Einstein's mass-energy equivalence relation: an explanation in terms of the *Zitterbewegung*

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Abstract: The radial velocity formula and the Planck-Einstein relation give us the *zbw* frequency (E = ħω = E/ħ) and *zbw* radius ($a = c/\omega = c\hbar/\mathrm{m}c^2 = \hbar/\mathrm{m}c$) of the electron. We interpret this by noting that the $c = a\omega$ identity gives us the $\mathrm{E} = \mathrm{m}c^2 = \mathrm{m}a^2\omega^2$ equation, which suggests we should combine the total energy (kinetic and potential) of *two* harmonic oscillators to explain the electron mass. We do so by interpreting the elementary wavefunction as a two-dimensional (harmonic) electromagnetic oscillation in real space which drives the pointlike charge along the *zbw* current ring. This implies a *dual* view of the reality of the real and imaginary part of the wavefunction:

1. The $x = a \cdot \cos(\omega t)$ and $y = a \cdot \sin(\omega t)$ equations describe the motion of the pointlike charge.
2. As an electromagnetic oscillation, we write it as $\boldsymbol{E}_0 = E_0 \cdot \cos(\omega t + \pi/2) + i \cdot E_0 \cdot \sin(\omega t + \pi/2)$.

The magnitudes of the oscillation a and E_0 are expressed in distance (m) and force per unit charge (N/C) respectively and are related because the energy of both oscillations is one and the same. The model − which implies the energy of the oscillation and, therefore, the effective mass of the electron is spread over the *zbw* disk − offers an equally intuitive explanation for the angular momentum, magnetic moment and the *g*-factor of charged spin-1/2 particles. Most importantly, the model also offers us an intuitive interpretation of Einstein's enigmatic mass-energy equivalence relation. Going from the stationary to the moving reference frame, we argue that the plane of the *zbw* oscillation should be parallel to the direction of motion so as to be consistent with the results of the Stern-Gerlach experiment.

Keywords: *Zitterbewegung*, mass-energy equivalence, wavefunction interpretations.

Contents

1 Introduction

The *Zitterbewegung* model of an electron is attractive because it offers a *physical* explanation for properties of matter which, in the mainstream interpretation of quantum mechanics, remain largely unexplained. Think, for example, of the angular momentum and the (related) magnetic moment of an electron. These just pop up as mathematical quantities in the mainstream Copenhagen interpretation of quantum physics, which basically tells us to *not* ask the question: what could they possible *be*? As such, the *Zitterbewegung* interpretation of quantum mechanics does hark back to Einstein's and Schrödinger's original intuition: if Nature is probabilistic, then *something must explain the probabilities.*

A related reason for the intuitive appeal of the *zbw* theory is its effortless integration of the idea of wave-particle duality: the *zbw* presents the electron as a pointlike electric *charge* which oscillates around some center. The charge itself is pointlike[1] and has no rest mass (it moves at the velocity of light). As such, the idea of a pointlike or dimensionless *charge* is separated from the concept of the particle, which does take up some space (the oscillation has a radius) and, because of the energy in the oscillation, acquires some inertia to motion. It must, therefore, have some (rest) mass.

The next step is to explain the *physical* nature of the oscillation and to explain the energy (or the *rest* mass) of the electron in terms of the energy of the oscillation. The oscillation in this model must be electromagnetic (as opposed to, say, gravitational) because the force can only grab onto an electric charge. However, we should not get ahead of ourselves here. Before we fully develop the model, we will review some math.

2 The basics of the model

Let us consider an electron traveling in the positive x-direction at constant speed v. Hence, its position, in our reference frame, as a function of time, is equal to $x(t) = v \cdot t$. Let us denote the position and time in the reference frame of the electron itself by x' and t'. The position of the electron in its own reference frame, , is $x'(t') = 0$ for all t', and the position and time in the two reference frames are related as follows:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{vt - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = 0$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If we denote the energy and the momentum of the electron in our reference frame as E_v and $p = \gamma m_0 v$, then the argument of the (elementary) wavefunction $a \cdot e^{-i\theta}$ can be re-written as follows:

[1] The idea of a pointlike charge – or an elementary particle – is that there is no underlying internal structure. We will not worry here about the question of whether or not its spatial dimension might actually be zero because the idea of the classical electron radius ($r_e \approx 2.82{\times}10^{-15}$ m) might still make sense. In this context, we may remind the reader that the ratio of the classical electron radius (aka Thomson or Lorentz radius) and the *zbw* radius (i.e. the Compton radius $r_c \approx 386{\times}10^{-15}$ m) is equal to the fine-structure constant $\alpha \approx 1/137 \approx 0.0073$, and that we can use the α ratio once again to get the size of the electron orbital (the Bohr radius of the hydrogen atom): $a_0 = r_{Bohr} = (386/\alpha){\times}10^{-15}$ m $\approx 53{\times}10^{-12}$ m. This ratio (the fine-structure constant α) between these three radii is arguably the most intriguing geometric relation in all of physics.

$$\theta = \frac{1}{\hbar}(\mathrm{E}_v t - \mathrm{p}x) = \frac{1}{\hbar}\left(\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}t - \frac{\mathrm{E}_0 v}{c^2\sqrt{1-\frac{v^2}{c^2}}}x\right) = \frac{1}{\hbar}\mathrm{E}_0\left(\frac{t}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{\mathrm{E}_0}{\hbar}t'$$

This well-known relativistic invariance of the argument of the wavefunction[2] makes one think that the wavefunction might be more real – in a *physical* sense, that is – than the various wave equations (Schrödinger, Dirac, Klein-Gordon) for which it is some solution.

However, should we equate the x and t variables with what we think might be the *actual* position, in space and in time, of our electron? It is surely *not* the standard interpretation of the wavefunction. In the standard interpretation, we will think of the (elementary) wavefunction $a \cdot e^{i\theta} = a \cdot \cos\theta + i \cdot a \cdot \sin\theta$ as a function from some *domain* $(\Delta\boldsymbol{x}, \Delta t)$ to an associated *range* of values $a \cdot e^{i\theta}$.

For example, if we limit Δt to *one* value only and, for simplicity, we reduce space to some line, then the domain reduces to $\Delta\boldsymbol{x} = [x_1, x_2]$ and $\Delta t = t_0$) and we will think of the absolute square of the wavefunction $|a \cdot e^{i\theta}|^2 = a^2$ as the probability *density* to find our particle at some point x in the $\Delta\boldsymbol{x} = [x_1, x_2]$ interval, as illustrated below.

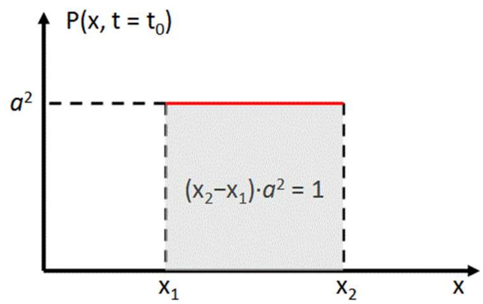

Figure 1: The standard interpretation of the elementary wavefunction

The interpretation assumes a *normalization* of the wavefunction. Normalization is a *mathematical* operation: it ensures the probabilities add up to one.[3] It does not answer Einstein's (or Schrödinger's) question to Born and Heisenberg: if Nature is probabilistic, what explains the probabilities? Any attempt to answer that question would probably want to relate the probability of actually finding the particle in some volume to the energy density in the volume. The energy of an oscillation is always proportional to the square of its amplitude. It, therefore, makes sense to write:

$$P \propto |\psi|^2 = |ae^{-i\theta}|^2 = |a|^2|\cos\theta - i\sin\theta|^2 = a^2(\cos^2\theta + \sin^2\theta) = a^2$$

Note that we took care to *not* write $\mathrm{P} = |\psi^2|$. There must a proportionality factor. What could it be? The energy will also be proportional to the square of the frequency. We write:

$$\mathrm{E} \propto a^2 \cdot \omega^2$$

What else could enter the equation? As we cannot think of anything else, we may write the proportionality relation as:

[2] E_0 is, obviously, the rest energy and, because p' = 0 in the reference frame of the electron, the argument of the wavefunction effectively reduces to $\mathrm{E}_0 t'/\hbar$ in the reference frame of the electron itself.

