nonzero given that a finite amount of money is only available and generically three X_e will be nonzero too and the sum rule $e_i + e_j > e_k$ usually holds too given that money is well distributed). The total dynamics of the system C must, so far, be build exclusively from the operators H, P_a, X_b which all preserve the number of bank accounts so that dimensional reduction in that regard remains possible. It is possible, of course to introduce creation and annihilation operators of bank accounts but we leave such extensions open for the interested reader. Moreover, in practice, only a finite number of black states exists given that there is a limited total amount of money circulating in a country. In contrast to the usual black-white setting, the pure states are finite and not distributional given that going from one account to another is defined by means of a finite incremental step and not an infinitesimally small one. Ideally, the operator C must be Hermitian in nature which we express this time as $C^\star = C$. This gives rise to the primitive evolution operator

$$U := e^{i\alpha C}$$

where α denotes a positive real number and mimics a primitive time step. This is not everything there is to the story as t Suppose now, we have M countries which are assumed to function independently according to the same principles but with possibly different C_μ. Suppose now that we are only interested in statistical distributions which solely depend upon the dynamics C_μ. Then it is logical to assume that the joint distribution should be equal to the product distribution governed by the Hamiltonian

$$C := \sum_{\mu=1}^{M} 1_1 \otimes \ldots 1_{\mu-1} \otimes C_\mu \otimes 1_{\mu+1} \otimes \ldots \otimes 1_M.$$

That is,

$$\rho(C) = \otimes_{\mu=1}^{M} \rho(C_\mu)$$

and the reader may easily verify that this implies that

$$\rho(C) = \frac{e^{-\beta C}}{\mathrm{Tr}(e^{-\beta C})}.$$

Here,

$$\beta = \frac{1}{k_B T}$$

where k_B is, in physics, the Boltzmann constant and T the usually positive temperature in Kelvin.

Given that in general a white eigenstate is approximately given by the function

$$|k^a\rangle : v \to \langle v|e^{ik^a(X_a - b_a)}|v\rangle_f = \mathrm{Tr}(e^{ik^a(X_a - b_a)}|v\rangle\langle v|^\star)$$

due to the Heisenberg algebra and $\langle v|P_b|v\rangle = 0$ for a pure state $|v\rangle$, one could argue that a white state is associated to a Hermitian energy operator $H_k := \sum_a k^a(X_a - b_a)$ which may be negative as well as positive. This suggests a new definition of blackness by means of the functions

$$|k^a\rangle^c : v \to \frac{\mathrm{Tr}(e^{-H_k}|v\rangle\langle v|^\star)}{\mathrm{Tr}(e^{-H_k})}$$

associated to the density matrices

$$\frac{e^{-H_k}}{\mathrm{Tr}(e^{-H_k})}$$

where k is variable, restrained by periodic boundary conditions as well as a monetary cutoff. Indeed, the Maxwell Boltzmann correspondance suggests a smearing of the delta functions by means of finite nonzero thermodynamic parameters instead of infinite ones. A new notion of whiteness may be developed from here by means of a Fourier transform over all k which amounts to integration over all possible kinds of thermodynamic variables. Indeed, the multivector k^a constitute the intensive parameters whereas the X_a the extensive ones; actually, they constitute distinct, possibly negative, temperatures associated to distinct "time directions" set by the X_a. The global temperature T is given by $k_B T = \frac{1}{\alpha}$ but each type of account may have its own popularity or hotness.

Chapter 3

Different kinds of Black and Whiteness.

Given that we have different country benchmarks of banking; how to reconsile those on a more international level? More in particular, in Europe one has the black male Italian, white dominant male Swiss and black female dominant English system at least on the retail level. At higher levels, with commercial banking being the most progressive one, exchange contracts become more risky and white meaning flexible and versatile. The question, to which we shall try to provide a universal answer in this chapter, is how to define mixed notions of blackness allowing one to rotate into the other. More specifically, assume two systems with black axis X_a, Y_α where $a : 1 \ldots N$ and $\alpha : 1 \ldots M$ representing the core metrics or values of the respective systems, some of which extensive whereas others intensive. A complete extensive system is defined as pure male whereas one with only intensive variables is pure female; mixed systems can be achieved by means of the respective Helmhotz tranformations. Therefore, it is useful to use the notation $(X_a)_{a=1}^{M} = (X_{a,I}, X_{a,E})$ and likewise for Y_α. Extensive variables are those, when two independent systems merge, who add up precisely such as the total amount of money in the country, the current number of accounts of each type or in general the X_a operators but *not* the P_a operators because cross terms between distinct accounts should be taken into account. This means that the definition of male blackness is additive but not of male whiteness

Chapter 4

Mixed black systems and complex banking.

Chapter 5

Afterword.

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