The Not-So Anomalous Magnetic Moment

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Abstract: This paper is a very short didactic exploration of the geometry of the experiments measuring the anomalous magnetic moment. It is argued that there is nothing anomalous about it. The Larmor precession invalidates the usual substitution that is made for the gyromagnetic ratio of the precessional motion. In fact, if the substitution is made, one gets a value of −1/2 instead of zero. We should, therefore, not wonder why the anomalous magnetic moment is not equal to zero, but why it is so *nearly* zero. We suggest the geometry of the situation – and the related classical calculations – explains all, except, of course, Schwinger's α/2π factor and the other quantum-mechanical corrections. However, we argue these might be explained by the *Zitterbewegung* model of an electron. That model is associated with a *form factor*: a disk-like structure, which relates the Bohr and the Compton radius through the fine-structure constant. We therefore argue that it would be worthwhile to re-attempt to explain the anomalous magnetic moment in terms of a classical explanation.

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The Not-So-Anomalous Magnetic Moment

Introduction

What is referred to as the electron's anomalous magnetic moment is actually *not* a magnetic moment. It is just some (real) number: it does not have a physical dimension – as opposed to, say, an actual magnetic moment, which – in the context of quantum mechanics – is measured in terms of the Bohr magneton $\mu_B = q_e\hbar/2m \approx 9.274\times10^{-24}$ *joule per tesla*.[1] To be precise, the electron's anomalous magnetic moment – denoted by a_e – is usually *defined* as the (half-)difference between (1) a supposedly *real* gyromagnetic ratio (g_e) and (2) Dirac's theoretical value for the gyromagnetic ratio of a spin-only electron (g = 2)[2]:

$$a_e = \frac{g_e - g}{2} = \frac{g_e - 2}{2} = \frac{g_e}{2} - 1$$

It is weird to use the g-factor for a spin-only electron, because the electron in the cyclotron (a Penning trap) is actually *not* a spin-only electron: it follows an orbital motion – as we will explain shortly. It is also routinely said (and written) that its *measured* value is 0.00115965218085(76). The 76 (between brackets) is the (un)certainty: it is equal to 0.00000000000076, i.e. 76 *parts per trillion* (ppt) and it is measured as a standard deviation.[3] However, the experiments do *not* directly measure a_e. What is actually being measured in the *Penning traps* that are used in these experiments are two slightly different frequencies – an orbital frequency and a precession frequency, to be precise – and a_e is then *calculated* as the fractional difference between the two:

$$a_e = \frac{f_p - f_c}{f_c}$$

Let us go through the motions here – literally. The orbital frequency f_c is the cyclotron frequency: a charged particle in a Penning trap will move in a circular orbit whose frequency depends on the charge, its mass and the strength of the magnetic field *only*. We write:

$$f_c = \frac{1}{2\pi} \cdot \frac{\mathrm{q}}{\mathrm{m}} \cdot \mathrm{B}$$

The subscript *c* stands for *c*yclotron – or *c*ircular, if you want. We should not think of the speed of light here! In fact, the orbital velocity is a (relatively small) fraction of the speed of light and we can, therefore, use non-relativistic formulas. The derivation of the formula is quite straightforward – but we

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[1] Needless to say, the tesla is the SI unit for the magnitude of a magnetic field. We can also write it as [**B**] = N/(m·A), using the SI unit for current, i.e. the ampere (A). Now, 1 C = 1 A·s and, hence, 1 N/(m·A) = 1 (N/C)/(m/s). Hence, the physical dimension of the magnetic field is the physical dimension of the electric field (N/C) divided by m/s. We like the [**E**] = [**B**]·m/s expression because it reflects the *geometry* of the electric and magnetic field vectors.

[2] See: Physics Today, 1 August 2006, p. 15 (https://physicstoday.scitation.org/doi/10.1063/1.2349714). The article also explains the methodology of the experiment in terms of the frequency measurements, which we explain above.