[3] The language is sometimes somewhat sloppy. Probabilities and probability densities are two different mathematical concepts. A probability is related to an *interval*. In contrast, the probability density is, effectively related to a specific *point* in spacetime. To calculate a probability, one should integrate probability *densities* over some larger or smaller interval. Hence, we should write the probability of finding a particle in some interval dx as $P \cdot dx$.

$$\mathrm{E} = m \cdot a^2 \cdot \omega^2$$

The m in this equation is just a proportionality coefficient. It is *not* a mass concept. Not yet, that is. Indeed, the structural equivalence between the formula above and Einstein's $\mathrm{E} = \mathrm{m}c^2$ equation makes us wonder: could m and m the same? An obvious implication of the Einstein's $\mathrm{E} = \mathrm{m}c^2$ equation is that the ratio between the energy and the mass of *any* particle is always equal to c^2:

$$\frac{E_{electron}}{m_{electron}} = \frac{E_{proton}}{m_{proton}} = \frac{E_{photon}}{m_{photo}} = \frac{E_{any\ particle}}{m_{any\ particle}} = c^2$$

Would it make sense to equate c^2 and $a^2\omega^2$? The physical dimensions are the same: $[a^2\omega^2] = [a^2\omega^2] = \mathrm{m}^2/\mathrm{s}^2$. Mass is a measure of inertia and, hence, the [E/m] dimensions works out too: $[\mathrm{E/m}] = (\mathrm{Nm})/(\mathrm{Ns^2/m}) = \mathrm{m}^2/\mathrm{s}^2$. This is interesting. If the m in the $\mathrm{E} = \mathrm{m}c^2$ and $\mathrm{E} = ma^2\omega^2$ are not the same, then they should only differ because of some scaling constant. Some *absolute* number – a number like the fine-structure constant, for example. Of course, we think m and m are, effectively, one and the same thing, but we will rest our case for the time being and first re-explore elementary wavefunction math.

Let us forget the standard interpretation of the elementary wavefunction for a while to explore the idea that the real and imaginary part of the elementary wavefunction $a \cdot e^{i\theta}$ might, perhaps, represent a *real* oscillation in space. To be precise, we will want to think of them as *two* orthogonal oscillations driven by the same function but with a phase difference of 90 degrees, as visualized below: the combination of the sinuisoidal and cosinusoidal motion makes the green dot go around in a circle.[4]

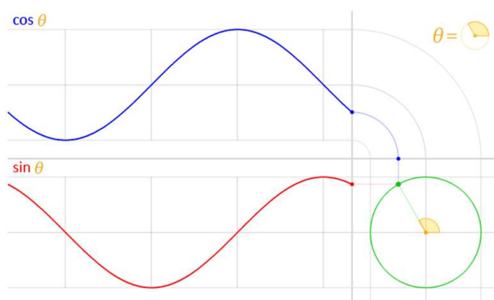

Figure 2: Euler's $e^{i\theta} = \cos\theta + i \cdot \sin\theta$ formula

In the *Zitterbewegung* model of an electron, we will be thinking of the green dot as a pointlike charge. However, if this two-dimensional oscillation is driven by an actual *force* on our charge, then we need to develop a *dual* view of what might be happening here. On the one hand, we can use the complex exponential (the elementary wavefunction, that is) to describe the motion of the green dot, which is the pointlike charge in the *zbw* model. In that case, we have two position variables x and y, whose physical dimension is just the distance unit (*meter*), and we write:

$$a \cdot e^{i\theta} = x + i \cdot y \text{ with } x = a \cdot \cos(\omega t) \text{ and } y = a \cdot \sin(\omega t)$$

Of course, we can also think of a position *vector* $\boldsymbol{r} = \boldsymbol{x} + \boldsymbol{y}$ here. Hence, we can represent $a \cdot e^{i\theta}$ as a point *or* as a vector.[5] We write:

$$\boldsymbol{r} \equiv \boldsymbol{x} + \boldsymbol{y} \equiv a \cdot e^{i\theta} \equiv (x, y) \equiv x + i \cdot y$$

[4] Papers do not do animations, unfortunately. For the animation, see https://en.wikipedia.org/wiki/Sine#/media/File:Circle_cos_sin.gif. For its source, see: https://commons.wikimedia.org/wiki/User:LucasVB.

[5] Boldface notation (e.g. $\boldsymbol{r} = \boldsymbol{x} + \boldsymbol{y}$) denotes a vector, i.e. an object with a *magnitude* $r = |\,\boldsymbol{r}\,|$ and, importantly, a *direction* in three-dimensional space.

But, as mentioned, if we think the oscillation is driven by some real force, then there is a *dual* view. Indeed, if the motion driven by two orthogonal (oscillating) fields, then we should associate some *force* with these fields. We have two obvious candidates for the *physical* dimension of the field here: force per unit mass (N/kg), or force per unit charge (N/C). The *zbw* model assumes a pointlike charge with no internal structure and, therefore, no mechanical mass. Hence, the force can only grab onto the charge and must, therefore, be electromagnetic in nature. We should, therefore, write something like this:

$$\boldsymbol{E}_0 = \boldsymbol{E}_x + \boldsymbol{E}_y = E_0 \cdot e^{i\theta} = E_0 \cdot \cos(\omega t) + i \cdot E_0 \cdot \sin(\omega t) = E_x + i \cdot E_y$$

This is familiar to us because it is the geometric representation of a circularly polarized electromagnetic: a rotating electric field vector ($\boldsymbol{E}$) which is analyzed as the sum of two orthogonal components: $\boldsymbol{E} = \boldsymbol{E}_x + \boldsymbol{E}_y$.[6]

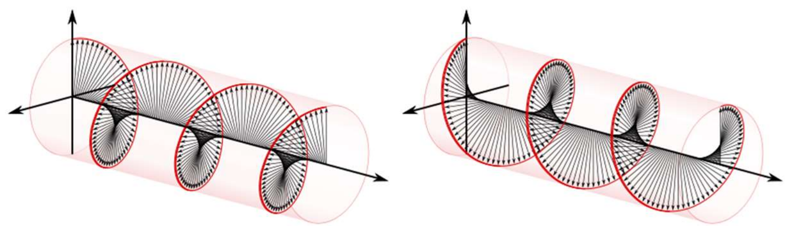

Figure 3: Left- and right-handed polarization[7]

However, there are a few issues here, which may or may not be difficult to deal with. First, if the motion is driven by an oscillating field, then the position and field vectors $\boldsymbol{r}$ and E_0 should be orthogonal to one another. To be specific, and exploiting the convention that a multiplication by the imaginary unit amounts to a rotation of 90 degrees in the counterclockwise direction, we write: $\boldsymbol{E}_0 = i \cdot \boldsymbol{r}$.[8] This is illustrated below. It should be noted that the mentioned convention establishes some absolute space: the notion of clockwise and counterclockwise implies a viewpoint – a line of sight between the subject and the object.[9]

[6] One should not confuse the electric field vector $\boldsymbol{E}$ with the energy E. Boldface is used to denote a vector. Of course, there is scope for confusion because we write E to denote the magnitude of the electric field. We cannot use the subscript (E_0 or E_0) to distinguish these concepts because we will use the subscript to (also) denote the rest energy (E_0). However, we will try to consistently use italics (or not). In any case, the context should make clear what we are talking about.

[7] Credit: https://commons.wikimedia.org/wiki/User:Dave3457.

[8] We make abstraction of the fact that E and r are measured in very different physical units here: force per unit charge (N/C) versus distance.

[9] It is a true two-dimensional *line* of sight because it has two directions: the observer is either in front of what is being observed or, else, he or she is looking at the object from behind. In the latter case, a clockwise rotation becomes a counterclockwise rotation, and vice versa. The reader may wonder why we would bother to mention this, but it is *not* a minor point: it resolves the issue of the weird 720-degree symmetry of the wavefunction – which we will discuss in a later section of this paper.

