Mach's Principle: the origin of the inertial mass (I)

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Abstract. We show that the forces of inertia acting on the accelerated bodies are forces of gravitational induction exerted by the whole of the Universe. Therefore, the phenomenon of inertia and the inertial mass of a body have a cosmic origin, as demanded by the Mach's principle. The calculations will be applied to a vector gravitational field theory. In a second part of this research we will apply these results to the General Theory of Relativity [1].

1 Introduction

We understand Mach's principle as the affirmation that the property of inertia, the inertial mass and the force of inertia acting on a body are the result of the gravitational action of the whole Universe [2].

The force of inertia defined by $-m_i\mathbf{a}$ is a real force that acts on all accelerated bodies [3], being m_i the inertial mass and **a** the acceleration of the body with respect to an inertial reference system. Then, the Mach's principle states that the force of inertia results from the gravitational action of the entire Universe on the accelerated body. The system of reference with respect to which the whole of the Universe is at rest or in uniform and rectilinear motion is an inertial reference system.

The force of inertia must be understood as a force of gravitational induction, since it only appears when the body is accelerated with respect to the whole of the Universe.

The gravitational induction is a gravitational force that arises as a result of movement of the field source or by the movement of the observer. We have to distinguish between active and passive induction. The first is caused by the movement of the source and the second by the movement of the observer. Only when the movements are uniform and rectilinear, both types of induction are equivalent [4].

In our research the Universe will be at rest with respect to an inertial reference system and the observer will move with respect to this system, therefore it is a passive gravitational induction. Then the velocity and acceleration that we have to use are those of the observer.

The phenomenon of induction occurs in any field theory and has its origin in the retarded action, that is, in the non-instantaneous propagation of the action of the field.

2 Vector gravitational theory

To simplify the calculations, we will consider a vector gravitational field compatible with the Special Relativity. To get this theory we have to adapt the electromagnetic theory to gravity. The gavitational potential is a tetravector that we define by

$$\phi^k = \left(\phi, c\mathbf{A}\right)$$

ϕ is the scalar potential and **A** the vector potential. The source of the field is the current tetradensity

$$j^k = \rho u^k = \left(j^o, \mathbf{j}\right)$$

ρ is the proper density of matter and u^k is the tetravelocity of the source defined by

$$u^k = \frac{dx^k}{d\tau}$$

$d\tau$ is the proper time of the source particle.

The field equation is

$$\nabla^2\phi^k - \frac{1}{c^2}\frac{\partial^2\phi^k}{\partial t^2} = \frac{4\pi G}{c} j^k . \tag{1}$$

This equation is valid in inertial reference systems and in Cartesian coordinates, although it is generalizable to non-inertial systems and to other coordinates, but then the partial derivatives must be substituted by covariant derivatives.

3 Potentials of Liénard-Wiechert

The inertial reference system with respect to which the Universe is at rest is K_0 [5]. The observer C is at rest at the origin of the system *K*, which has a velocity $-\mathbf{u}$ with respect to K_0. This reference system, in general, will not be inertial (that is, it will have an acceleration with respect to K_0).

We consider a small mass *m* at rest with respect to K_0. For an observer at rest at K_0 there is only the scalar potential, which will correspond to the Newtonian potential

$$\phi^k = \left(-G\frac{m}{r_0}, 0\right) \tag{2}$$

where r_0 is the distance from the mass *m* to the observer C. Now we calculate the tetrapotential of the mass *m* for the observer C that is at rest in *K*, for which we express (2) in tetravectorial notation.

