A simple field-theoretical treatment of the magnetodynamic nature of mass for baryons.

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Abstract:
Baryons are considered as dressed protons, resulting from perturbations of magnetodynamic origin built upon a background sea of excitations at about 3.7 GeV. In order to simulate perturbations from such a state a sum over the energy spectrum of excitations is necessary. A Zeta-function regularization procedure previously adopted for the Casimir Effect is applied to eliminate divergencies when the sum upon the energy spectrum states is carried out. States of negative energy compared to the background state are obtained and represent the baryons. The periodic behavior of the baryons masses with confined magnetic flux is reproduced with no further forms of energies required besides the magnetodynamic terms.

1. Introduction.

In recent work [1] we have shown that through the imposition of gauge invariance conditions to the wavefunctions of a particle(represented in energy terms by a closed loop of current and performing zitterbewegung motion), it is possible to relate rest energy to magnetic energy for the baryons. Gauge covariance was imposed by making the magnetic flux linked through the region covered by the particle "orbit" be quantized in units n of $\phi_0= hc/e$, the flux quantum. We therefore *adopted* integer (allowing also for half-integer; which case depends upon the actual boundary conditions) values of n in the analysis for the baryons, guided also by the criterion that n should be proportional to the magnetic moment (in n.m. units) in the classical limit of flux generated by self-fields. Such model is essentially based upon heuristic arguments, and in particular the assumption that zitterbewegung currents flow inside complex particles like the baryons is the extension of a similar proposal made for the electron. The model predicts an inverse dependence of mass with the fine structure constant alpha, in agreement with experimental data analysis reported in the literature[1]. The model produces a reasonable agreement between the calculated magnetic(plus kinetic) energies and the rest energies, revealing also a clear dependence of rest mass upon the square root of the spin angular momentum, of the form predicted and observed in the literature. However, a noticeable scattering of data around the theoretical line still remained. The meaning of such scattering was not addressed in the previous work.

To better understand if such deviations might have a physical meaning rather than indicating possible limitations of the model, we decided that the data should be analyzed again in a slightly different way, by

avoiding any previous assumption about the values of n. The number of flux quanta is now objectively determined through the model, from the product of the known values of mass and magnetic moment (through the same eq.(3) of [1]; see below). The relation of mass with such "model-adapted" values of n, calculated from the available data, become the object of this new analysis.

To make the model expressions applicable to a sizeable number of particles it is necessary to eliminate the effects on the rest energies, of kinetic energy contributions specifically attributable to the "excess" spin angular momenta of decuplet particles(spin-3/2 particles) as compared to the spin-1/2 octet particles, which were evident in our previous paper[1]. In fact, the finding that the double-logarithmic plots for different values of spin in Figure 1 of [1] are parallel to each other indicates that the difference in mass has no magnetic origin(just notice that the $1/n$ term in the vertical scale of that Figure essentially *balances* the $1/\mu$ in the horizontal scale for all points; since these are the terms responsible for magnetic effects, the shift between the parallel lines of data is not therefore attributable to unbalanced magnetic energy differences between octet and decuplet). Following London´s calculation that for perfect shielding of a (flux containing) hole in a superconductor by circulating currents on the hole rim, the kinetic and magnetic parts of the energy should be equal to each other, the excess kinetic energy due to the greater spin 3/2 should be about $(\sqrt{3} - 1)/2$ times the smallest spin-1/2 baryon mass, i.e., 340 Mev/c^2. Therefore, for the range of mass values covered by the decuplet particles the elimination of such excess kinetic energy can be made by subtracting from the masses of the decuplet particles the average difference between the actual masses of decuplet and octet particles (244 Mev/c^2, which is smaller than the idealized estimate presented

above). The resulting "*transformed masses*" m_t of the decuplet thus have the same average as the masses of the octet particles. This should eventually make all baryons fit the mass-energy expression derived for spin-1/2 in [1]. These particles are considered as comparable for the purposes of this theoretical analysis .

As expected, the new values of n are not substantially different from the ones adopted previously(see [1] for details in the Tables there). In this way, the margin of arbitrariness in the choice of n inherent to the previous criterion is eliminated and the determination of this parameter for each baryon becomes an objective for the model. From the new analysis it should therefore be possible to better evaluate the internal consistency of the model itself, including the evaluation on whether the proposed interpretation of n as a true number (integer or not) of magnetic flux quanta is physically meaningful, as well as analysing how appropriate is the utilization of closed currents as a means of representing complex particles.