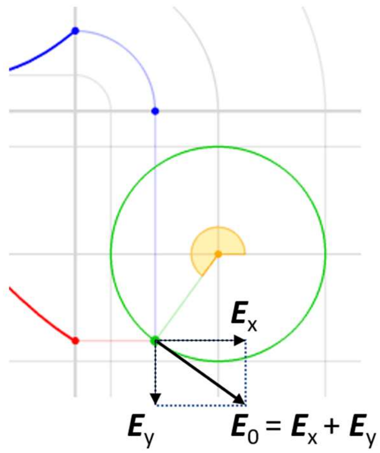

Figure 4: Orthogonal field and position vectors

Hence, the field vector $\boldsymbol{E}_0 = i \cdot \boldsymbol{r}$ can be written as[10]:

$$\boldsymbol{E}_0 = a \cdot i \cdot \cos(\omega t) + a \cdot i^2 \cdot \sin(\omega t) = a \cdot \cos(\omega t + \pi/2) - a \cdot i \sin(\pi/2 - \omega t)$$

$$= a \cdot \cos(\omega t + \pi/2) + i \cdot a \cdot \sin(\omega t + \pi/2)$$

The attentive reader should immediately note a mistake here: for no reason whatsoever, we assumed that the *numerical* value of the (maximum) amplitude of the field (E_0) would be equal to the (maximum) amplitude of the *physical* oscillation (a). There is no obvious reason for that and we should, therefore, correct the mistake and write our field vector $\boldsymbol{E}_0$ as:

$$\boldsymbol{E}_0 = E_0 \cdot \cos(\omega t + \pi/2) + i \cdot E_0 \cdot \sin(\omega t + \pi/2)$$

Of course, we will want to relate the two amplitudes (note that E_0 is expressed in N/C units, while a is just a simple distance). In Section 5, we will show we can do this because the energy in the two oscillations should, obviously, be the same. We will show there that we get the following elegant formula for the *force* (F) on the charge (q_e):

$$F = q_e E_0 = \frac{mc^2}{2\pi a} = \frac{\mathrm{E}}{\lambda_e}$$

This is a very elegant formula which we should probably relate to the results of the Compton scattering experiments, which use light of a similar wavelength. We will come back to this. As for now, we should note that we associate *one* wavefunction with *two* very different things: the motion of the charge, and the oscillation of the field. However, their frequency is the same. It is given by the Planck-Einstein relation: $\omega = \mathrm{E}/\hbar$.

This solves our problem in regard to m and m. The $c^2 = a^2\omega^2$ identity suggests we can, somehow, distribute the energy over the amplitude and the frequency. It suggests, for example, that we could have an oscillation with twice the frequency but half of the amplitude (which would give it the same energy), but we cannot. The Planck-Einstein relation gives us the frequency. The obvious question is: why can we use the Planck-Einstein relation in this context? The answer is equally obvious: because the fundamental *nature* of the oscillation is electromagnetic. We may, therefore, propose a very elegant derivation of the *Zitterbewegung* radius:

$$\mathrm{E} = \hbar\omega = ma^2\omega^2 \Leftrightarrow a^2 = \frac{\hbar}{m\omega} = \frac{\hbar^2}{m \cdot mc^2} \Leftrightarrow a = \frac{\hbar}{mc}$$

[10] See: http://mathonweb.com/help_ebook/html/trigids.htm.

The derivation answers a typical question of a freshman in physics: the energy of an oscillation is always proportional to the *square* of its amplitude, so why is there no *square* in the $E = \hbar\omega$ formula? The answer is: the energy of an electromagnetic oscillation is proportional to both ω and ω^2. We just have a different proportionality constant in the two formulas: $\hbar$ versus $\mathrm{m}a^2$.

We have presented the basics of the model, but the skeptical reader will probably need more convincing. The next sections are intended to walk him or her through the model at a much more leisurely pace. They will also explore some conceptual issues which we have not dealt with. These issues include the following:

1. The model assumes a force field, but the line of action of the force is the oscillating charge. Hence, there is no real center for the oscillation.
2. What about the magnetic force? The moving charge should generate a magnetic field.
3. More generally, the moving charge should radiate the energy away.

These questions are valid concerns, and we do not have any definite answer to them. However, the remarks below may help the reader to develop his or her own view on them. Before getting into the nitty-gritty of it all, we will first remind the reader of the history of the *Zitterbewegung* hypothesis.

3 Schrödinger's Zitterbewegung

The equations for the relativistic transformation of the space and time coordinates in the argument of the wavefunction – which show its argument is invariant – assume the relativistically correct definition of energy and momentum is used: $E = \mathrm{m}c^2 = \gamma \mathrm{m}_0 c^2$ and $p = \mathrm{m}v = \gamma \mathrm{m}_0 v$. Schrödinger's (non-relativistic) wave equation does *not* use the relativistically correct energy equation. Instead, it incorporates kinetic energy only[11] and, therefore, excludes most of the energy of our electron, which is the rest mass. However, Schrödinger also explored solutions to Dirac's wave equation for free electrons, whose energy operator does include the rest energy. These explorations led to the discovery of the *Zitterbewegung*: a local very high-frequency oscillatory motion of the electron.[12] In 1933, he shared the Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth quoting Dirac's summary of Schrödinger's discovery:

> "The variables [in Dirac's wave equation] give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."[13]

Hestenes (1990, 2008) and Celani et al. (2017) may be credited with a revival of what is sometimes referred as the *Zitterbewegung* interpretation of quantum mechanics. The idea is visualized in the illustration below , which depicts an *accelerating* electron reaching relativistic speeds: as it is assumed that the velocity of the pointlike charge cannot exceed the speed of light, the *radius* of the circulatory motion must effectively diminish as the electron gains speed.

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[11] Of course, it may also include potential energy because of the presence of a positively charged nucleus, but we will not consider this for the moment.
[12] The term can be translated as a shaking or trembling motion. It is often abbreviated as *zbw* or just *Zitter*.
[13] See: Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf.

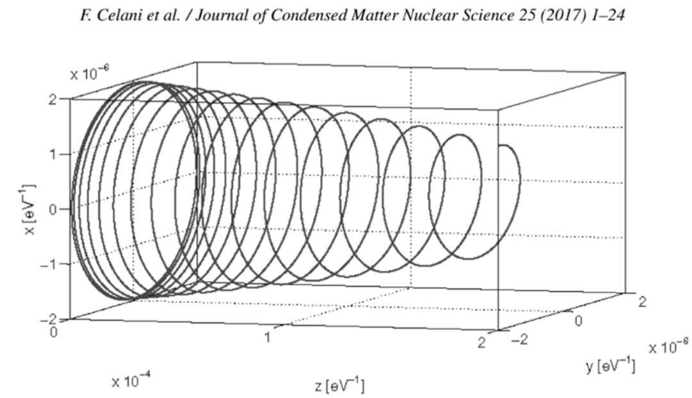

Figure 5: The *presumed* Zitterbewegung (*zbw*) of an electron[14]

The rather peculiar length scale (1/eV) in the illustration above is based on the E = hf = hc/λ formula for electromagnetic radiation. As such, it is a natural distance unit. However, one might – and probably *should* – make the case for considering the radius of the circulatory motion itself as a natural unit, as it is equal to the reduced Compton wavelength ħ/mc ≈ 386×10^{-15} m = 0.386 pm (picometer). Hence, the *circumference* of the loop corresponds to the non-reduced Compton wavelength, which is equal to λ_e = h/mc ≈ 2.452 pm. This is an extremely small distance: it is the order of magnitude of the wavelength of hard gamma rays. Also, because the angular velocity of the pointlike charge is equal to the speed of light, the angular frequency of the rotation is an equally astronomic number: ω_e = E/ħ ≈ 0.776×10^{-21} rad/s. These are, effectively, values that cannot be measured. The model is, therefore, theoretical only. Why, then, is there so much interest in the model?

As Dirac already notes, the idea of the *Zitterbewegung* is very intuitive – and, therefore, very attractive – because it seems to give us a geometric (or, we might say, *physical*) explanation of the (reduced) Compton wavelength as the Compton *scattering radius* of an electron (a = ħ/mc).[15] However, if we think of an actual physical interpretation, then it is quite obvious that the suggested plane of circulatory motion is *not* consistent with the measured direction of the magnetic moment – which, as the Stern-Gerlach experiment has shown us, is either up or down. Hence, we may want to think the plane of oscillation might be parallel to the direction of propagation, as drawn below.

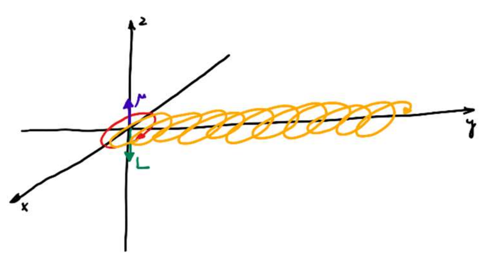

Figure 6: The *actual* Zitterbewegung of an electron?

[14] Source: Francesco Celani et al., *The Electron and Occam's Razor*, November 2017, https://www.researchgate.net/publication/320274514_The_Electron_and_Occam's_Razor.
[15] The term physical (or geometric) explanation is used here as a contrast to the merely mathematical proof (based on the principles of the conservation of energy and momentum) of the Compton scattering formula which, we may remind the reader, is equal to λ'-λ=h/mc (1-cosθ).

We should add that the derivation of the presumed *Zitterbewegung* as a mathematical solution to a wave equation is also not very intuitive: it does not give us an explanation for its *nature*. This paper aims to present a simpler and more intuitive model.