Now we make use of the relative movement. Instead of considering the movement of *K* with respect to K_0, we now consider the movement of K_0 with respect to *K*. Then, the mass *m* moves with velocity **u** with respect to C. For the observer K this is a retarded velocity, that is to say, it is the velocity that the mass *m* had when it emitted the gravitational action at the moment t' (retarded instant), which reaches observer C at the current time *t*. r' is the distance at which the mass *m* was at time t', that is to say, the distance at which C observes the mass *m* at the time *t*: $r' = c(t - t')$ and $\mathbf{r}'$ is the position of C with respect to the position retarded of *m* and therefore $-\mathbf{r}'$ is the vector of position of *m* with respect to C. Therefore the tetrapositions of *m* and the point C with respect to the *K* system are

$$x^k(m) = (ct', -\mathbf{r}'); \quad x^k(C) = (ct, 0),$$

the tetravelocity of *m* with respect to *K* is

$$u^k = \frac{dx^k(m)}{d\tau'} = \frac{dx^k(m)}{dt'\sqrt{1-u^2/c^2}} = \left(\frac{c}{\sqrt{1-u^2/c^2}}, \frac{\mathbf{u}}{\sqrt{1-u^2/c^2}}\right)$$

$d\tau'$ is the proper time of the mass *m* at the time retarded and

$$\mathbf{u} = -\frac{d\mathbf{r}'}{dt'}.$$

We define the tetravector

$$R^k = x^k(m) - x^k(C) = \left[c(t'-t), -\mathbf{r}'\right] = (-r', -\mathbf{r}')$$

then the tetrapotential (2) in a tetravectorial form is

$$\phi^k = Gm\frac{u^k}{u^i R_i} \tag{3}$$

since it is a covariant expression it is the same for all reference systems (inertial or non-inertial), therefore it is the tetrapotential in the system *K*. From the above definitions we find

$$u^i R_i = \frac{\mathbf{r}'\cdot\mathbf{u}}{\sqrt{1-u^2/c^2}} - \frac{r'c}{\sqrt{1-u^2/c^2}} = -\frac{cs}{\sqrt{1-u^2/c^2}}$$

we have defined

$$s = r' - \frac{\mathbf{r}'\cdot\mathbf{u}}{c} \tag{4}$$

therefore the tetrapotential (3) is

$$\phi^{k} = -Gm\left(\frac{1}{s}, \frac{\mathbf{u}}{cs}\right)$$

which corresponds to the following scalar and vector potentials

$$\mathbf{A} = -\frac{G}{c^{2}} m \frac{\mathbf{u}}{s}; \quad \phi = -G\frac{m}{s}, \tag{5}$$

(5) are the potentials of Liénard-Wiechert [6]. The equations (5) are valid for any observer at rest with respect to *K*. In the equations (5) **u** is the velocity with which the particle *m* moves with respect to the system *K,* system in which the observer is at rest. However, *m* is at rest with respect to the system K_0, which is an inertial reference system, which means that mass *m* does not emit gravitational radiation.

For the observer C the mass *m,* that has a velocity **u**, emitted a gravitational signal at the previous moment t'. Therefore, in (4) $\mathbf{r}'$ is the position vector of C with respect to the retarded position of *m.*

C would observe all the masses of the Universe move with the same velocity **u**, since this movement is the reflection of the movement of C with respect to K_0 with velocity $-\mathbf{u}$.

4 Calculation of potentials

As in the electromagnetic theory, fields strengths are defined in our vector gravitational theory by the relations

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}; \quad \mathbf{B} = \nabla \wedge \mathbf{A}. \tag{6}$$

The techniques for calculating the gravitoelectric field strength **E** and the gravitomagnetic field strength **B** are the same as those used in electromagnetism, for which we must use the relations [7]

$$\begin{aligned} \nabla\left(\frac{1}{s}\right) &= -\frac{\mathbf{r}'}{s^{2}r'} + \frac{\mathbf{u}}{cs^{2}} - \frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{u})}{cs^{3}r'} - \frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{a})}{c^{2}s^{3}} + \frac{u^{2}\mathbf{r}'}{c^{2}s^{3}} \\ \frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right) &= \frac{r'}{s}\left[\frac{\mathbf{a}}{s} + \frac{\mathbf{u}(\mathbf{r}'\cdot\mathbf{u})}{s^{2}r'} + \frac{\mathbf{u}(\mathbf{r}'\cdot\mathbf{a})}{cs^{2}} - \frac{\mathbf{u}u^{2}}{cs^{2}}\right]. \end{aligned} \tag{7}$$