As shown in the following sections, the approach proved valuable. As far as results are concerned new important features have arisen from the analysis. By plotting against n both the octet baryons masses and the transformed rest masses m_t of the decuplet baryons, we obtain the novel result that a simple periodic function, with n in the argument, is capable of fitting the points. That is, the rest energy (given by magnetodynamic terms) is periodic on magnetic flux.

Such result seems quite revealing since it has actually been repeatedly associated in the literature with the effect of confined flux upon the magnetic energies due to currents, flowing around multiply-connected paths, which is exactly what this research proposes to demonstrate happens inside particles. The Aharonov-Bohm effect of interfering electron beams surrounding a solenoid, as well as superconducting

currents in rings[2,3], charge density waves in dielectric structures[4], and even currents around normal metallic rings[5] (all surrounding confined magnetic flux) have been reported to display such periodic dependence of energy and current on magnetic flux.

Starting from a Lagrangian suitable to fermion fields[6], we obtain an energy spectrum for the possible current-carrying states around the closed path confining magnetic flux. In order to simulate self-field perturbations involving pair creation\ annihilation from vacuum, a sum over the states in the energy spectrum is necessary. An Epstein-Riemann zeta-function regularization procedure previously adopted for the Casimir Effect is applied to eliminate divergencies when the sum upon the energy spectrum states is carried out[7], and the periodic behavior of the baryons masses with magnetic flux is reproduced with no further forms of energies required besides the magnetodynamic terms.

It is a basic assumption of the model adopted in this treatment [1]that currents generate magnetic moments, which give rise to self-magnetic fields and flux within particles. An "anomalous" magnetodynamic energy is generated, which we identify with the additional rest energy of the "dressed" particles. It appears that the resulting trapped magnetic flux modulates the currents obtained from wavefunctions running around the closed path, through the imposition of a phase factor, and such phases vary from one baryon to another. The magnetic energy depends on such modulation, and thus also the mass along the baryons family.

All these results are considered in detail in the Theory section. A review of previous results of the model is also added for the sake of completeness of exposition.

2. Theory

2.a. Phenomenological determination of the parameter *n*.

Isolated current-loops containing a single quantum of flux of value $\phi_0/2 = hc/2e$ are well known from type-II superconductivity. The formation of superconductor current loops is a many-body effect, though. In a series of papers we have investigated if there might exist single-particle systems confining flux in a similar manner[1]. It is essential that such proposal be quantitatively supported by experimental data. Let's consider the actual case of particles of the baryon octet. All the eight particles have well-established rest masses and magnetic moments. E.J. Post [8]considered how to write an energy-mass relation in a tentative model for *the electron*. Post showed that the magnetic moment for the electron could be obtained up to the first-order correction (from QED) with the equation:

$$ mc^2 = \phi\, i\, /c + eV \qquad (1) $$

Here the left side is the rest energy of the electron, which from the right side is considered as fully describable by electromagnetic quantities. The first term on the right side is the energy of an equivalent current ring of value *i* linking an amount of flux ϕ, that should occur in a number *n* of flux quanta ϕ_0. In spite of the adopted parameters from electromagnetic theory, such term contains similar amounts of magnetic and kinetic energy contributions of moving charges, as discussed by London[9], and thus the kinetic effects are already included. The second (electrostatic energy) term is much smaller than the first (it will be neglected hereafter) and accounts for the radiation-reaction correction for the magnetic moment which is

proportional to the fine structure constant α [8]. Post associates the current with the magnetic moment μ and the size R of the ring with the equation:

$$\mu = \pi R^2 i/c \qquad (2)$$

One then inserts equation (2) into equation (1) (without the electrostatic small term) and thus eliminates the current. The parameter R has been calculated/measured for the nucleons only, but it remains part of the final expression for all baryons obtained after the combination of (1) and (2). We may conveniently eliminate R from this treatment by adopting for all baryons an expression which is valid for the leptons (assuming in that case $R= \lambda$, the Compton wavelength), and for the proton[1] (from experiments evidence), namely:

$$\mu = eR/2 \qquad (3)$$

In the present case we are interested in assessing a sufficiently large group of particles in order that the proposed association between mass and confined flux can be properly investigated, and the baryons form such a group.

