Author :Pankaj Mani,FRM,CQF, (India) manipankaj9@gmail.com

Abstract :

Riemann Hypothesis is TRUE if we look at the Functional Equation satisfied by the Riemann Zeta function upon analytical continuation in Game Perspective way as visualized by David Hilbert. It uses technical game theoretical concepts e.g. Nash Equilibrium to confidently assert that Riemann Hypothesis has to be True. Needs to be looked at the Foundational Principles underlying Mathematics. In other words,it's the game of arranging Zeros in the complex plane using the functional equation.

INTRODUCTION:

In this paper, I will be looking functional equation satisfied by Riemann zeta function actually a non-cooperative game between its constituent terms(here different mathematical functional symbols) in which the best strategy adopted by each player to locate zeros on mathematical field leads to discovering the most stable arrangement of physical location non-trivial zeros of Riemann zeta function, which in turn leads to TRUTHFULNESS OF RIEMANN HYPOTHESIS..

As visualized by David Hilbert- Advanced mathematics is actually a game between different mathematical symbols, where different symbols follow certain defined rules. I am going to extend his view stating that entire(number theory in particular) itself is actually a game, where different players play a non-cooperative game to reach at the most stable equilibrium stage.

The mathematical theory of games was invented by John von Neumann and Oskar Morgenstern (1944).Game theory is the study of the ways in which *strategic interactions* among *agents* produce *outcomes* with respect to the *preferences* (or *utilities*) of those agents, where the outcomes in question might have been intended by none of the agents..All situations in which at least one agent can only act to maximize his utility through anticipating (either consciously, or just implicitly in his behavior) the responses to his actions by one or more other agents is called a *game*. Agents involved in games are referred to as *players*. If all agents have optimal actions regardless of what the others do, as in purely parametric situations or conditions of monopoly or perfect competition we can model this without appeal to game theory; otherwise, we need it.

Each player in a game faces a choice among two or more possible *strategies*. A strategy is a predetermined 'programme of play' that tells her what actions to take in response to *every possible strategy other players might use*. The significance of the italicized phrase here will become clear when we take up some sample games below.

I will prominently use the tools of game theory to find out different Nash equilibrium stage in this functional game played between mathematical symbols

Here, in particular, I visualize the functional equation satisfied by Riemann zeta function as game between different constituent terms which are connected through multiplication sign on both side of equality sign.. I would be finding the Nash Equilibrium which will be the solution and prove the Riemann Hypothesis to be True.

. As this has exactly 1 NE stage corresponding to the location of non-trivial zeros on the critical line in $0<R(s) <1$.

So, what I would be doing is- finding the locations of trivial & non-trivial zeros by looking the arithmetic structure of Riemann zeta function and by applying the two basic arithmetic of numeric '0' to find out different set of possibilities of taking zero value by different constituent terms.

In a nutshell, I will NOT go into finding the zeros of this functiuon,rather I will be visualizing the arithmetic structure of FUNCTIONAL EQUATION ,in which different constituent terms are connected through multiplicative sign and using game theory find the NE stage to locate zeros. So, it has hardly anything to do with anything else than game theory and slight arithmetic of numeric 0.

The Riemann zeta function ((s) is a function of a complex variable s = o + it (here, s, o and t are traditional notations associated to the study of the Ç-function). The following infinite series converges for all complex numbers s with real part greater than 1, and defines ((s) in this case:

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots \qquad \sigma = \Re(s) > 1.$$

The Riemann zeta function is defined as the analytic continuation of the function defined for o > 1 by the sum of the preceding series.

The Riemann zeta function satisfies the functional equation

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\, \zeta(1-s),$$

where T(s) is the gamma function which is an equality of meromorphic functions valid on the whole complex plane. This equation relates values of the Riemann zeta function at the points s and 1 – s. The gamma function has a simple pole at every non-positive integer, therefore, the functional equation implies that ((s) has a simple zero at each even negative integer s = – 2n Pi these are the trivial zeros of ((s).

$$\eta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = (1 - 2^{1-s})\zeta(s).$$

Incidentally, this relation is interesting also because it actually exhibits ((s) as a Dirichlet series (of the y-function) which is convergent (albeit non-absolutely) in the larger half- plane o > 0 (not just o > 1), up to an elementary factor.