The force acting on a body of gravitational mass m_g and velocity **w** is

$$\mathbf{F} = m_g\mathbf{E} + m_g\mathbf{w}\wedge\mathbf{B}. \tag{8}$$

5 Induction force on an accelerated body

We suppose a body C of gravitational mass m_g that has a velocity $-\mathbf{u}$ and an acceleration $-\mathbf{a}$ with respect to the Universe, that is to say, with respect to the inertial reference system K_0. The Liénard-Wiechert potentials (5) act on the body C, from which the gravitoelectromagnetic field strengths (6) are calculated using the relations (7). Finally by (8) we calculate the force of induction that acts on the body C.

In order to see the basic concepts without technical difficulties, we consider a simplified cosmic model. We assume that the Universe is large enough, of finite age, static and with a constant and uniform density.

To make the integration of the induction force acting on C we divide the Universe into spherical layers of negligible thickness dr', of center in the body C and of radius r'. Let *dm* be the gravitational mass of a portion of a spherical shell, its position vector with respect to the system K in which it is at rest C in spherical coordinates is

$$-\mathbf{r}' = -r'\sin\theta\cos\varphi\mathbf{i} - r'\sin\theta\sin\varphi\mathbf{j} - r'\cos\theta\mathbf{k}.$$

The divergence of the potential $d\phi$ produced by *dm* is

$$\nabla d\phi = -Gdm\nabla\left(\frac{1}{s}\right)$$

and the time derivative of the potential vector $d\mathbf{A}$ is

$$\frac{\partial d\mathbf{A}}{\partial t} = -\frac{G}{c^{2}}dm\frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right).$$

Now integrate over the whole mass of the spherical shell considered

$$\frac{\partial \delta \mathbf{A}}{\partial t}=-\frac{G}{c^2}\int \frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right)dm; \quad \nabla \delta\phi=-G\int \nabla\left(\frac{1}{s}\right)dm, \tag{9}$$

the relation between mass and density is given in [8]

$$dm=\left(1-\frac{\mathbf{r}'\cdot\mathbf{u}}{r'c}\right)[\rho]dV'$$

$[\rho]$ is the density in the retarded moment, which in our cosmological model always has the same value ρ Therefore of (9) we obtain in spherical coordinates

$$\begin{aligned}\frac{\partial \delta \mathbf{A}}{\partial t}&=-\frac{G}{c^2}\iint\left(1-\frac{\mathbf{r}'\cdot\mathbf{u}}{r'c}\right)\frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right)\rho r'^2\sin\theta dr'd\theta d\varphi\\ \nabla\delta\phi&=-G\iint\left(1-\frac{\mathbf{r}'\cdot\mathbf{u}}{r'c}\right)\nabla\left(\frac{1}{s}\right)r'^2\sin\theta dr'd\theta d\varphi,\end{aligned} \tag{10}$$

then the integration is made on all the spherical shells and the force induced is calculated by (8).

We limit ourselves to non-relativistic velocities, because we intend is to identify the inertial force of classical mechanics with the force of gravitational induction (8). So $u\ll c$, therefore $s\approx r'$.

Since r' has cosmic dimensions, we neglect in equation (7) the terms that depend on $1/r'^2$ versus the terms that depend on $1/r'$. With this simplification the only terms we consider are the fourth of the first equation (7) and the first and third of the second equation (7).

The direct calculation gives us

$$\frac{\partial \delta \mathbf{A}}{\partial t}=-4\pi\frac{G}{c^2}\rho\mathbf{a}r'dr'; \quad \nabla\delta\phi=\frac{4\pi}{3}\frac{G}{c^2}\rho\mathbf{a}r'dr',$$

now we integrate for all the spherical shells, from the radius 0 to the radius r' that is at the distance *ct*, where *t* is the age of the Universe and finally we apply (8), resulting

$$\mathbf{F}=\frac{4\pi}{3}G\rho t^2 m_g\mathbf{a} \tag{11}$$

which is the force of induction that the whole of the Universe exerts on a particle of gravitational mass m_g that has an acceleration $-\mathbf{a}$ with respect to the whole of the Universe [9]. The body C is at rest relative to the system *K*, then the second term of (8) is zero.