The model by Post was devised to fit a single fundamental particle, the electron, and there was actually no discussion about the application to other particles. We are now able to justify (see section 2.b) the proposal that the collective motion of constituents inside baryons can also be described in terms of currents, so that a similar model should apply.

The combination of equations (1)-(3) with $\phi = n(hc/e)$ can therefore be cast in the form (inserting $\alpha = e^2/\hbar c$):

$$n = (2c^2\alpha/e^3) \, \mu \, m. \qquad\qquad (4)$$

Tables 1 and 2 bring the mass and magnetic moments data for all baryons of the octet and decuplet, alongside the values for n from eq.(4). It should ne noticed that according to the present treatment the proton corresponds to $n \approx 3$ (see Table 1). In a semiclassical treatment Barut [10] considered baryons and mesons as resulting from stabilized configurations of constituents linked together by dipolar magnetic forces. A quantum number is introduced and the rearrangement of parameters makes Barut´s final formulas for mass quite similar to the ones obtained in ref. [1]. In particular Barut obtains $n=3$ for the proton, by associating one unit of angular momentum for each of three unit-charged constituents

2.b. Heuristic model based upon Field-Theoretic concepts.

Equation (4) stresses the fact that in this work n is the parameter to be determined from the data available for mass and moment (note that it is the same eq.(3) of [1] written in another form). In addition, equation (4) can be rewritten in a useful form by isolating in it the expression for the nuclear magneton (n.m.), $e\hbar/2m_\mathrm{p}c$, yielding $n= (m/m_\mathrm{p}) \, \mu$(n.m.). Here m_p is the proton mass and the magnetic moment is given in n.m. units.

All the parameters on the right side of eq.(4) are known for the eight baryons of the octet, and are listed in Table 1(data from [11]). Figure 1 shows the plot of the calculated n against the magnetic moment for each particle, which mirrors the dependence of mass on magnetic

properties for each octet baryon. Note the presence of a diagonal line. There is a tendency to form Shapiro-like steps at integer numbers of flux quanta, but the approach to the steps has an undulating shape rather than being sharply defined (note: such "Shapiro" steps for superconducting rings characterize the penetration of flux inside the ring in units of flux quanta).

The existence of a diagonal *baseline* , $n= \mu$(n.m.), experimentally characterizes the presence of a *minimum* amount of mass in all baryons. From eq.(4), it becomes clear that the proton mass would be this minimum mass. Barut in the 1970s proposed that the other baryons might be considered perturbations built upon a proton "fundamental-state", thus providing a minimum amount of mass. The undulations in the figure lie above the diagonal line since it characterizes a stable, fundamental-like state.

In fact the undulations can be thought as a consequence of the confinement of magnetic flux inside a multiply connected path described by each particle charge motion. Gauge covariance of a Lagrangian which describes such particle ends up imposing such periodic dependence on the magnetic properties of the particles. Similar problems have extensively been dealt with by condensed matter physics groups[2-6].

Let´s consider a fermion field confined to a circular path of length L, enclosing an amount of self induced magnetic flux ϕ, in a potential A . We need to show that such a system corresponds to a state detached from a higher state associated with a sea of excitations in equilibrium, and therefore might be used to represent a "quasiparticle". The relativistic Lagrangian for such a fermion can be modelled through the dressing of a proton of mass m_p in view of the presence of magnetodynamic terms[6]:

$$\mathcal{L} = \bar{\psi}\left\{i\alpha_\mu\left(\hbar\partial_\mu - i\frac{e}{c}A_\mu\right) - \alpha_4 m_p c\right\}\psi \qquad (5)$$