4. $\mathbf{E = m}\boldsymbol{c}^2 \mathbf{= m}\boldsymbol{a}^2\boldsymbol{\omega}^2 = \hbar\boldsymbol{\omega}$

Combining (1) the idea of a pointlike *charge* and (2) the idea of an oscillation in two orthogonal directions – resulting in circular or rotational motion – one might be tempted to bluntly equate the $E = m \cdot c^2$ and $E = m \cdot a^2 \cdot \omega^2$ equations, also using the $\omega = E/\hbar = m \cdot c^2/\hbar$ equation. If we do so, we immediately get the radius for our *Zitterbewegung*, which – as mentioned above – is equal to the (reduced) Compton wavelength:

$$E = m \cdot c^2 = m \cdot a^2 \cdot \omega^2 = m \cdot a^2 \cdot (E/\hbar)^2 \Leftrightarrow a = \hbar/mc$$

The $m \cdot a^2 \cdot \omega^2$ adds the kinetic and potential energy of *two* oscillators working in tandem as a perpetuum mobile.[16] The $E = m \cdot a^2 \cdot \omega^2 = \hbar\omega$ equation follows naturally from the $c^2 = a^2 \cdot \omega^2$ identity. Thinking of the *Zitterbewegung* as being caused by some real oscillation in two dimensions, we also get an intuitive explanation of why the effective mass of our electron (m_e) will be spread over a disk, as opposed to viewing the electron as a current ring only. To be precise, *the effective mass of our electron is explained here as the equivalent mass of the energy in this two-dimensional oscillation.*

We can now use the correct *form factor* for the angular momentum formula. It *must* be 1/2, because we do think of the effective mass of our electron as being spread over a disk. Hence, we get the value we would want to get for a spin-1/2 particle:

$$L = I \cdot \omega = \frac{ma^2}{2}\frac{c}{a} = \frac{mc}{2}\frac{\hbar}{mc} = \frac{\hbar}{2}$$

Of course, the model also yields the correct value for the magnetic moment. Indeed, if our pointlike charge is, effectively, going around in a loop, the effective current will be equal to the charge (q_e) divided by the period (T) of the orbital revolution: $I = q_e/T$. The period of the orbit is the time that is needed for the electron to complete one loop, so T is equal to the circumference of the loop ($2\pi \cdot a$) divided by the tangential velocity (v). Using our results, we should substitute v for c and a for the Compton radius $a = \hbar/(m \cdot c)$. The formula for the area is $A = \pi \cdot a^2$ and, hence, we get:

$$\mu = I \cdot A = (q_e/T) \cdot (\pi \cdot a^2) = [(q_e \cdot c)/(2\pi \cdot a)] \cdot (\pi \cdot a^2) = [(q_e \cdot c)/2] \cdot a = [(q_e \cdot c)/2] \cdot [\hbar/(m \cdot c)]$$

$$= (q_e/2m) \cdot \hbar$$

This is great, because we explained the mysterious g-factor for the pure spin moment of an electron: 2. Moreover, we did so without having to invoke the notion of (Larmor) precession.[17]

Last but not least, the explanation above offers an intuitive understanding of Einstein's enigmatic mass-energy equivalence relation ($E = m \cdot c^2$) which – in our humble view – is the single biggest advantage of this rather simplistic model. Of course, we have used the *rest* mass of the electron. This is where the exact wording of Dirac's description of the Zitterbewegung becomes very relevant: a very high frequency motion (of small amplitude) which is *superposed* on the regular motion. The *regular* motion is given by the kinetic

[16] We presented the metaphor of a V-2 engine, or springs that are connected to a crankshaft in a 90-degree angle in previous papers and, hence, we will not repeat ourselves here. See: http://vixra.org/author/jean_louis_van_belle.

[17] See Feynman (*Lectures*, II-34-2) for a good conceptual discussion of the g-factor in classical and quantum mechanics. The reader will know quantum physicists calculate a slightly different g-factor (about 2.0023193) but that is because they use a formula based on the fine-structure constant. In other words, their model is "deep down in relativistic quantum mechanics", as Feynman would put it, and, therefore, not relevant in the context of the discussion here.

energy and/or the *momentum* of the electron. Can we combine them in a single wavefunction – or a single wave equation?

5. Spacetime elasticity: c^2 as the defining property of spacetime

The $E = m \cdot a^2 \cdot \omega^2$ is intuitive: the energy of any oscillation will be proportional to the square of (1) the (maximum) amplitude of the oscillation and (2) the frequency of the oscillation, and the mass appears as the proportionality coefficient. But what does it mean?

When everything is said and done, we should admit that the bold $c^2 = a^2 \cdot \omega^2$ assumption interprets spacetime as a relativistic aether – a term that is taboo but that is advocated by Nobel Prize Laureate Robert Laughlin[18]. It is inspired by the most obvious implication of Einstein's $E = mc^2$ equation, and that is that the ratio between the energy and the mass of *any* particle is always equal to c^2:

$$\frac{E_{electron}}{m_{electron}} = \frac{E_{proton}}{m_{proton}} = \frac{E_{photo}}{m_{photo}} = \frac{E_{any\ particle}}{m_{any\ particle}} = c^2$$

This reminds us of the $\omega^2 = C^{-1}/L$ or $\omega^2 = k/m$ of harmonic oscillators[19] – with one key difference, however: the $\omega^2 = C^{-1}/L$ and $\omega^2 = k/m$ formulas introduce *two* (or more[20]) degrees of freedom. In contrast, $c^2 = E/m$ for *any* particle, *always*. The reader will probably say: so what? However, this is the point that we are making here: we can modulate the resistance, inductance and capacitance of electric circuits, and the stiffness of springs and the masses we put on them, but we live in *one* physical space only: *our* spacetime. Hence, the speed of light c emerges here as *the* defining property of spacetime. It is, in fact, tempting to think of it as some kind of *resonant frequency* but the $c^2 = a^2 \cdot \omega^2$ hypothesis tells us it defines both the frequency as well as the amplitude of what we will now refer to as *the rest matter oscillation*.[21] The energy state of the particle gives us the frequency through the $\omega = E/\hbar = m \cdot c^2/\hbar$ equation, and then we get the amplitude (or the *radius of the oscillation*, we should say) from the $a = c/\omega$ identity, which is just the formula for the angular velocity ($c = a \cdot \omega$).

The obvious question is: what is the *nature* of this two-dimensional oscillation? If the two perpendicular (and, therefore, independent) oscillations are *real*, then they should be driven by some real *field*. What field? What physical dimension would it have? There are two obvious candidates:

1. Force per unit charge (*coulomb*), which is the physical dimension of the electric field.
2. Force (*newton*) per unit mass (*kg*), which is the physical dimension of a gravitational field – which reduces to the physical dimension of an acceleration ($N/kg = m/s^2$).

When we first started thinking about this model, we briefly entertained the latter idea.[22] However, it does not seem to be compatible with the *Zitterbewegung* model: the force needs something to grab onto and, in

[18] Robert Laughlin (2005), as quoted in the Wikipedia article on aether theories (https://en.wikipedia.org/wiki/Aether_theories).
[19] The $\omega^2 = 1/LC$ formula gives us the natural or resonant frequency for an electric circuit consisting of a resistor (R), an inductor (L), and a capacitor (C). Writing the formula as $\omega^2 = C^{-1}/L$ introduces the concept of elastance, which is the equivalent of the mechanical stiffness (k) of a spring. Needless to say, the k in the $\omega^2 = k/m$ equation is the mechanical stiffness which has, obviously, nothing to do with wavenumber $k = p/\hbar$ in the wavefunction.
[20] We will usually include a resistance in an electric circuit to introduce a damping factor. Also, when analyzing a mechanical spring, one may also want to introduce a drag coefficient. Both are usually defined as a fraction of the *inertia*, which is the mass for a spring and the inductance for an electric circuit. Hence, we would write the resistance for a spring as γm and as $R = \gamma L$ respectively.
[21] It is, of course, the *Zitterbewegung* idea but, importantly, complemented with the idea of a two-dimensional oscillation.
[22] See: Jean Louis Van Belle, 22 October 2017, *The Quantum-Mechanical Wavefunction as a Gravitational Wave*, http://vixra.org/abs/1709.0390.