We identify the force (11) with the force of inertia $-m_i(-\mathbf{a})$ so we find the relation between the inertial mass and the gravitational mass

$$m_i=\frac{4\pi}{3}G\rho t^2 m_g. \tag{12}$$

to the coefficient of proportionality between the inertial mass and the gravitational mass $\xi(t)=4\pi/3G\rho t^2$ we call it the coefficient of inertia and it is a magnitude that depends on the cosmic model and that in general depends on the age of the Universe.

6 Proportionality of the inertial and gravitational mass

According to equation (11) when a body moves with an acceleration with respect to the whole of the Universe, a gravitational induction force acts on it, proportional to the acceleration of the body and in the opposite direction, exactly like the force of inertia. Therefore we must identify both forces, concluding that the force of inertia acting on a body is the inductive force of the Universe.

(11) also shows us that the force of inertia does not depend on the velocity, at least at the classical level, and is proportional to the acceleration. Note that **a** is any type of acceleration, therefore (11) also explains the centrifugal force.

By the equation (12) we find that the inertial mass is proportional to the gravitational mass, although both magnitudes are conceptually independent. The universal gravitation constant is chosen so that the inertial and gravitational mass are equal in the current epoch, that is to say

$$G=\frac{3}{4\pi}\frac{1}{\rho t_0^2}$$

t_0 is the current age of the Universe.

The inertial mass varies as the square of the age of the Universe

$$m_i(t)=\left(\frac{t}{t_0}\right)^2 m_i(t_0).$$

In our simplified model this increase in inertial mass is explained because as time passes there are more masses of the Universe that are causally connected to the body that undergoes the force of induction.

Of (12) we deduce that gravity has to be an exclusively attractive force. In fact, if gravity were repulsive the scalar potential of equation (2) would be positive and the same will happen with the vector and scalar potentials of (5). From these new equations we find again (12) but with a negative sign. But then the inertial mass would be negative, contrary to the observation that inertia is opposed to the change of movement of a body.

The previous reasoning is valid even in the case in which gravitational mass was negative. By equation (12) the gravitational mass can only have one sign, either positive or negative. If the gravitational mass had two signs (like the electric charge), there would be bodies with a negative inertial mass, which is absurd. Indeed, if the active gravitational mass (which produces the force) had a sign and the passive gravitational mass (on which the force acts) had the opposite sign, then its inertial mass would be negative, which is not observed in nature. Or in other words, if there were gravitational masses of the two signs, there would be bodies with a negative inertial mass.

The relation (12) produces effects that could be detectable, and which we will examine in a subsequent investigation. Among others, we point out that the variation of the inertial mass with time will affect to the emission frequency of the spectral lines, producing a shift of these lines, an effect that would overlap the shift caused by the cosmic expansion. The variation of the inertial mass will affect the orbital movements, so the rotation of distant galaxies will have a different law from the rotation of nearby galaxies. The equivalence between mass and the energy would also be affected by the variation of the inertial mass, in fact in the equation $E=mc^2$, m is the inertial mass, therefore its variation will affect the nuclear processes, which would have an impact on to stellar evolution.

7 Participation of the Universe in the formation of the inertial mass

According to Mach's principle, the inertial mass of a body is generated by the action of the whole Universe. On a body C act forces that are produced in all parts of the observable Universe. Some of these forces originate in very distant objects, that is, they were produced a long time ago; while the forces exerted by nearby objects were produced a short time ago.

Now we study the participation in the generation of the inertial mass of the different epochs of the Universe. We consider the Universe formed by N spherical shells in whose center is the body C. To the furthest shell we give the numeration $n=0$ and in it are the objects that produce the force at the beginning of the Universe, whose effects reach the body C in the present moment, when the age of the Universe is t_0.