[3] See: G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, New Determination of the Fine Structure Constant from the Electron g Value and QED, Phys. Rev. Lett. 97, 030802 (2006). More recent theory and experiments may have come up with an even more precise number.

find it useful to recap it. It is based on a simple analysis of the Lorentz force, which is just the magnetic force here: $\mathbf{F} = q(\boldsymbol{v} \times \mathbf{B})$.[4] Note that the frequency does *not* depend on the velocity or the radius of the circular motion. This is actually the whole idea of the trap: the electron can be inserted into the trap with a precise kinetic energy and will follow a circular trajectory if the frequency of the alternating voltage is kept constant. This is why we italicized *only* when writing that the orbital frequency depends on the charge, the mass and the strength of the magnetic field *only*. So what is the derivation? The Lorentz force is equal to the centripetal force here. We can therefore write:

$$\mathrm{q} \cdot v \cdot \mathrm{B} = \frac{mv^2}{r}$$

The v^2/r factor is the centripetal acceleration. Hence, the $\mathrm{F} = \mathrm{m} \cdot v^2/r$ does effectively represent Newton's force law. The equation above yields the following formula for v and the v/r ratio:

$$v = \frac{\mathrm{q} \cdot r \cdot \mathrm{B}}{\mathrm{m}} \Rightarrow \frac{v}{r} = \frac{\mathrm{q} \cdot \mathrm{B}}{\mathrm{m}}$$

Now, the cyclotron frequency f_c will respect the following equation:

$$v = \omega \cdot r = 2\pi \cdot f_c \cdot r$$

Re-arranging and substituting v for $\mathrm{q} \cdot r \cdot \mathrm{b/m}$ yields:

$$f_c = \frac{v}{2\pi \cdot r} = \frac{\mathrm{q} \cdot \mathrm{B}}{2\pi \cdot \mathrm{m}}$$

The associated *current* will be equal to:

$$\mathrm{I} = \mathrm{q} \cdot f = \mathrm{q}\frac{v}{2\pi \cdot r} = \frac{\mathrm{q}^2 \cdot \mathrm{B}}{2\pi \cdot \mathrm{m}}$$

Hence, the magnetic moment is equal to:

$$\mu = \mathrm{I} \cdot \pi \cdot r^2 = \mathrm{q}\frac{v}{2\pi \cdot r} \cdot \pi \cdot r^2 = \frac{\mathrm{q} \cdot v \cdot r}{2}$$

The angular momentum – which we will denote by – is equal to[5]:

$$\mathrm{J} = I \cdot \omega = \mathrm{m}r^2 \cdot \frac{v}{r} = \mathrm{m} \cdot r \cdot v$$

Hence, we can write the g-factor as:

$$g_c = \frac{2\mathrm{m}}{\mathrm{q}}\frac{\mu}{\mathrm{J}} = \frac{2\mathrm{m}}{\mathrm{q}} \cdot \frac{\mathrm{q} \cdot v \cdot r}{2\mathrm{m} \cdot r \cdot v} = 1$$

It is what we would expect it to be: it is the gyromagnetic ratio for the *orbital* moment of the electron. It is *one*, not 2. Because g_c is 1, we can write something very obvious:

[4] Our derivation is based on the following reference: https://www.didaktik.physik.uni-muenchen.de/elektronenbahnen/en/b-feld/anwendung/zyklotron2.php.

[5] J is the symbol which Feynman uses. In many articles and textbooks, one will read L instead of J. Note that the symbols may be confusing: I is a current, but *I* is the moment of inertia. It is equal to $\mathrm{m} \cdot r^2$ for a rotating mass.

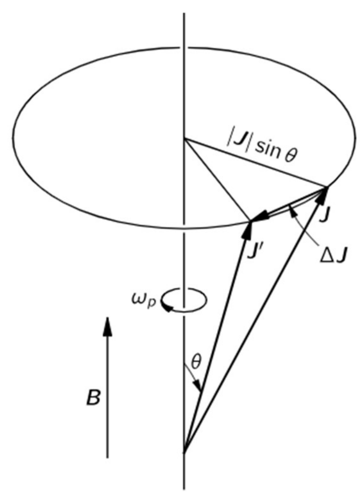
$$f_c = g_c \frac{\mathrm{q} \cdot \mathrm{B}}{2\pi \cdot \mathrm{m}}$$

We should also note another equality here:

$$\frac{2\mathrm{m}}{\mathrm{q}} \frac{\mu}{\mathrm{J}} = 1 \Leftrightarrow \frac{\mu}{\mathrm{J}} = \frac{\mathrm{q}}{2\mathrm{m}}$$

Let us now look at the other frequency f_s. It is the *Larmor* or precession frequency. It is (also) a classical thing: if we think of the electron as a tiny magnet with a magnetic moment that is proportional to its angular momentum, then it should, effectively, *precess* in a magnetic field.