Where the α_i are Dirac matrices. For A a constant around the ring path, the spectrum of possible energies for a confined fermion will be:

$$\varepsilon_k = c\{(p_k - eA/c)^2 + m_p^2 c^2\}^{1/2} \qquad (6)$$

which comes straight from the orthonormalized definition of the Dirac matrices and diagonalization of the Hamiltonian. If one takes the Bohr-Sommerfeld quatization conditons the momentum p_k (for integer k) is quantized in discrete values $2\pi\hbar k/L$. We start from this assumption but the true boundary conditions to close the wave loop might impose corrections to this rule in the form of a phase factor(see below). The potential A can be replaced by ϕ/L . Such charge motion is affected by vacuum polarization and the effects on the kinetic energy are accounted for in a way similar to that used in the analysis of the Casimir Effect, by summing over all possible integer values of k in eq.(6) [6,7]. This summation diverges. According to the theory of functions of a complex variable the removal of such divergencies requires that the analytic continuation of the terms be taken, which reveals the diverging parts which are thus considered as contributions from the infinite vacuum reservoir. A successful technique for this purpose begins with the rewriting of eq.(6) in terms of Epstein-Riemann Zeta functions[7], including the summation over k from minus to plus infinity integers, and making a regularization (Reg) transformation. Here $M(\phi)$ is the flux-dependent dressed mass of a baryon, and $s \rightarrow -1$:

$$M c^2 = U_0 + \text{Reg} \sum_k c\{(p_k - e\phi/Lc)^2 + m_p^2 c^2\}^{-s/2} \qquad (7)$$

where we have allowed for the existence of a finite energy U_0 to represent an hypothetical state from which the individual baryons would condense, since they would correspond to lower energy states. Such particles should be characterized as states of energy Mc^2 lower than U_0. It is convenient to define from L a paramer with units of mass $m_0 = 2\pi\hbar/cL$, which will be used to define a scale in the fit to the data. We notice that m_0 is related to the parameter L in the same way field-theories regard mass as created from broken symmetries of fields, establishing a range for an otherwise boundless field distribution(e.g., as happens at the establishment of a superconductor state with the London wavelength related to an electromagnetic field "mass" by a similar expression). For convenience, we define the ratios $m' = m_p / m_0$ and $u_0 = U_0 / m_p c^2$. For comparison with the data analysis in our previous work[1], we must introduce also the number of flux quanta n (integer or not) associated to ϕ, such that $n = \phi/\phi_0$. In terms of these parameters one may write (7) in the form:

$$M(n)/m_p = u_0 + (1/m') \mathrm{Reg} \sum_k \{(k-n)^2 + m'^2\}^{-s/2} \qquad (8)$$

In the analysis of data the experimental values of M/m_p for baryons will be plotted against n. The sum in the right side of (8) is a particular case of an Epstein Zeta function $Z(s)$, and becomes a Riemann Zeta function, since the summation is over one parameter k only. The summation diverges but it can be analytically continued over the entire complex plane, since the Epstein Zeta function displays the so-called reflection property. It has been shown that after the application of reflection the resulting sum is already regularized, with the divergencies eliminated. The reflection formula is[7]:

$$\pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) Z(s) = \pi^{\frac{s-1}{2}} \Gamma\left(\frac{1-s}{2}\right) Z(1-s) \qquad (9)$$

This replaces the diverging $Z(s)$ straight away by the regularized $Z(1-s)$, which converges(since $\Gamma(-1/2) = -2\sqrt{\pi}$ we see that the regularized sums are negative, like in the Casimir Effect solution). For the sake of clarity we describe now the regularization of eq.(8) below as eq.(10), step by step (note that $s \to -1$, and the "reflected" exponent $-(1 - s)/2$ replaces $-s/2$ of (8)). In the first passage from the left the entire summation argument is replaced by the Mellin integral which results into it. This creates a convenient exponential function to be integrated later. In the second passage the Poisson summation formula is used, in which the summed exponential function is replaced by its Fourier Transform (note that the same notation k is used for the index to be summed in the Fourier Transformed quantity). The objective is to replace the $k^2 t$ in the initial exponential by k^2/t. In this way, when the integration over t is carried out a modified Bessel function K is obtained. In the final line the $k=0$ term in the sum is separately worked out and appears as the first term between brackets. The remaining summation in k therefore does not include 0 ("/0" as shown). The influence of the parameter n is, as we wanted to prove, to introduce a periodicity depending on the amount of flux confined by the current ring, and the regularized energy is therefore periodic in n.   Therefore, $Z(1-s)$ is given as:

$$\sum_k \{(k-n)^2 + m'^2\}^{-(1-s)/2} =$$

$$= \frac{2}{\Gamma\left(\frac{1-s}{2}\right)} \int_0^\infty t^{\frac{1-s}{2}-1} \left(\sum_k e^{-(k-n)^2 t - m'^2 t} \right) dt =$$