The thickness of the spherical shells is $c\tau$ where $\tau=t_0/N$. The inertial mass produced by shell n is by (12)

$$\delta m_i=\frac{8\pi}{3}G\rho t\tau m_g \tag{13}$$

t is the time takes the gravitational interaction to travel from the spherical shell n to the body C, therefore

$$t=t_0-n\tau \tag{14}$$

of (12) and (13) we get

$$\frac{\delta m_i}{m_i(t_0)}=\frac{2}{N}\left(1-\frac{t'}{t_0}\right) \tag{15}$$

$t'=n\tau$ is the time retarded.

We must remember that (15) is applicable to the cosmic model that we are considering which has a constant and uniform density of matter. But independently of the cosmic model, (15) shows us that each epoch of the Universe contributes differently to the formation of the inertial mass of a body. (15) shows that the first moments of the Universe (when $t'=0$) have a greater contribution to the inertial mass; while the present Universe ($t'=t_0$) the contribution is minimal. This result can be more pronounced in a realistic cosmic model.

8 Mach's principle and the Big Bang

Now we will use a more realistic cosmic model with the exclusive idea of studying the influence that the Big Bang has on the formation of the inertial mass, and considering mainly the qualitative aspects.

Now we suppose, for example, a Universe where the density depends on the cosmic age according to the law

$$\rho(t') = \rho_0 \frac{t_0}{t'} \tag{16}$$

ρ_0 is the density at the present moment and t' is the age of the Universe at the time of generating the force or retarded time. By (16) the density of the Universe would be very high when its age is small, a situation similar to what happens in the Big Bang, therefore (16) will inform us about how the inertial mass is generated in a Universe that has a high density at its beginning.

By (12) the contribution to the inertial mass of a spherical shell located at a distance σ and thickness $\delta\sigma$ is

$$\delta m_i = \frac{8\pi}{3}\frac{G}{c^2} m_g [\rho] \sigma\delta\sigma = \frac{8\pi}{3}\frac{G}{c^2} m_g \frac{\rho_0 t_0}{t'} \sigma\,\delta\sigma .$$

t' is the moment when the signal was generated and t the moment when the signal reaches the observer, it means that the distance it travels is $c(t-t')$ and $\delta\sigma \equiv |\delta\sigma| = c\delta t'$ [10], therefore,

$$\delta m_i = \frac{8\pi}{3} G m_g \rho_0 t_0 \frac{t-t'}{t'} \delta t', \tag{17}$$

integrating (17) for all the spherical shells we find a singularity at $t'=0$. To avoid it we do the integration from the moment t_c and we get

$$m_i = \frac{8\pi}{3} G m_g \rho_0 t_0 t \left(\ln\frac{t}{t_c} - 1 + \frac{t_c}{t} \right) \tag{18}$$

which is the inertial mass when the age of the Universe is t. We notice that when $t_c = 0$ the inertial mass of the body would be infinite.

With the current values of ρ_0 and t_0 is found

$$\frac{8\pi}{3} G \rho_0 t_0^2 \approx 0.92,$$

we verified that the inertial mass observed in the current epoch would be generated by the forces produced by the Universe from $t_c = 0.1432 t_0$. But taking into account ancient cosmic actions, the relationship (18) increases, which means that the inertial mass becomes much greater than the gravitational mass.

For example, if we take the limit at $t_c = 0.0003 t_0$, which corresponds to an age of the Universe of about 4.5 million years, the ratio between the inertial mass and the gravitational mass is approximately 7,1.

The conclusion is that if the density in the past was greater than at present (as it must happen in a cosmic model with Big Bang), the participation in the inertial mass of the primitive Universe increases. That is to say, that the inertial mass would be generated mainly by the first stages of the Universe, until being very superior to the mass observed.

We conclude that if the Universe had a Big Bang, the Mach's principle would not be fulfilled and vice versa.

9 Cosmic expansion

Now we study the modification of the previous results, assuming that there is cosmic expansion. For reasons of symmetry, the cosmic expansion, which is isotropic, does not produce induction forces.