The analysis of precession is quite straightforward. The geometry of the situation is shown below and we may refer to (almost) any standard physics textbook for the derivation.[6]



$$\omega_p = \frac{\mu}{\mathrm{J}} \cdot \mathrm{B} \Leftrightarrow \frac{\mu}{\mathrm{J}} = \frac{\omega_p}{\mathrm{B}}$$

It is tempting to use the equality above and write this as:

$$\omega_p = \frac{\mathrm{q}}{2\mathrm{m}} \cdot \mathrm{B}$$

However, we should *not* do this. The precession causes the electron to wobble: its plane of rotation – and, hence, the axis of the angular momentum (and the magnetic moment) – is no longer fixed. This wobbling motion changes the orbital and, therefore, we can no longer trust the values we have used in our formulas for the angular momentum and the magnetic moment. There is, therefore, nothing anomalous about the anomalous magnetic moment. In fact, we should not wonder why it is not zero, but – as we will argue – we should wonder why it is so *nearly* zero.

Let us continue our analysis. It is, in fact, a bit weird to associate a gyromagnetic ratio with this motion, but that is what the physicists doing these experiments do. We will denote this g-factor by g_p:

[6] We like the intuitive – but precise – explanation in Feynman's *Lectures* (II-34-3), from which we also copied the illustration.

$$g_p = \frac{2\mathrm{m}}{\mathrm{q}}\frac{\mu}{\mathrm{J}} = \frac{2\mathrm{m}}{\mathrm{q}} \cdot \frac{\omega_p}{\mathrm{B}}$$

Hence, we can write the following *tautology*:

$$\omega_p = g_p \frac{\mathrm{q}}{2\mathrm{m}} \cdot \mathrm{B} = \frac{2\mathrm{m}}{\mathrm{q}} \cdot \frac{\omega_p}{\mathrm{B}} \cdot \frac{\mathrm{q}}{2\mathrm{m}} \cdot \mathrm{B} = \frac{\mu}{\mathrm{J}} \cdot \mathrm{B}$$

You can verify that this is nothing but a tautology by writing it all out:

$$\omega_p = g_p \frac{\mathrm{q}}{2\mathrm{m}} \cdot \mathrm{B} = \frac{2\mathrm{m}}{\mathrm{q}} \cdot \frac{\omega_p}{\mathrm{B}} \cdot \frac{\mathrm{q}}{2\mathrm{m}} \cdot \mathrm{B} = \omega_p$$

We can, of course, measure the frequency in cycles per second (as opposed to radians per second):

$$f_p = \frac{\omega_p}{2\pi} = g_p \frac{\mathrm{q} \cdot \mathrm{B}}{2\pi \cdot 2\mathrm{m}}$$

Hence, we get the following expression for the so-called anomalous magnetic moment of an electron a_e:

$$a_e = \frac{f_p - f_c}{f_c} = \frac{g_p \frac{\mathrm{q} \cdot \mathrm{B}}{2\pi \cdot 2\mathrm{m}} - g_c \frac{\mathrm{q} \cdot \mathrm{B}}{2\pi \cdot \mathrm{m}}}{g_c \frac{\mathrm{q} \cdot \mathrm{B}}{2\pi \cdot \mathrm{m}}} = \frac{1}{2}\frac{g_p}{g_c} - 1$$

Hence, the so-called anomalous magnetic moment of an electron is nothing but the ratio of two mathematical factors – definitions, basically – which we can express in terms of *actual* frequencies:

$$g_c = f_c \frac{2\pi \cdot \mathrm{m}}{\mathrm{q} \cdot \mathrm{B}}$$

$$g_p = f_p \frac{2\pi \cdot 2\mathrm{m}}{\mathrm{q} \cdot \mathrm{B}}$$

Our formula for a_e now becomes:

$$a_e = \frac{1}{2}\frac{g_p}{g_c} - 1 = \frac{1}{2}\frac{f_p \frac{2\pi \cdot 2\mathrm{m}}{\mathrm{q} \cdot \mathrm{B}}}{f_c \frac{2\pi \cdot \mathrm{m}}{\mathrm{q} \cdot \mathrm{B}}} - 1 = \frac{f_p}{f_c} - 1 = \frac{f_p - f_c}{f_c}$$

Of course, if we use the $\mu/\mathrm{J} = 2\mathrm{m}/\mathrm{q}$ equality, then the f_p/f_c ratio will be equal to ½, and a_e will not be zero but –1/2.

$$\frac{f_p}{f_c} - 1 = \frac{\frac{\mathrm{q} \cdot \mathrm{B}}{2\pi \cdot 2\mathrm{m}}}{\frac{\mathrm{q} \cdot \mathrm{B}}{2\pi \cdot \mathrm{m}}} - 1 = \frac{1}{2} - 1 = -1/2$$

However, as mentioned above, we should not do that. The precession causes the magnetic moment and the angular momentum to wobble. Hence, we should ask ourselves: what is anomalous about the anomalous magnetic moment, *really*? Let us try a classical explanation.