$$= \frac{2\sqrt{\pi}}{\Gamma\left(\frac{1-s}{2}\right)} \int_0^\infty t^{-\frac{s}{2}-1} \left(\sum_k e^{-2\pi i k n} \; e^{-\frac{\pi^2 k^2}{t} - m'^2 t} \right) dt =$$

$$= \frac{2\sqrt{\pi}}{\Gamma\left(\frac{1-s}{2}\right)} \left(\frac{\Gamma\left(-\frac{s}{2}\right)}{m'^{-s}} + 2\pi^{-s/2} \sum_{k/0} \left(\frac{k}{m'}\right)^{\frac{-s}{2}} K_{\frac{s}{2}}(2\pi m' k) e^{-2\pi i k n} \right) \qquad (10)$$

for s → −1. From eq. (9), the "Reg" summation in (8) becomes $(\pi^{\frac{2s-1}{2}} / \Gamma\left(\frac{s}{2}\right)) \, \Gamma\left(\frac{1-s}{2}\right) Z(1-s)$, and the exponential produces a cosine term.

Since Γ(−1/2)= −2√π we see that the regularized sum is negative, corresponding to energies lower than U_0. In the fitting to the data we will admit that both *m'* and u_0 are adjustable parameters.

Figure 2 shows the data for all baryons in Tables I and II, and the plot of eq.(8) regularized by eq.(10), for u_0 = 3.96 and *m'*= 0.347 (corresponding to m_0= 2.88 m_p and U_0= 3710 MeV). The energy 3710 MeV would represent the sea of excitations from which the baryons would evolve.

3. Analysis and Conclusions.

The present paper provides a theoretical background for the phenomenological analysis of ref.[1]. Such previous analysis has been improved through the redefinition of the parameter *n* in terms of

the experimental data on mass and magnetic moments for baryons. The basic idea has been the modeling of such particles by means of confined currents. The present work has shown that this is theoretically sensible. Closed currents are associated with confined magnetic flux. Since the represented particle is immersed in a sea of excitations, the energy spectrum of closed currents is summed up over all possible values of a Bohr-Sommerfeld kinetic quantum number, leaving the previously defined magnetic n as the parameter to dictate the mass differences among the baryons, in view of the fulfillment of gauge-covariance conditions. A regularization procedure is necessary since the original sums diverge. The model regards particles as the result of a type of condensation from a sea of excitations of top energy U_0, which is the accepted picture in field theories of the origin of mass(however no phase-transitions or broken symmetries are explicitly introduced in the present treatment). The lowest energy particles are the nucleons in this picture. The magnetic flux introduces a modulation of rest energy which is quite well reproduced and the parameter m' is defined with such a magnitude to cover all baryons up to the omega particle. No other kinds of forces are necessary for such theoretical treatment to reproduce data, neither is necessary a detailed knowledge about inner constituents of baryons.

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Table 1: Data for the baryon octet(moments μ from ref. [11]).
According to equation (4) in gaussian units: $n= 1.16 \times 10^{47} \mu\, m$.
The plot of m/m_p (m_p the proton mass) against n are shown in
Figure 2.

	abs μ (n.m.)	μ(erg/G) x 10^{23}	m(Mev/c^2)	m(g) x 10^{24}	n from eq.(4)
p	2.79	1.41	939	1.67	2.73
n	1.91	0.965	939	1.67	1.9
Σ^+	2.46	1.24	1189	2.12	3
Σ^0	0.82(theor.)	0.414	1192	2.12	1
Σ^-	1.16	0.586	1197	2.12	1.5
Ξ^0	1.25	0.631	1314	2.34	1.7
Ξ^-	0.65	0.328	1321	2.34	0.9
Λ	0.61	0.308	1116	1.98	0.7

Table 2: Data for the baryon decuplet (moments μ from ref. [11]). The average difference between the decuplet and octet particles masses is discounted as discussed in the text and the resulting mass is put in columns 4 and 5. According to equation (4) in gaussian units: $n = 1.16 \times 10^{47} \mu m$. The plot of m_t / m_p against n are shown in Figure 2.