We will suppose that the cosmic substrate has the same spatial coordinates and that there is a cosmic scale factor, being the Euclidean three-dimensional space, that is, the element of line temporal space is

$$ds^2 = c^2dt^2 - R^2(t)\left(dr + r^2\sin^2\theta d\varphi^2 + r^2 d\theta^2\right)$$

$R(t)$ is the cosmic scale factor and *t* is the cosmic time, that is, the coordinate time of the reference system with respect to which the Universe is at rest.

The radial distance proper is

$$d\sigma = R(t)dr$$

and the equation of a ray that has a radial movement is

$$cdt = -R(t)dr$$

therefore

$$\sigma\left(\zeta,\hat{\zeta}\right) = ct_0 a(\zeta)\int_{\zeta}^{\hat{\zeta}} \frac{d\zeta'}{a(\zeta')} \tag{19}$$

we have made the following definitions

$$\zeta = \frac{t}{t_0}; \quad a(\zeta) = \frac{R(\zeta)}{R_0}$$

t_0 is the current age of the Universe and R_0 the cosmic factor at the moment t_0. ζ corresponds to a retarded moment or moment of signal departure and $\hat{\zeta}$ it is the moment of arrival the signal to the observer.

To specify the calculation, we suppose that the coefficient *a* has the same value as in the cosmic model of Einstein-de Sitter

$$a(\zeta) = \zeta^{2/3}. \tag{20}$$

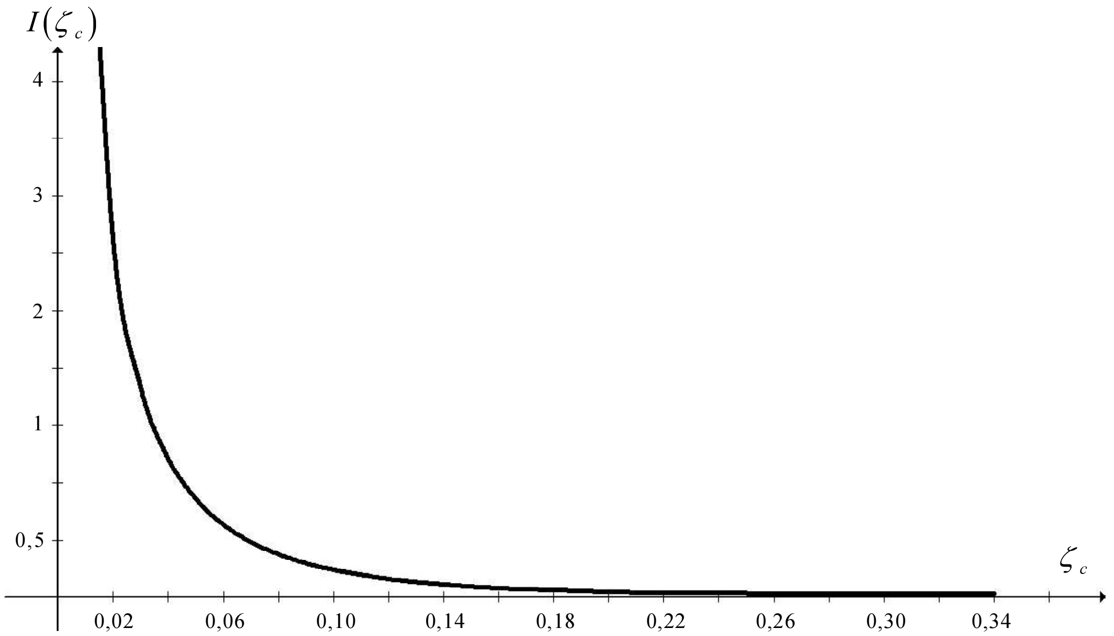
By (19) we find

$$\sigma\left(\zeta,\hat{\zeta}\right) = 3ct_0\left(\hat{\zeta}^{1/3}\zeta^{2/3} - \zeta\right)$$

function that is canceled in its extreme points $\zeta = 0$ and $\zeta = \hat{\zeta}$. That is, it has a maximum, so its derivative $\sigma' = d\sigma/d\zeta$ at the beginning has a positive value and then it is negative

By the cosmic expansion the density of matter in the model of Einstein-de Siter decreases according to the law

$$\rho(\zeta) = \frac{\rho_0}{a^3(\zeta)} = \rho_0\zeta^{-2} = \frac{1}{6\pi G}\frac{1}{t_0^2}\zeta^{-2} \tag{21}$$



ρ_0 is the density in the present moment.