What needs to be explained, *exactly*?

Quantum physicists explain the anomaly in the magnetic moment is expressed as a series of first-, second-, third-, n^{th}-order *corrections*, which are written as follows:

$$a_e = \sum_n a_n \left(\frac{\alpha}{\pi}\right)^n$$

The first coefficient (a_1) is equal to 1/2 and the associated first-order correction is, therefore, equal to:

$$\alpha/2\pi \approx 0.00116141$$

Using "his renormalized QED theory", Julian Schwinger had already obtained this value back in 1947. He got it from calculating the "one loop electron vertex function in an external magnetic field." I am just quoting here from a much better-informed article than mine (Todorov, 2018[7]). Indeed, Todorov's article is an article that beautifully describes the *math* behind this "tennis match between experiment and theory" – as Brian Hayes referred to it.[8] We will come back to Todorov's insights in a moment.

Let me first note that, yes, I recognize Julian Schwinger as one of the most prominent representatives of the second generation of quantum physicists, and he has this number on this tombstone. Hence, we surely do not want to question the depth of his understanding of this phenomenon. However, the difference that needs to be explained by the 2nd, 3rd, etc. corrections is only 0.15%, and Todorov's work shows all of these corrections can be written in terms of a sort of exponential series of α/2π and a *phi*-function ϕ(*n*) which had intrigued Euler for all of his life. We copy the formula for the (the sum of) the first-, second- and third-order term of the theoretical value of a_e as calculated in 1995-1996 (*th* : 1996).[9]

$$\begin{aligned} a_e(th:1996) &= \frac{1}{2}\frac{\alpha}{\pi} + \left[\phi(3) - 6\,\phi(1)\,\phi(2) + \phi(2) + \frac{197}{2^4\,3^2}\right]\left(\frac{\alpha}{\pi}\right)^2 \\ &+ \left[\frac{2}{3^2}\left(83\,\phi(2)\,\phi(3) - 43\,\phi(5)\right) - \frac{50}{3}\,\phi(1,3) + \frac{13}{5}\,\phi(2)^2\right. \qquad (19) \\ &+ \left.\frac{278}{3}\left(\frac{\phi(3)}{3^2} - 12\,\phi(1)\,\phi(2)\right) + \frac{34202}{3^3\,5}\,\phi(2) + \frac{28259}{2^5\,3^4}\right]\left(\frac{\alpha}{\pi}\right)^3 + \ldots \\ &= 1.159652201(27)\times 10^{-3} \end{aligned}$$

We also quote Todorov's succinct summary of how this result was obtained: "Toichiro Kinoshita of Cornell University evaluated the 72 [third-order loop Feynman] diagrams numerically, comparing and combining his results with analytic values that were then known for 67 of the diagrams. A year later, the

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[7] See: Ivan Todorov, *From Euler's play with infinite series to the anomalous magnetic moment*, 12 October 2018 (https://arxiv.org/pdf/1804.09553.pdf).

[8] See: Brian Hayes, *Computing Science: g-ology*, in: *American Scientist*, Vol. 92, No. 3, May-June 2004, pages 212-216. The subtitle says it all: it is an article 'on the long campaign to refine measurements and theoretical calculations of a physical constant called the *g* factor of the electron.' (https://pdfs.semanticscholar.org/4c12/50f66fc1fb799610d58f25b9c1e1c2d9854c.pdf).

[9] It is worth quoting Todorov's succinct summary of how this result was obtained: Toichiro Kinoshita of Cornell University evaluated the 72 [Feynman] diagrams [corresponding to the third-order loop] numerically, comparing and combining his results with analytic values that were then known for 67 of the diagrams. later the last few diagrams were calculated analytically by Stefano Laporta and Ettore Remiddi of the University of Bologna.