this model, the only thing it can grab onto is a pointlike charge which has no internal structure and, therefore, no mechanical mass whatsoever. Hence, the remaining option is to think that our pointlike charge is effectively being driven by some weird two-dimensional electromagnetic field. In fact, that is not too difficult, because the geometric representation of a circularly polarized electromagnetic wave already offers us such picture: the rotating electric field vector E can be analyzed as the sum of two orthogonal components: $\boldsymbol{E} = \boldsymbol{E}_x + \boldsymbol{E}_y$.[23]

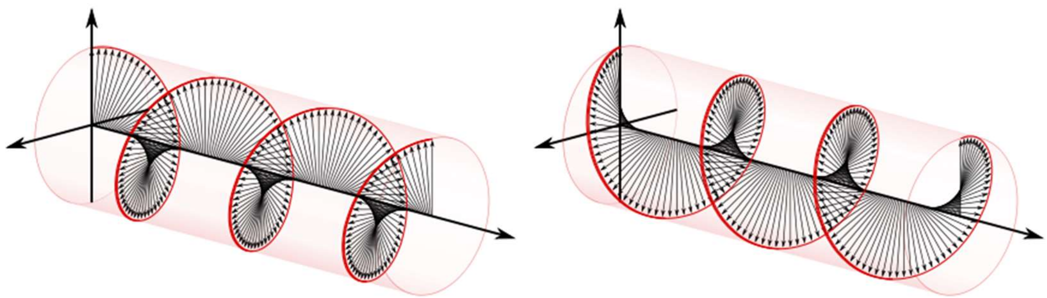

Figure 7: Left- and right-handed polarization[24]

If the motion is driven by an electromagnetic oscillation, then we should be able to relate the radius a to the amplitude of the electric field. How can we do that? The angular frequencies of the electromagnetic oscillation and the circular motion of our pointlike charge should be the same. Fortunately, we know that is the case. Referring to the results obtained in section 1, we write:

$$|\boldsymbol{E}_x| = E_x = E_0 \cdot \cos(\omega t + \pi/2) \text{ and } |\boldsymbol{E}_y| = E_y = E_0 \cdot \sin(\omega t + \pi/2), \text{ with } \omega = \mathrm{E}/\hbar$$

However, the *numerical value* of the amplitudes may differ. In fact, they are *very likely* to differ because we describe the *Zitterbewegung* in actual distance units:

$$x = a \cdot \cos(\omega t) \text{ and } y = a \cdot \sin(\omega t), \text{ with } a = \hbar/\mathrm{m}c \approx 386 \times 10^{-15}\,\mathrm{m}$$

In contrast, the magnitude (or amplitude) of the electric field (E_0, E_x or E_y) is measured in force per unit charge (N/C) units. How can we relate the two? We have a *dual* view of the reality of the wavefunction here:

1. The $x = a \cdot \cos(\omega t)$ and $y = a \cdot \sin(\omega t)$ equations describe the motion of our pointlike charge.
2. As electromagnetic oscillation, we write it as $E_0 = E_x + i \cdot E_y = E_0 \cdot \cos(\omega t + \pi/2) + i \cdot E_0 \cdot \sin(\omega t + \pi/2)$.

The magnitudes of the oscillation a and E_0 are expressed in distance (m) and force per unit charge (N/C) respectively and must be related because the energy of both oscillations is one and the same (E = ħω). Now, we know the energy *density* in an electromagnetic oscillation is given by:

$$u = \frac{\epsilon_0}{2}\boldsymbol{E} \cdot \boldsymbol{E} + \frac{\epsilon_0 \cdot c^2}{2}\boldsymbol{B} \cdot \boldsymbol{B} = \frac{\epsilon_0}{2}E^2 + \frac{\epsilon_0 \cdot c^2}{2}B^2 = \frac{\epsilon_0}{2}E^2 + \frac{\epsilon_0 \cdot c^2}{2}\frac{E^2}{c^2} = \epsilon_0 \cdot E^2$$

It might be useful to remind ourselves of the energy density unit: E^2 – which we will write as E_0^2 to avoid confusion with the E for energy – is expressed in $\mathrm{N}^2/\mathrm{C}^2$, while ε_0 is expressed in $\mathrm{C}^2/\ \mathrm{N} \cdot \mathrm{m}^2$. Hence, we get the expected *force per unit area* unit ($\mathrm{N/m}^2$).[25] To get the total energy, we need to integrate this – but over what? A point, a line, a volume?

[23] One should not confuse the electric field vector $\boldsymbol{E}$ with the energy E. Boldface is used to denote a vector. Of course, there is more scope for confusion when we will use E to denote the magnitude of the electric field, which we will do shortly. The context should make clear what we are talking about.
[24] Credit: https://commons.wikimedia.org/wiki/User:Dave3457.
[25] Multiplication by m/m gives us joule (or energy) per unit volume.

This is where the above-mentioned conceptual issue arises: the model assumes a force field, but the line of action of the force is the oscillating charge. Hence, there is no real center for the oscillation. It is not easy to answer this question. In fact, perhaps we cannot. But we can make some remarks which may or may not help us to think it through.

First, we should note the concept of an energy density itself is somewhat ambiguous: we define the energy density at a zero-dimensional point, but its physical dimension is force per unit *area*. As such, it resembles the concept of a probability density: we need to integrate it over a line, or over some volume, in order to get the more meaningful concept of a probability.[26] Hence, the following may or may not make sense. If the force needs to grab onto the charge and, hence, the line of action of the force is the current loop. Now, the force is the field times the charge ($F = q_e \cdot E_0$), and the circumference is $2\pi \cdot a$. Hence, the work done (or the energy) over one cycle is equal to:

$$W = F \cdot 2\pi \cdot a = 2\pi \cdot q_e \cdot E_0 \cdot a$$

We know need to think about the energy concepts (kinetic and potential) that are associated with a harmonic oscillator. We know the energy in such oscillator is constant. However, we also know that it is the *sum* of the kinetic and potential energy that is constant: over one cycle, the kinetic energy will go from 0 to its maximum and then back to zero, while the potential energy will go from its maximum value to zero, and the back to its maximum value, as shown below.

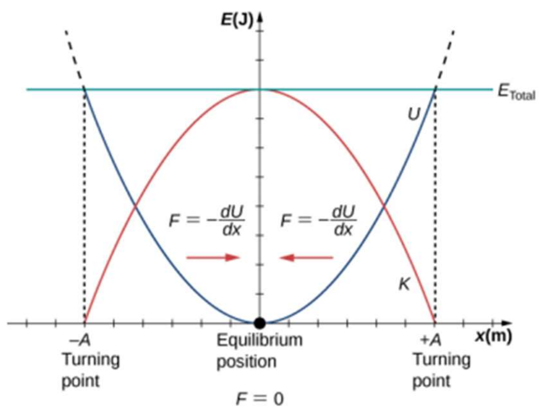

Figure 8: Kinetic (T) and potential energy (U) in a harmonic oscillator[27]

Now, we also know that the kinetic energy and potential energy will vary with the *square* of the sine or cosine function that describes the motion[28], and that the average value of the squared sine (or cosine) is equal to 1/2.[29] To make a long story short, one can show that the energy *that will be expended over one cycle* will be equal to the energy of the oscillator. We can, therefore, equate W and $E = mc^2$ to get the following value for E_0:

$$2\pi a q_e E_0 = mc^2 \Leftrightarrow E_0 = \frac{mc^2}{q_e}\frac{mc}{2\pi\hbar} = \frac{m^2c^3}{2\pi q_e \hbar}$$

---

[26] The probability of finding a particle at a point is zero because the volume of a point ($d\boldsymbol{x}$) is zero. This is why we say the concept of a probability is more meaningful than the concept of a probability density.

[27]

[28] In case the reader would want to see some proof here, we can refer him to section 6, where we develop the math associated with a two-dimensional oscillator.

[29] One can easily show this by taking the average value of the $\cos^2(\omega t) + \sin^2(\omega t) = 1$ identity, as the average value of both functions over a full cycle should be the same.