	abs μ (n.m.)	μ(erg/G) x 10^{23}	$m_t = m - 244$ (Mev/c^2)	m_t(g) x 10^{24}	n from eq.(4)
Δ^{++}	4.52	2.28	986	1.75	4.64
Δ^+, Δ^-	2.81,2.81	1.42	990	1.75	2.9 , 2.9
Σ^+	3.09	1.56	1135	2.02	3.65
Σ^0	0.27	0.136	1136	2.02	0.32
Σ^-	2.54	1.28	1138	2.02	3
Ξ^0	0.55	0.28	1281	2.28	0.73
Ξ^-	2.25	1.14	1283	2.28	3
Ω^-	2.02	1.02	1428	2.54	3

Figure 1: Plot of *n* against the magnetic moment for the octet following eq (4) and Table 1. The diagonal line is the classical prediction of one flux quantum per nuclear magneton (n.m.). Nucleons are on the line. Horizontal (Shapiro-like) steps at integer values of *n* are shown. The data display undulations, and a tendency to reach for the steps (traced line as guide).
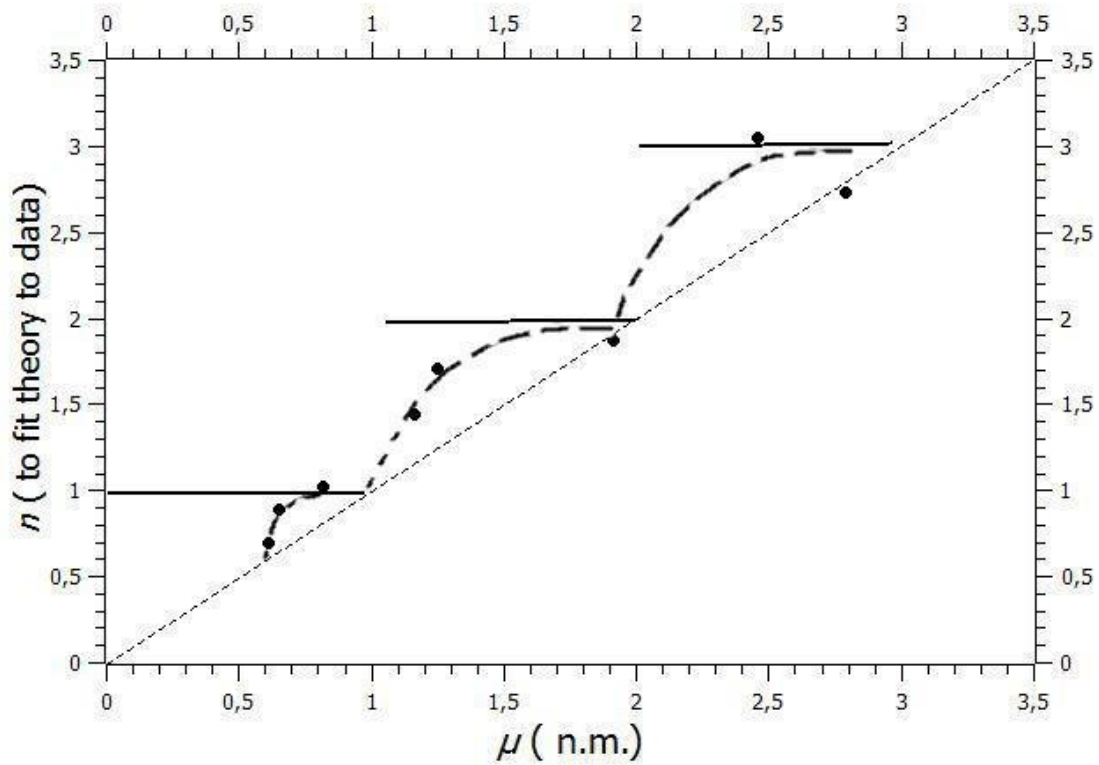
Figure 2: Comparison of baryons masses calculated from eq. (8) as a function of confined flux n , with data from Tables 1 and 2 for octet (open circles) and decuplet particles (m_t used, stars). The phenomenological eq.(4) provides values for n as a function of mass and moment, and the relation between these quantities(data points) agrees quite well with the field-theoretical calculations (curve) of mass as function of n from eq.(8). Nucleons are on the basis of the figure.