The integral that we have to do is

$$\int_0^1 \rho(\zeta)\sigma(\zeta)d\sigma = \int_0^1 \rho(\zeta)\sigma(\zeta)\left|\sigma'(\zeta)\right|d\zeta \qquad (22)$$

that we evaluate from the origin of time to the current moment. Note that the absolute value of σ' has been taken since $d\sigma$ is always positive.

By (20) and (21) the integral (22) is for $\hat{\zeta}=1$

$$\int_0^1 \rho(\zeta)\sigma(\zeta)d\sigma = 9c^2t_0^2\rho_0\int_0^1\left|\frac{2}{3}\zeta^{-5/3}-\frac{5}{3}\zeta^{-4/3}+\zeta^{-1}\right|d\zeta = 9c^2t_0^2\rho_0 I$$

but this integral has a singularity in the origin of time. To avoid it we will integrate from a value ζ_c until the current moment, resulting

$$I=\int_0^1\left|\frac{2}{3}\zeta^{-5/3}-\frac{5}{3}\zeta^{-4/3}+\zeta^{-1}\right|d\zeta=\int_{\zeta_c}^{\zeta_m}\left(\frac{2}{3}\zeta^{-5/3}-\frac{5}{3}\zeta^{-4/3}+\zeta^{-1}\right)d\zeta-\int_{\zeta_m}^{1}\left(\frac{2}{3}\zeta^{-5/3}-\frac{5}{3}\zeta^{-4/3}+\zeta^{-1}\right)d\zeta$$

$\zeta_m=(2/3)^3$ is the moment when σ reaches its maximum value. From (12) and (21) it is found that the inertial mass is

$$m_i = 26\pi G\rho_0 t_0^2 I(\zeta_c)m_g = \frac{13}{2}I(\zeta_c)m_g,$$

to explain the identity of inertial and gravitational mass observed the present moment has to be fulfilled

$$I(\zeta_c)=\frac{2}{13} \quad \Rightarrow \quad \zeta_c = 0.13 \quad \Rightarrow \quad t = 0.13t_0$$

which corresponds approximately to 1,900 million years. Therefore, all gravitational actions produced by the Universe before this date generate an excess of mass.

In the previous graph the integral I is represented as a function of the parameter ζ_c. We verify that when approaching the moment of the Big Bang the inertial mass increases in an unlimited way, a sample of the incompatibility between Mach's principle and the theory of Big Bang.

10 Conclusions

We have used a vector gravitational theory, from which we deduce induction forces. The results show that it is possible to identify the induction force produced by the Universe with the force of inertia acting on an accelerated body.

We have verified that there are no forces of induction on a body that has a uniform movement, as is required by the principle of inertia. We verify that the cosmic expansion does not produce induction forces.

We have calculated how the different epochs of the Universe contribute to the formation of the inertial mass.

We have found incompatibility between Mach's principle and a cosmic model with Big Bang. In this model the inertial mass of a body is mainly produced by the first moments of the Universe and the generated mass significantly exceeds the observed inertial mass.

The conclusion is that although the vector gravitational theory is an approximation to the correct theory, and the cosmic model that we have used is very simple, we find that the induction forces produced by the whole Universe are comparable in sense and magnitude to the forces of inertia that are observed in accelerated bodies, which is a strong support for Mach's principle.

From the theory we have developed, we find that the Mach's principle is correct, but we also see that classical Newtonian mechanics is correct, at least at its application level. As in the mechanics of Newton, we can speak of a preferred system: the system in which the whole of the Universe is at rest. This system is equivalent to the Newtonian absolute space. The laws of dynamics are still valid, but are now deduced from the laws of gravity. We have verified that these laws are exclusively valid in a system at rest or in uniform and rectilinear movement with respect to the Universe.