The reader will be able to verify that a dimensional analysis of this formula makes sense, and that we also get the same formula when using the $E = ma^2\omega^2$ formula. The formula looks rather horrible but makes more sense if we write it in terms of the (constant) force that is acting on the pointlike charge:

$$F = q_e E_0 = \frac{m^2c^3}{2\pi\hbar} = \frac{mc}{\hbar}\frac{mc^2}{2\pi} = \frac{mc^2}{2\pi a} = \frac{\mathrm{E}}{\lambda_\mathrm{e}}$$

It is an elegant formula: the force is the ratio of the energy and the Compton wavelength of the electron, but what does it mean? We are not so sure. It must, somehow, explain the nitty-gritty of the Compton scattering experiments – which use light of a similar wavelength – but we need to work this out. It is interesting to calculate its actual value:

$$F = q_e E_0 = \frac{\mathrm{E}}{\lambda_\mathrm{e}} \approx \frac{8.187 \times 10^{-14}\ \mathrm{J}}{2.246\ \times 10^{-12}\ \mathrm{m}} \approx 3.3743 \times 10^{-2}\mathrm{N}$$

This force is equivalent to a force that gives a mass of about 37.5 *gram* (1 g = 10^{-3} kg) an acceleration of 1 m/s per second. In light of the distance scale, this force is *huge* and, as such, we would need to think through the implications in terms of the distortion of spacetime caused by the presence of such energy in such tiny volume. However, this would require a study of general relativity theory and is, therefore, outside of the scope of this introductory paper.

6. Occam's Razor and the wavefunction

We will make a small but necessary digression in this section. We wrote the elementary wavefunction as $a \cdot e^{i\theta} = a \cdot (cos\theta + i \cdot sin\theta) = a \cdot cos\theta + i \cdot a \cdot cos(\pi/2 - \theta) = a \cdot cos\theta + i \cdot a \cdot cos(\theta - \pi/2)$ above. A minus sign in front of our $\exp(i\theta)$ function – or, what amounts to the same, taking the complex conjugate – reverses the direction of the oscillation:

$$-\psi = -\exp(i\theta) = -(cos\theta + i \cdot sin\theta) = cos(-\theta) + i \cdot sin(-\theta) = \exp(-i\theta) = \psi^*$$

This is obvious. However, we would like to note that we started out with a wavefunction that does *not* respect the usual convention: physicists usually write ψ as $a \cdot e^{-i\theta}$. Just like Celani et al., we would like to invoke Occam's Razor, which tells us that we should not have any redundancy in a theoretical explanation, and the above-mentioned convention may, effectively, be analyzed as a redundancy. Indeed, most introductory courses in quantum mechanics will show that only $\psi = \exp(i\theta) = \exp[i(\mathrm{kx}-\omega\mathrm{t})]$ *or* $\psi = \exp(-i\theta) = \exp[-i(\mathrm{kx}-\omega\mathrm{t})] = \exp[i(\omega\mathrm{t}-\mathrm{kx})]$ would be acceptable waveforms for a particle that is propagating in the x-direction – as opposed to, say, some real-valued sinusoid. We would then think some proof should follow of why one would be better than the other, or some discussion on why they might be different, but that is not the case. The professor usually concludes that "the choice is a matter of convention" and, that "happily, most physicists use the same convention."[30]

This is very surprising because we *know*, from *experience*, that theoretical or mathematical possibilities in quantum mechanics often turn out to represent real things. Here we should think of the experimental verification of the existence of the positron (or of anti-matter in general) after Dirac had predicted its existence based on the mathematical possibility only. So why would that *not* be the case here? *Occam's Razor* tells us that we should not have any redundancy in the description. Hence, if there is a physical interpretation of the wavefunction, then we should not have to choose between the two mathematical possibilities: they would represent two different physical situations. Of course, the only characteristic that can make the difference here would be spin. Hence, we would *not* agree with the mainstream view that "the choice is a matter of convention" and that "happily, most physicists use the same convention"[31] but, instead, dare to suggest that the two mathematical possibilities may represent identical particles with

[30] See, for example, the MIT's edX Course 8.04.1x, Lecture Notes, Chapter 4, Section 3.

[31] See the reference above.

opposite spin (i.e. *real* spin-1/2 particles as opposed to non-existing spin-zero particles), in which case we get the following table.

Spin and direction of travel	**Spin up ($J = +\hbar/2$)**	**Spin down ($J = -\hbar/2$)**
Positive *x*-direction	$\psi = \exp[i(kx-\omega t)]$	$\psi^* = \exp[-i(kx-\omega t)] = \exp[i(\omega t-kx)]$
Negative *x*-direction	$\chi = \exp[-i(kx+\omega t)] = \exp[i(\omega t-kx)]$	$\chi^* = \exp[i(kx+\omega t)]$

Figure 9: Occam's Razor: mathematical possibilities versus physical realities

Of course, the reader will wonder why this point should matter. The answer is that the redundancy in the description is directly related to the logic which leads us to the rather uncomfortable conclusion that the wavefunction of spin-1/2 particles have a 720-degree symmetry in space. This conclusion is uncomfortable because we cannot imagine such objects in space without invoking the idea of some kind of relation between the subject and the object (the reader should think of the Dirac belt trick here), which we want to avoid. We have written at length about this and other objections to a geometric interpretation of the wavefunction before, so we will just refer the reader there.[32]

To wrap up the paper, we will present the reader with a metaphor that may help him or her to consider this particular physical interpretation of the (elementary) wavefunction.

7. The metaphor: two-dimensional oscillators

The reader will be familiar with the formulas for the kinetic and potential energy for *one* oscillator: these energies add up to $ma^2\omega^2/2$. The visualization of Euler's formula shows that we can get rid of the 1/2 factor by thinking of an oscillation in two dimensions – provided we ensure a phase difference of 90 degrees between the two oscillators. Let us quickly go over the math.

If we refer to the two orthogonal dimensions as the *x* and *y* direction[33], then we can add the energies of both oscillations:

$$\begin{gathered} E_{total} = E_{kinetic} + E_{potential} = T + U = E_y + E_z = (T_y + U_y) + (T_z + U_z) \\ = m \cdot \omega^2 \cdot a^2 \cdot [\sin^2(\omega \cdot t + \Delta) + \cos^2(\omega \cdot t + \Delta)]/2 + m \cdot \omega^2 \cdot a^2 \cdot [\cos^2(\omega \cdot t + \Delta) + \sin^2(\omega \cdot t + \Delta)]/2 \\ = m \cdot a^2 \cdot \omega^2/2 + m \cdot a^2 \cdot \omega^2/2 = m \cdot a^2 \cdot \omega^2 \end{gathered}$$

To focus the mind, we may think of a metaphor: some *mechanism* which illustrates the principle of energy conservation. For example, we can think of a V-2 engine with the pistons at a 90-degree angle, as illustrated below. The 90° angle makes it possible to perfectly balance the counterweight and the pistons, thereby ensuring smooth travel – always.[34] With permanently closed valves, the air inside the cylinder compresses and decompresses as the pistons move up and down. It provides, therefore, a restoring force. As such, it will store potential energy, just like a spring. In fact, the motion of the pistons will also reflect that of a mass on

[32] Such objections usually also include the idea that the coefficient (a) of the wavefunction $a \cdot e^{i\theta}$ may be complex-valued, whereas in any *real* interpretation this (maximum) amplitude should be real-valued. This objection is also rejected. See: Jean Louis Van Belle, 30 October 2018, *Euler's wavefunction: the double life of −1*, http://vixra.org/abs/1810.0339.

[33] The *x*-direction would then be the direction of propagation of the wave. This follows the usual convention in quantum mechanics, according to which we will measure something (e.g. angular momentum) along the *z*-direction, which is perpendicular to the direction of propagation, i.e. the *x*-direction. The *y*-direction is then determined by the right-hand rule. We may say this establishes a reference frame that combines the object and the subject (the measurement apparatus).

[34] Motorbike lovers will know this is why a Ducati engine (which has the cylinders in a 90-degree angle) is more efficient than the 45-degree set-up of, say, a Harley-Davidson. In contrast, the irregular sound of a Harley is, obviously, less ordinary than that of an engine designed for efficiency and speed.

a spring: it is described by a sinusoidal function, with the zero point at the center of each cylinder. We can, therefore, think of the moving pistons as harmonic oscillators, just like mechanical springs.[35]

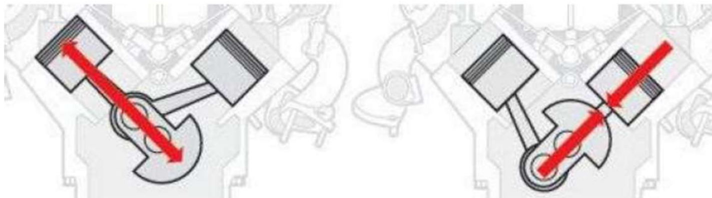

Figure 10: Propagation and energy conservation: the V-2 metaphor

Of course, instead of two cylinders with pistons, one may also think of connecting two springs with a crankshaft, and the analogy can also be extended to include two *pairs* of springs or pistons, in which case the springs or pistons in each pair would help drive each other. Making abstraction of friction and other worldy imperfections (such as the evacuation of heat), we have a *perpetuum mobile*. Somehow, in this beautiful interplay between linear and circular motion, energy is borrowed from one place and then returns to the other, cycle after cycle. While transferring kinetic energy from one piston to the other, the crankshaft will rotate with a constant angular velocity: linear motion becomes circular motion, and vice versa. Let us briefly review the math.