Riemann also found a symmetric version of the functional equation, given by first defining

$$\xi(s) = \frac{1}{2}\pi^{-s/2}s(s-1)\Gamma\left(\frac{s}{2}\right)\zeta(s).$$

STATEMENT of Riemann Hypothesis: All non-trivial zeros of Riemann zeta function in the critical space 0<R(s)<1 lies on R(s)=1/2 .

Here we look at Game theoretic aspects of how to arrange the Zeros on this plane.

Given the physical aspects of numbers and its complex field, I visualize numbers and their mathematical functions playing the non-cooperative game.

In context of functional equation game played by Riemann zeta functions in the ebthere are two players A & B where A corresponds to sin()=0 and B corresponds to sin() NOT=0.

A solution concept in game theory :
Nash Equilibrium which corresponds to the solution,here the physical location of non-trivial zeros of Riemann zeta function.

PROOF:

Functional equation satisfied by Players Ç (s) & Ç(1-s) in the entire complex domain 'C' is

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\, \zeta(1-s),$$

As one and only one term on each side of "=" sign can and must be zero as 0*0 = 0 & 0 *non-zero number= 0

2^s(Pi)^(s-1) and Gamma function terms can never be equal to 0 ,so we can skip that here as they will not contribute to becoming 0 using the functional equation.
And by coordinate transformation, s & 1-s can be transformed to ½-s and ½+s .

- A ={ C: s :: sin(Pi*s/2)=0,s‡0} as s=0 is the location for pole

i.e. those values of s for which Sin(Pi*s/2) is not equal to 0.

- B = { C-A,s‡0} i.e.
  those values of s for
  which Sin(Pi*S/2) =0

Player A (for which Sin () term is not 0) has also two options .It can also exercise one of the two.

1. Ç(s)=0 for R(s)> 1/2 and simultaneously for R(s)<1/2 (Both sides 0 simultaneously)

2. Ç(s)=0 for s= ½+it But, Ç(s)‡ 0 for R(s)>1 (Or none of the sides will be 0)
   i.e. C(s) ‡0 for R(s)<1/2 and C(s) ‡0 for R(s) >1/2

Player B (for which Sin() term =0) has two options to exercise in the game .It can exercise only one of the two.

1. Ç(s) =0 for R(s)>1/2, Ç(s)= 0 for R(s)<1/2 i.e.(Both sides will be 0)

2. Ç(s)=0 for R(s)<1/2, Ç(s)‡0 for R(s)>1/2 (Left side of R(s)=1/2 will be Zero ,Right side will not be zero)

Similarly,

Now, we look at the different permutations of strategies adopted in this game and find their payoff matrix.

Payoff matrix of this game

	Player A exercises 1st option	Player A exercises 2nd option

Player B exercises 1 option	0,0(All the points 0)Impossible as it means f(s)=0 for all s.	0,0(0 on Right and Left side both for Sin()=0 Impossible because it is already proven that there is no zeros for R(s)>1..

Player B exercises 2nd option	0,0(All left side points = 0)Impossible as only trivial zeros are only limited points	1,1(Possible location for 0) The only possible way to gain the stability and maximizes the payoff. Nash Equilibrium Stage.

By looking at the table Payoff is maximum i.e.(1,1) when A exercises 2^{nd} and B also exercises 2^{nd} option to locate Zeros.
That's the Nash equilibrium state by looking when both the players exercise the 2^{nd} options.

Which means that f(s) =0 in the critical strip 0< R(s)<1/2 will not exist either on the left side of R(s)=1/2 nor right side. So, the only possible location for the Non-Trivial Zeros would be R(s)=1/2

This asserts the truthfulness of the Riemann hypothesis that trivial zeros lie on the points s=2k,k<0 and non-trivial zeros lie on the R(s)=1/2 .Thus,

It implies that

Ç(s) =0 for R(s)=1l2+it for 0<R(s)<1 and
also,Ç(s) ‡ 0 for R(s)>1l2
QED

Reference:

1. *GAMETHEORY* http://plato.stanford.edu/entries/game-theory/#Games
2. Mathematics as Game https://www.marxists.org/reference/subject/philosophy/works/ge/hilbert.htm

3. *Riemann hypothesis by Enrico Bombieri https://www.issnaf.org/inside-issnaf/enrico-bombieris-lectures-on-the-riemann-zeta-functions.html*