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The elements for a classical explanation

In light of what we wrote above, it is obvious that our suggestion that there might be some rather simple classical explanation for the anomalous magnetic moment is quite disrespectful. However, that is what we are going to do: we are going to think of the elements that might go into a classical explanation.

In a very first, but extremely native, attempt at it, we might think Ptolemean physics – circular motion within circular motion – might do the trick, as illustrated below.[10]

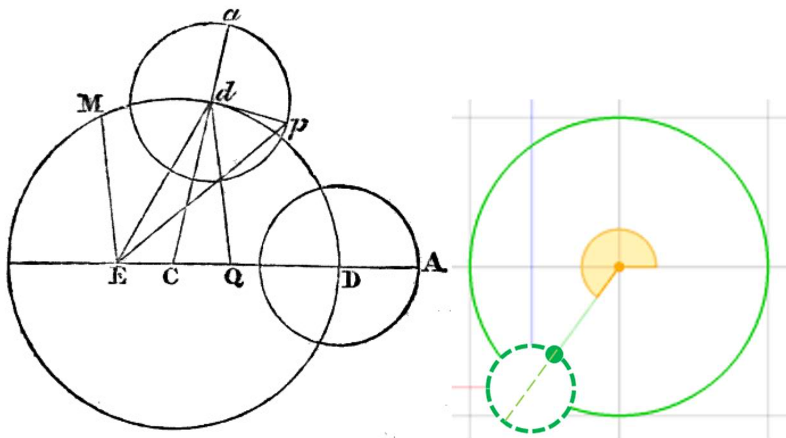

Figure 1: Ptolemean loops

The trajectory of a charged particle in a Penning trap will, effectively, resemble a trajectory we would get from adding two circular motions, as shown below. The large circular motion is referred to as the deferent, while the smaller circular motion as the epicycle. If we denote the (angular) frequency of the deferent and epicyclical motion as ω_- and ω_+ respectively, then the illustrations below show what we get from combining these two motions.

[10] The first illustration is taken from the *Project Gutenberg* e-publications: *The Life of Galileo*, written by John Elliot Drinkwater Bethune (https://www.gutenberg.org/files/43877/43877-h/43877-h.htm). It is supposed to represent the motion of planets in the Ptolemean system. The second is the author's simpler rendition of the pretty much the same thing – but in the context of presumed electron orbitals.

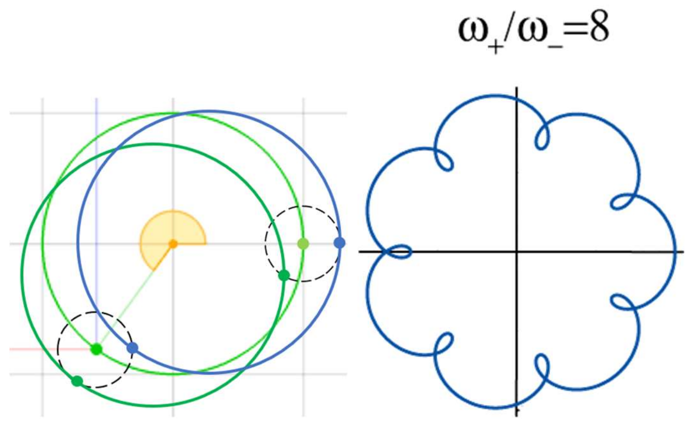

Figure 2: Ptolemean physics?

The combined motion is referred to as an *epitrochoid*, and the illustration on the left shows the motion for $\omega_{+} = 0$. Note that, if we are rotating a disk which can freely rotate itself, its inertia will *not* cause any rotation of the disk itself. Any point on that disk will, therefore, just cover the same distance as any point on the deferent. Hence, the epitrochoid will just describe the same circle as the deferent, but its center will move about. This leads to something interesting. Carefully look at the illustration on the right-hand side: the ratio of the two frequencies is equal to 8, but we do *not* have 8 loops within the larger loop. There are only 7.