Because of the 90-degree angle between the two oscillators, their motion is given by $a \cdot \cos(\omega \cdot t + \Delta)$ and $a \cdot sin(\omega \cdot t + \Delta) = a \cdot cos(\omega \cdot t + \Delta - \pi/2)$ respectively.[36] The kinetic and potential energy of the first oscillator can be calculated as:

$$\text{K.E.} = T = m \cdot v^2/2 = (1/2) \cdot m \cdot \omega^2 \cdot a^2 \cdot sin^2(\omega \cdot t) = (1/2) \cdot m \cdot \omega^2 \cdot a^2 \cdot sin^2\theta$$
$$\text{P.E.} = U = k \cdot x^2/2 = (1/2) \cdot k \cdot a^2 \cdot cos^2(\omega \cdot t) = (1/2) \cdot m \cdot \omega^2 \cdot a^2 \cdot cos^2\theta$$

The coefficient k in the potential energy formula characterizes the restoring force: $F = -k \cdot x$. From the dynamics involved, it is obvious that k must be equal to $m \cdot \omega^2$, which is a fact we use in the formula. Hence, the total energy is equal to: $E = T + U = (1/2) \cdot m \cdot \omega^2 \cdot a^2 \cdot (sin^2\theta + cos^2\theta) = m \cdot a^2 \cdot \omega^2/2$.

For the second oscillator, we just switch *sin* for *cos* in the formulas, and vice versa. However, we get the same result when adding kinetic and potential energy. Hence, adding the *total* energy of the *two* oscillators, we have a perpetuum mobile storing an energy that is equal to twice this amount: $E = m \cdot a^2 \cdot \omega^2$. Indeed, it is easy to show this engine is, effectively, a perpetuum mobile. The (instantaneous) change of the kinetic energy of the first oscillator, as a function of the phase angle θ, will be equal to:

$$d(T_1)/d\theta = d[(1/2) \cdot m \cdot \omega^2 \cdot a^2 \cdot sin^2\theta]/d\theta = (1/2) \cdot 2 \cdot m \cdot \omega^2 \cdot a^2 \cdot sin\theta \cdot [d(sin\theta)/d\theta] = m \cdot \omega^2 \cdot a^2 \cdot sin\theta \cdot cos\theta$$

The motion of the second oscillator is given by the $a \cdot sin\theta$ function, and its kinetic energy is equal to $(1/2) \cdot m \cdot \omega^2 \cdot a^2 \cdot \cos^2\theta$. Hence, how it changes – as a function of θ – will be equal to:

$$d(T_2)/d\theta = d[(1/2) \cdot m \cdot \omega^2 \cdot a^2 \cdot cos^2\theta]/d\theta = (1/2) \cdot 2 \cdot m \cdot \omega^2 \cdot a^2 \cdot cos\theta \cdot [d(cos\theta)/d\theta] = -m \cdot \omega^2 \cdot a^2 \cdot sin\theta \cdot cos\theta$$

We can calculate the same energy conservation equation for the *potential* energies of both oscillators.

Of course, the attentive reader will immediately object we should use relativistically correct equations, but that can be done easily. The relativistically correct force equation for one oscillator is: F = dp/dt = F = –

[35] Instead of two cylinders with pistons, one may also think of connecting two springs with a crankshaft. The analogy can also be extended to include two *pairs* of springs or pistons. The two springs or pistons in each pair

[36] The phase factor Δ only depends on our zero point for time. We will assume our zero point is chosen such that $\Delta = 0$ and, hence, $\theta = \omega \cdot t$.

kx with p = mv = γm$_0v$. Multiplying both sides with v = dx/dt yields the following energy conservation expression:

$$v\frac{d(\gamma m_0 v)}{dt} = -kxv \Leftrightarrow \frac{d(m_v c^2)}{dt} = -\frac{d}{dt}\left[\frac{1}{2}kx^2\right] \Leftrightarrow \frac{dE}{dt} = \frac{d}{dt}\left[\frac{1}{2}kx^2 + m_v c^2\right] = 0$$

We recognize the potential energy (it is the same kx^2/2 formula). However, the m$_0v^2$/2 term that we would get when using the non-relativistic formulation of Newton's Law is now replaced by the mc^2 = γ · m$_0$ · c^2 term.

8. Electromagnetic mass: (not) the right concept?

The gymnastics above may have annoyed the reader. The blunt equation of the E = m · c^2 and E = m · a^2 · ω^2 equations lead to wonderful results, but are we actually using the same mass concept in both? The mass concept in the E = m · c^2 is, obviously, the *effective* mass of an electron. Hence, that is a measure of *inertia* to motion, and it has been measured to be equal to about 9.1×10^{-31} kg or – a more familiar measure – about 0.511 MeV/c^2.[37] In contrast, when using the E = m · a^2 · ω^2 formula, we think of some mass on a spring. Of course, the oscillator metaphor is just what it is, but we still need to answer the question: what is the mass concept here?

According to current *Zitterbewegung* theorists[38], it is an electromagnetic mass – which is also a measure of inertia, because an electric charge – with zero *mechanical* mass – will also resist motion. However, if so, then the electromagnetic mass should also increase with velocity, shouldn't it? As Feynman writes: "No matter what the origin of the mass, it all should vary as $m_0/\sqrt{1-v^2/c^2}$."[39] Hence, it should, therefore, be impossible for our pointlike charge to whizz around at the speed of light – which is what it does according to our model.

The objection is everything but philosophical. Can we say anything about it? We are not sure. Of course, we could – and should – note that the classical calculations of electromagnetic mass are based on the idea of assembling the elementary charge from an infinite number of infinitesimally small charges, which is an inconsistent idea. Why? Because the calculations blow up when we try to calculate the energy that is needed to assemble a true *pointlike* elementary charge – a charge with *zero* dimension, that is: our integral blows up, showing an *infinite* amount of energy would be needed for that. The easiest way to illustrate that is to just remind the reader of the formula for the energy of a charged sphere with some finite charge q and some radius r, which is equal to U = (1/2) · q^2/4πε$_0r$. It is clear that U goes to infinity as r goes to 0.[40] This problem is solved by assuming we do not have a true pointlike charge: if we compress all charge into a very tiny ball, whose radius is equal to the *classical* electron radius, then we get a more sensible result: U = (1/2) · q_e^2/4πε$_0r$ = (1/2) · e^2/r.[41] Equating the energy U to the total energy E = m$_e c^2$, gives us the following radius:

$$\mathrm{m_e}c^2 = \frac{1}{2}\frac{\mathrm{e}^2}{r} \Leftrightarrow r = \frac{1}{2}\frac{\mathrm{e}^2}{\mathrm{m_e}c^2}$$

This radius is *half* of what is known as the *Lorentz* radius of an electron, which is also known as the Thomson scattering length or *classical* electron radius. The 1/2 factor does bother (some) physicists. If we

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[37] Mass and energy are equivalent but not the same. Hence, while the (rest) energy of an electron is equal to 0.511 MeV, we should be carefully to use MeV/c^2 units when expressing its mass. Mass and energy do have different *physical* dimensions – because they are equivalent but not the same: the mass and energy *concept* are different.
[38] See references above.
[39] See: Feynman, *Lectures*, II-28. The coefficient depends on the assumption with regard to the charge distribution. For a uniformly charged sphere, the coefficient will be equal to 3/5 (instead of 1/2).
[40] See: Feynman *Lectures*, II-28-3.
[41] The e^2 in the formula here is equal to e^2 = q$_e^2$/4πε$_0$.

assume the electron is a uniformly charged sphere, then the energy expression becomes $U = (3/5) \cdot q^2/4\pi\varepsilon_0 r$ and, hence, we find a radius that matches the classical electron radius somewhat better:

$$m_e c^2 = \frac{3}{5}\frac{e^2}{r} \Leftrightarrow r = \frac{3}{5}\frac{e^2}{m_e c^2}$$

As Feynman jokes, "rather than to argue over which distribution is correct, it was decided to define $r_0 = e^2/m_e \boldsymbol{c}^2$ as the 'nominal' radius. Then different theories can supply their pet coefficients."[42] However, the point is that we should *not* try to think of the mass (m) in the $E = m \cdot a^2 \cdot \omega^2$ as an electromagnetic mass – not in the classical sense, at least. Otherwise it cannot move at the speed of light, and the whole idea of the Zitterbewegung – seen as a current ring – collapses.

