Time intervals measured by two observers in relative motion

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Abstract

It has been shown previously that when for two inertial observers A and B in relative motion, the value of the distance between A and B measured by observer B in a frame where B is stationary is the same as that obtained by observer A in a frame where A is stationary. The present study will show that the time interval for observer A to meet observer B measured by observer B in a frame where B is stationary is the same as that for observer B to meet observer A obtained by observer A in a frame where A is stationary. The proof is based on two fundamental conditions of special relativity: 1) the space time interval between two events in the Minkowski space is independent of the inertial reference frame chosen; and 2) there is no privileged reference frame and all inertial reference frames are equal.

Keywords: Minkowski diagram; time interval; special relativity; Lorentz ether theory; privileged frame.

It has been show previously that when for two inertial observers A and B in relative motion (Fig.1), the value of the distance between A and B measured by observer B in a frame where B is stationary is the same as that obtained by observer A in a frame where A is stationary (Ma 2018). What is the time interval for them to meet each other measured by two observers A and B respectively? Will the value measured in the observer A stationary frame be equal to the value measured in the observer B stationary frame? In the following analysis, I will the previous result that the value of the distance between A and B measured by observer B in a frame where B is stationary is the same as that obtained by observer A in a frame where A is stationary.

Fig.1 Two inertial observers A and B in relative motion. The velocity $v_{B,A}$ is that of observer B as measured by observer A, $v_{A,B}$ is that of observer A as measured by observer B.

According to the Einsteinian principles that there is no privileged inertial reference frame and space-time intervals are invariant across different frames, we can draw Murkowski diagrams for both observer A being stationary (Fig.2A) and observer B being stationary (Fig.2B).

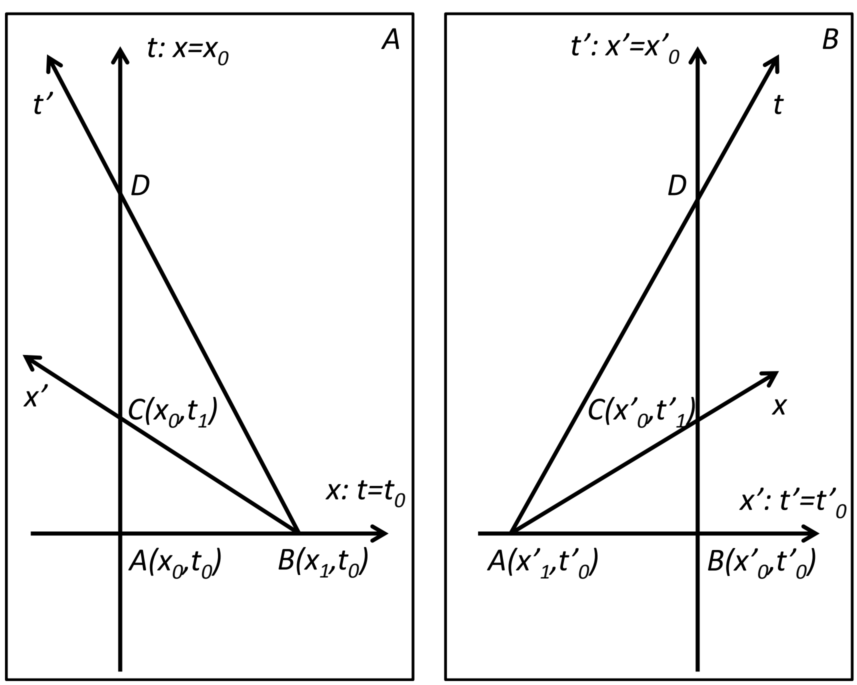

Fig.2 Minkowski diagrams representing the worldlines of observers A and B. Observer A's worldline is along a line parallel to the t-axis, $t|x = x_0$, and its line of simultaneity at its time $t = t_0$, $x|t = t_0$, is line AB in Fig.2A and line AC in Fig.2B. Observer B's wordline is along a line parallel to the t'-axis, $'|x' = x'_0$, and its line of simultaneity at its time $t' = t'_0$, $x'|t' = t'_0$, is line BC in Fig.2A and line AB in Fig.2B.

To avoid any confusion caused by ambiguity in notation, we will rewrite $x_{A,A}$ and $t_{A,A}$ in Fig.2A, the space and time coordinates of point A in observer A's reference frame when observer A's frame is stationary frame, as $x^{A}_{A,A}$ and $t^{A}_{A,A}$; $x'_{A,B}$ and $t'_{A,B}$ in Fig.2A, the space and time coordinates of point A in observer B's reference frame when observer A's frame is stationary frame, as $x'^{A}_{A,B}$ and $t'^{A}_{A,B}$; $x'_{B,B}$ and $t'_{B,B}$ inFig.2B, the space and time coordinates of point B in observer B's reference frame when observer B's frame is stationary frame, as $x'^{B}_{B,B}$ and $t'^{B}_{B,B}$; $x_{B,A}$ and $t_{B,A}$ in Fig.2B, the space and time coordinates of point B in observer A's reference frame when observer B's frame is stationary frame, as $x^{B}_{B,A}$ and $t^{B}_{B,A}$. The notations of other points will follow the same rule. The slope of BC in Fig.2A will be designated by k and the slope of AC in Fig.2B by k'.

From the Einsteinian principle of relativity and the stipulations for Minkowski diagrams,

$$v_{BA,A} = v_{AB,B}, \tag{1}$$

Therefore, $$k = k'. \tag{2}$$

In Fig.2A, $$c\left(t^{A}_{C,A} - t^{A}_{A,A}\right) = k\left(x^{A}_{B,A} - x^{A}_{A,A}\right) \quad . \tag{3}$$

In Fig.2B, $$c\left(t'^{B}_{C,B} - t'^{B}_{B,B}\right) = k(x'^{B}_{B,B} - x'^{B}_{A,B}) \,. \tag{4}$$

Since it has been proved previously (Ma 2018),

$$\left(x_{B,A}^{A} - x_{A,A}^{A}\right)^2 = ({x'}_{B,B}^{B} - {x'}_{A,B}^{B})^2 \,,$$

$$x_{B,A}^{A} - x_{A,A}^{A} = {x'}_{B,B}^{B} - {x'}_{A,B}^{B} \,. \tag{5}$$

Therefore,
$$t_{C,A}^{A} - t_{A,A}^{A} = {t'}_{C,B}^{B} - {t'}_{B,B}^{B} \tag{6}$$

$$\Delta {t'}_{CB,B}^{B} = \Delta t_{CA,A}^{A} \qquad . \tag{7}$$

This proves that according to the Einsteinian special relativity, the time interval for observer A to meet observer B measured by observer B in a frame where B is stationary is the same as that for observer B to meet observer A obtained by observer A in a frame where A is stationary. This results can be obtained directly from the principle of relativity. Just one moment of thinking should be enough to realize that how can the same time interval measured in one frame be always shorter than that measured in another frame when the two observers have the exactly same background?

References

Ma, Q.P. (2018) The distance between two inertial observers in relative motion in special relativity. http://vixra.org/abs/1811.0107.