The question is: what is the impact on the (electric) current? We ask this question because it is the current that determines the magnetic moment. Think of the two formulas:

$$\mathrm{I} = \frac{\mathrm{q}}{\mathrm{T}} = \mathrm{q} \cdot f = \mathrm{q}\frac{v}{2\pi \cdot r}$$

$$\mu = \mathrm{I} \cdot \pi \cdot r^2 = \mathrm{q}\frac{v}{2\pi \cdot r} \cdot \pi \cdot r^2 = \frac{\mathrm{q} \cdot v \cdot r}{2}$$

We are not very well versed in the math of epitrochoids but, intuitively, it would seem the superposition of the two motions would *not* change anything in regard to the current: the velocity (v) and the distance (r) will constantly change, of course, but the charge that goes round and round is the same and, hence, there will be some *effective* velocity and radius that will give us the same current we get from simple orbital motion.[11] Hence, Ptolemean physics are probably *not* going to help to explain the anomalous frequency.

Hence, we need to move on and think about the precession. If there is precession, then it is going to cause some extra movement *along the direction of the magnetic field* (***B***) (see the visualization of the formula for the precession frequency). Taking a radial view, the motion is going to look like this:

[11] We may also note that charge – unlike mass or energy – is relativistically invariant.

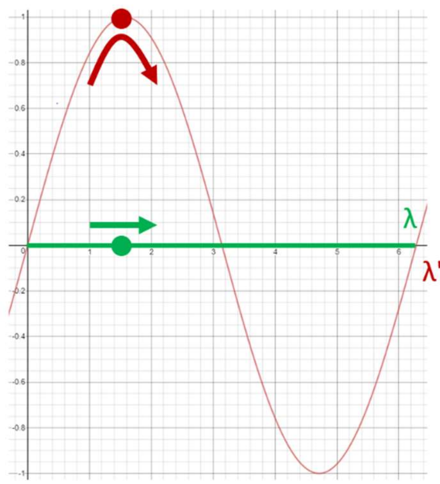

Figure 3: A radial view of the precessional rotation

The analysis is complicated, because we should wonder whether this vertical motion – with vertical, we mean perpendicular to the plane of rotation – will be linear. When taking a view from the side – along the plane of rotation – the up-and-down motion might follow some arc-like trajectory – as illustrated below.

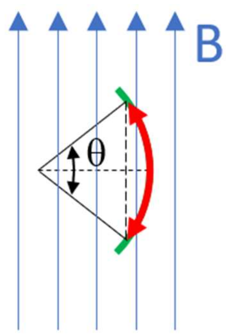

Figure 4: A *sideway* view of the precessional rotation

The angle that is to be associated with this arc is, of course, the angle of the precession: we just turned it by 90 degrees. We have a formula for that angle, of course. We get it from equation the rate of change of the angular momentum with the torque:

$$\frac{d\mathrm{J}}{d\mathrm{t}} = \omega_p \cdot \mathrm{J} \cdot sin(\theta) = \tau \cdot \mu \cdot \mathrm{B} \cdot sin(\theta)$$

In fact, it is this equation that gives us the formula for the precession frequency. Now, we are not quite sure if the up-and-down motion follows some arc-like trajectory because we should remember the set-up of this experiment: we do *not* have a positive charge at the center and there is, therefore, no electrostatic potential that would keep the radius (r) constant. The analysis here becomes quite complicated and we should refer to more advanced literature here, such as a detailed analysis of

Larmor's Theorem, which details radial, tangential, centrifugal and various other components of the motion here.[12]

However, no matter how rich the analysis, these equations will also *not* give you that strange $\alpha/2\pi$ factor. It is an anomalous factor, and it can only be explained by some form factor. In other words, we can (probably) explain it if we would *not* think of the electron as a pointlike charge.

We have detailed that model elsewhere and, hence, we will not go into too much detail here.[13] It is an interpretation of an electron which goes back to Schrödinger and Dirac[14], and which combines the idea of motion with the idea of a pointlike *charge*, which has no inertia and can, therefore, move at the speed of light. The illustration below described the presumed circular oscillatory motion of the charge (the *Zitterbewegung*). The most spectacular result is the explanation for the *rest* mass of an electron: it is the equivalent mass of what we referred to as the *rest matter oscillation*.

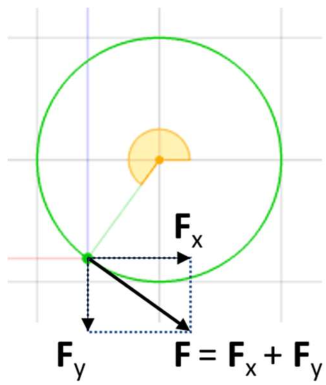

Figure 5: The *Zitterbewegung* model of an electron

The model also gives the right formulas for all the measured properties of a free electron, such as angular momentum, magnetic moment, g-factor, etcetera:

[12] See, for example, Feynman's *Lectures* (II-34-4 and II-34-5).