There is another reason why the oft-invoked concept of electromagnetic mass – in the context of the *Zitterbewegung* model – should *not* be invoked: a classical oscillating charge radiates all of its energy away – and very rapidly so. As such, the *Zitterbewegung* models suffers from the same defect as the Rutherford-Bohr model of an atom: a rotating charge, such as the electron classically orbiting around the nucleus, should radiate its energy away.[43] However, one should admit that the quantum-mechanical picture of an electron does not really answer this question either: we may say the quantum-mechanical picture avoids the question altogether.

The third conceptual issue that we have raised was about the magnetic force: Maxwell's equations tell us that this moving charge must generous an incredibly strong magnetic field. In fact, if we would look at opposite ends of the current ring as parallel wires, then the current is in opposite directions and would, therefore, the force between the two would tend to push them away. We have no answer to this, but it is a burning question. We will try to address it in our next paper.

9. Conclusions: what does it all mean?

We can only say what it means for us – which is, perhaps, not all that much. The results that come of our model – such as correct values for the g-factor (2), the angular momentum (ħ/2), and the Compton scattering radius (a = ħ/mc) – are very intuitive, but we find the idea of a disk-like electron itself quite artificial. If we can think of the *structure* of an electron – because that is what we are talking about – can effectively be described as some two-dimensional oscillation, then we should probably not assume these oscillations are perfectly linear. We would probably want to introduce some uncertainty here and think of the plane of oscillation as something that is rotating in space itself. Of course, once we *measure* the magnetic moment of an electron – or, when applying the model to a photon, the polarization state of a photon – we should probably think of it as sort of snapping into place again: a nice oscillation in a plane – rather than in 3D space – but something that will drift and rotate in space again when it leaves the Stern-Gerlach apparatus.

This is why – in our original paper[44] – we also considered circular rather than linear oscillations. Indeed, we should, perhaps, think of a tiny ball, whose center of mass stays where it is: any rotation – around any axis – will then be some combination of a rotation around the two other axes. Hence, we may want to think of a two-dimensional oscillation as an oscillation of a polar and an azimuthal angle. Having said this, the Uncertainty Principle tells us we should probably not think of nice circular oscillations either: any direction – linear or circular – is bound to be imprecise. In short, the mystery remains deep.

The most intriguing idea, in our humble view, is the idea of a superimposition of various *motions* – linear and circular, obviously – as opposed to the usual approach of superimposing wavefunctions. Indeed, we have

[42] See: Feynman *Lectures*, II-28-3.

[43] One should admit, however, that the quantum-mechanical picture of an electron does not really answer this question either.

[44] See: Jean Louis Van Belle, 22 October 2017, *The Quantum-Mechanical Wavefunction as a Gravitational Wave*, http://vixra.org/abs/1709.0390.

offered a model for the motion of a stationary electron. If we give it kinetic energy – momentum – then we should introduce some linear motion. How can we relate this to the $c^2 = a^2 \cdot \omega^2$ equation? For example, we may speed up an electron to, say, about one tenth of the speed of light, so the Lorentz factor is equal to γ = 1.005. This means we added 0.5% (about 2,500 eV) – to the rest energy E_0: $E_v = \gamma\ E_v \approx 0.5135$ MeV. The relativistic momentum will then be equal to $m_v v = (0.5135\ \text{eV}/c^2) \cdot (0.1 \cdot c) = 5.135\ \text{eV}/c$.

$$\theta = \frac{1}{\hbar}(E_v t - px) = \frac{1}{\hbar}\left(\frac{E_0}{\sqrt{1-\frac{v^2}{c^2}}}t - \frac{E_0 v}{c^2\sqrt{1-\frac{v^2}{c^2}}}x\right) = \frac{1}{\hbar}E_0\left(1.005 \cdot t - 1.005 \cdot \frac{v}{c^2} \cdot x\right)$$

The v/c^2 factor is equal to $0.1/c$ and, therefore, $1.005 \cdot v/c^2$ equals 0.335×10^{-9} s/m. We see what happens: we do get linear motion. How should we interpret this? If we look at the $\psi = a \cdot \cos(p \cdot x/\hbar - E \cdot t/\hbar) - i \cdot a \cdot \sin(p \cdot x/\hbar - E \cdot t/\hbar)$ once more, we can write $p \cdot x/\hbar$ as Δ and think of it as a phase factor. We will, of course, be interested to know for what x this phase factor $\Delta = p \cdot x/\hbar$ will be equal to 2π. Hence, we write:

$$\Delta = p \cdot x/\hbar = 2\pi \Leftrightarrow x = 2\pi \cdot \hbar/p = h/p = \lambda$$

We now get a meaningful interpretation of the *de Broglie* wavelength. It is the distance between the crests (or the troughs) of the wave, so to speak, as illustrated below.

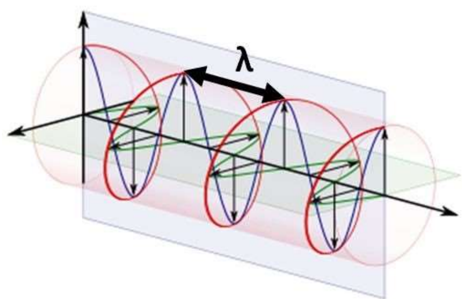

Figure 11: The meaning of the *de Broglie* wavelength in the *Zitterbewegung* model[45]

If this interpretation makes sense, then the challenge ahead is obvious: how should we think of potential energy, so as to add the missing layer to the motion of our electron motion: the electron orbitals in an atomic system? Here we should probably further explore Dirac's suggestion that the real motion of a pointlike charge may be a superposition of motions related to its rest energy, its kinetic energy and its potential energy *respectively*. Hence, a focus on the superposition of *motions* – as opposed to a superposition of ill-understood wavefunctions – may help us to find *the* wave equation – as opposed to the various *ad hoc* equations (such as the Schrödinger equation) that are used to explain one aspect of the motion (electron orbitals for the Schrödinger equation) only.

Finally, the next step would be to think about how the various diffraction and interference phenomena can be explained. The key to these in any *Zitterbewegung* interpretation would probably include an exploration of how we can build up and/or split the composite circularly polarized wave from/into linearly polarized waves.[46]

Jean Louis Van Belle, 24 November 2018

[45] Credit: https://commons.wikimedia.org/wiki/User:Dave3457. The author only added the wavelength, which can be interpreted as the *de Broglie* wavelength for a particle. For more details, see http://vixra.org/pdf/1709.0390v5.pdf.

[46] See, for example: Jean Louis Van Belle, 5 November 2018, *Linear and Circular Polarization States in the Mach-Zehnder Interference Experiment*, http://vixra.org/abs/1811.0056.

References

This paper discusses general principles in physics only. Hence, references were mostly limited to references to general physics textbooks. For ease of reference – and because most readers will be familiar with it – we often opted to refer to

1. Feynman's *Lectures* on Physics (http://www.feynmanlectures.caltech.edu). References for this source are per volume, per chapter and per section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

Specific references – in particular those to the mainstream literature in regard to Schrödinger's *Zitterbewegung* – were mentioned in the footnotes. We should single out the various publications of David Hestenes and Celani et al., because they single-handedly provide most of the relevant material to work on here:

2. David Hestenes, Found. Physics., Vol. 20, No. 10, (1990) 1213–1232, *The Zitterbewegung Interpretation of Quantum Mechanics*, http://geocalc.clas.asu.edu/pdf/ZBW_I_QM.pdf.

3. David Hestenes, 19 February 2008, *Zitterbewegung in Quantum Mechanics – a research program*, https://arxiv.org/pdf/0802.2728.pdf.

4. Francesco Celani et al., *The Electron and Occam's Razor*, November 2017, https://www.researchgate.net/publication/320274514_The_Electron_and_Occam's_Razor.

In addition, it is always useful to read an original:

5. Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf

The illustrations in this paper are open source or – in one or two case – have been created by the author. References and credits – including credits for open-source Wikipedia authors – have been added in the text.

One reference that has not been mentioned in the text is:

6. *How to understand quantum mechanics* (2017) from John P. Ralston, Professor of Physics and Astronomy at the University of Texas.

The latter work is one of a very rare number of exceptional books that address the honest questions of amateur physicists and philosophers upfront. We love the self-criticism: "Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research." (p. 1-10)

Last but not least, we pre-published a series of more speculative arguments on the viXra.org site. See: http://vixra.org/author/jean_louis_van_belle. The author intends to further develop these in future articles.