In the second part of this investigation we will apply the results that have obtained to the General Theory of Relativity.

11 References and bibliography

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[2] There are several ways to define the Mach's principle, BONDI Hermann and SAMUEL Joseph: "The Lense-Thirring Effect and Mach's Principle", arXiv: gr-qc / 9607009v1, 1996.

[3] There is great confusion about the forces of inertia and not only in elementary texts. The problem may be that there are two types of forces of inertia, ASSIS Andre KT: *Relational Mechanics*, Apeiron, 1999. To extend the second principle of the dynamics to a non-inertial reference system, one must consider forces of inertia, which are fictitious. For us the forces of inertia are the real forces that act on any body that is accelerated with respect to an inertial reference system, it is the force $\mathbf{F}_i = -m\mathbf{a}$. We assume that the principle of dynamic equilibrium is valid, which states that the sum of all the forces acting on a body is zero

$$\mathbf{F} + \mathbf{F}_i = 0$$

where **F** is the applied force, SEGURA GONZÁLEZ, Wenceslao: *Gravitoelectromagnetismo y principio de Mach*, eWT Ediciones, 2013, pp. 3-16. Mathematically, the previous expression is identical to the usual way in which the second principle of Newtonian dynamics is expressed: $\mathbf{F} = m\mathbf{a}$, but conceptually both expressions are very different.

[4] When a mass is accelerated with respect to an inertial reference system it emits gravitational radiation. Therefore it is not the same thing that a mass is accelerated, that the observer has the acceleration, in the first phenomenon there would be gravitational radiation and there would not be in the second. The system of reference with respect to which the average matter of the Universe is at rest (system K_0), has a function similar to the absolute space in Newtonian mechanics. In the sense of being a special reference system, from which the inertial reference systems are defined.

[5] We define an inertial reference system as a system in which "the whole of the Universe" is at rest or in uniform and rectilinear motion. Therefore, the laws of dynamics are correct in this reference system. The velocity of rotation of the Earth measured by terrestrial dynamic experiments is the same as the speed measured astronomically, this is explained because the Universe is an inertial reference system and therefore has no rotation. In effect, the acceleration that is measured in these experiments is an acceleration with respect to an inertial reference system, therefore an acceleration with respect to the Universe; which means that the two measurements (dynamic and astronomical) are made with respect to the same reference system.

[6] There are two sets of Liénard-Wiechert potentials that are formally identical, but conceptually different. One of these sets is derived with the technique of the retarded potentials applied to the field equation. The other type of Liénard-Wiechert potentials are derived exclusively from relativistic considerations, without the need to use the field equations, as we have done in section 3. But there is a very sensitive difference between both sets of potentials. The first applies when the source is moving with respect to an inertial reference system, while the second set is applicable when the observer moves, while the source is at rest in an inertial system, PANOFSKY W.; PHILLIPS, M.: *Classical Electricity and Magnetism*, Addison-Wesley, 1972, pp. 240-245 y pp. 326-327.

[7] Idem, pp. 341-344.

[8] Idem, pp. 354-357. The vector $\mathbf{r}'$ has the dependence $\mathbf{r}' = \mathbf{r}'\left(x^{\alpha}, t'\right)$, where x^{α} are the coordinates of the point C and t' is the retarted time. But the operator ∇ are spatial derivatives with respect to the coordinates of point C in $t = \text{cte}$, therefore it must be put in function of ∇' that are derivatives with respect to x^{α} a t' constant. Equations (7) corresponds to this conversion.

[9] The acceleration of Coriolis that is observed in a reference system in rotation and that acts on a particle in motion, is not a real acceleration, that is to say, it is not an acceleration with respect to an inertial reference system. Therefore, the Coriolis acceleration is not the result of the gravitational induction of the whole Universe.

[10] The thickness of the spherical layer is always positive, for this reason we take the absolute value of $\delta\sigma$.