[13] See, for example, Jean Louis Van Belle, *Einstein's Mass-Energy Equivalence Relation: an Explanation in Terms of the Zitterbewegung*, 24 November 2018 (http://vixra.org/abs/1811.0364).

[14] Erwin Schrödinger derived the *Zitterbewegung* as he was exploring solutions to Dirac's wave equation for free electrons. In 1933, he shared the Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth quoting Dirac's summary of Schrödinger's discovery: "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

Spin-only electron (*Zitterbewegung*)
$\mathrm{S} = \mathrm{h}$
$\mathrm{E} = \mathrm{m}c^2$
$r = r_{\mathrm{C}} = \frac{\hbar}{\mathrm{m}c}$
$v = c$
$\mathrm{L} = I \cdot \omega = \frac{\hbar}{2}$
$\mu = \mathrm{I} \cdot \pi {r_{\mathrm{C}}}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}} \hbar$
$\mathrm{g} = \frac{2\mathrm{m}}{\mathrm{q_e}} \frac{\mu}{\mathrm{L}} = 2$

Table 1: The properties of the free electron (spin-only)

The reader should keep his wits about him[15] here: the *Zitterbewegung* model should not be confused with the model for the Bohr orbitals. We do not have any centripetal force here. There is no nucleus or other charge at the center of the *Zitterbewegung*. Instead of a tangential momentum vector, we have a tangential *force* vector (**F**), which we thought of as being the resultant force of two perpendicular oscillations.[16] This led us to boldly equate the E = mc^2, E = m·a^2·ω^2 and E = $\hbar$·ω equations – which gave us all the results we wanted. The *zbw* model – which, as we have mentioned in the footnote above, is inspired by the solution(s) for Dirac's wave equation for free electrons – tells us the velocity of the pointlike *charge* is equal to *c*. Hence, *if* the *zbw* frequency would be given by Planck's energy-frequency relation ($\omega = \mathrm{E}/\hbar$), *then* we can easily combine Einstein's E = mc^2 formula with the radial velocity formula ($c = a \cdot \omega$) and find the *zbw* radius, which is nothing but the (reduced) Compton wavelength:

$$r_{\mathrm{Compton}} = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_{\mathrm{e}}}{2\pi} \approx 0.386 \times 10^{-1} \quad \mathrm{m}$$

By now, the reader will probably wonder: what is the point here? What is the relation with the anomalous magnetic moment. The point is that the calculations also relate the Bohr radius to the Compton radius through the fine-structure constant:

$$r_{\mathrm{Bohr}} = \frac{\hbar^2}{\mathrm{me}^2} = \frac{4\pi\varepsilon_0\hbar^2}{\mathrm{m}{\mathrm{q_e}}^2} = \frac{1}{\alpha} \cdot r_{\mathrm{Compton}} = \frac{\hbar}{\alpha \mathrm{m}c} \approx 53 \times 10^{-12} \mathrm{~m}$$

The same fine-structure constant also relates the respective velocities, frequencies and energies of the Bohr and Compton oscillations. Indeed, one easily show the following:

[15] The him could be a her, of course.

[16] A metaphor for such oscillation is the idea of two springs in a 90-degree angle working in tandem to drive a crankshaft. The 90-degree ensures the independence of both motions. See: Jean Louis Van Belle, *Einstein's mass-energy equivalence relation: an explanation in terms of the Zitterbewegung*, 24 November 2018 (http://vixra.org/pdf/1811.0364v1.pdf).

$$v = \alpha \cdot c = r_{\mathrm{B}} \cdot \omega_{\mathrm{B}} = \frac{\hbar}{\alpha \mathrm{m}c} \cdot \frac{\alpha^2 \mathrm{m}c^2}{\hbar} = \alpha \cdot c \Leftrightarrow \omega_{\mathrm{B}} = \frac{\alpha^2 \mathrm{m}c^2}{\hbar}$$

The fact that the fine-structure constant pops up naturally here – as a dimensional constant, so to speak – makes us feel that Schwinger's factor – and the successive corrections – might just come out of a more classical approach to the calculations: we just need to acknowledge the form factor of the electron.

However, an intuition is something else than a full-blown proof, of course. We will work on it.

References

All references are given in the footnotes.