Kaluza hypothesis as a key to the “hidden” geometry of quantum electrodynamics

We cannot solve our problems with the same thinking we used when we created them. Albert Einstein

Victor Paromov, PhD

Email: vikus68@yahoo.com

Subjects: GR, KK, QED, QCD, spacetime, extra dimensions, unification

This letter attempts to show that the Kaluza hypothesis of five-dimensional (5D) spacetime is sufficient to explain the quantum electrodynamics (QED). It is a short version of the article “Fractal Structure of the Spacetime, the Fundamentally Broken Symmetry” (http://vixra.org/abs/1806.0181) written as an attempt to start a discussion in the physics community. To author’s regret, it has been rejected by *Nature Physics* on 9/24/2018 and by arxiv.org (gr-qc) on 10/23/18 in despite of the endorsement from Dr. Sergei M. Kopeikin.

Introduction

It is a common belief that the original Kaluza hypothesis **[1]** adding a fifth extra dimension to the Einsteinian spacetime model cannot lead to a self-consistent quantum theory of electromagnetism, even in its Klein’s interpretation **[2]**. Indeed, the Kaluza-Klein (KK) extensions **[3]** of the General Relativity (GR) theory remain essentially classical field theories and are not quantized naturally. Conversion of any given KK theory into a self-consistent quantum field theory like the quantum electrodynamics (QED) seems highly problematic. In addition, taking to the account the experimental evidence **[4]** of the Bell’s inequalities’ brake **[5]** at the “quantum scale”, the classical field theoretical approach cannot be applied to particles’ interactions in general. Thus, it is likely that the classical KK approach to unification of electromagnetism with gravity based on the Kaluza hypothesis cannot lead to a self-consistent quantum field theory.

However, the classical KK approach may not be the only way of building a quantum field theory based on the Kaluza hypothesis. The argument below suggests a completely different approach. The presented reasoning is not a formal proof as it contains a number of assumptions related to the extradimensional geometry. Nevertheless, it shows a simple self-consistent way of constructing the QED based on the Kaluza’s model of 5D spacetime.

Results

Let us start with the original Kaluza’s description of the 5D spacetime disregarding the cylinder condition **[1]**. An additional geometrical condition for the 5D spacetime model is that the extradimensional 1D circles mapped to each point of the ordinary 4D spacetime are uniform and maximal sections.

Let us assume that an elementary electric charge (point-like test particle) induces 5D spacetime curvature just like masses induce 4D spacetime curvature. The two main differences of the charge-induced curvature compared to the mass-induced one should be taken to the account: 1) the effect of mass appears in the 4D spacetime only, whereas the effect of electric charge influences all the five dimensions; 2) the latter effect is dual, i.e. one type of charges induces positive geometric curvature, and another type induces negative geometric curvature. Let us describe the movement of a charged test particle in space and time. Obviously, the particle movement in the 5D spacetime should be somehow translated into a point movement in 4D spacetime or in 3D space and absolute time. The latter description is preferable due to the simplicity. Notably, as the fifth dimension size is presumably microscopic [2], one cannot describe the charge-induced curved local 5D spacetime with a classical field (e.g. as a KK GR extension) due to the fifth coordinate (related to the fourth spatial dimension) inevitably collapsing into a point. Let us call this phenomenon the undetectability condition. However, instead of describing the actual 5D spacetime curvature in $\mathbf{R}^{4+1}$ (as all the KK models do) one can try to describe the 4D-space curvature as an additional parameter of the ordinary 3D space (i.e. a scalar field in $\mathbf{R}^3$) and take to the account the notion that particles always move by geodesics (the geodesic condition). If all possible 4D (spatial) geodesics are translated into 3D geodesics, one can possibly describe the 4D-space curvature as a field in the ordinary 3D space. Thus, if the extradimensional curvature does determine the charged particle movement, it can be described in $\mathbf{R}^3$ (flat 3D space and absolute time) with the two conditions: 1) scalar 4D-space curvature is described by a scalar field, and 2) all possible 4D geodesics are properly translated into 3D geodesics.

As there are two types of electric charges, it is logical to assume that one type induces positive geometrical curvature, and another type induces negative curvature. First, let us consider the case of a positive 4D spatial curvature (time is disregarded for the simplicity). If an elementary charge induces a stable curvature of the flat 4D space, this local space can be modeled by a small hypersphere S^4. Although the global 4D space cannot be a hypersphere (as its topology presumably is $S^3 \times S^1$), the local 4D space perfectly can, if its size does not exceed the fifth dimension's diameter. In reality, this curved space may have some curvature gradient and shape imperfections, which can be disregarded. For the simplicity, one can treat this local space as an ideal hypersphere embedded in flat $\mathbf{R}^5$. Next, one needs to find a proper transition from this local S^4 to global $\mathbf{R}^3$. First, S^4 can be mapped to S^3, the intersection of S^4 with hyperplane $\mathbf{R}^4$ containing the center of S^4. The latter condition assures that S^3 is the geodesic of S^4 and is predetermined by the Kaluza model's additional condition (see above). Then, the S^4 original scalar curvature lost in the transition can be described as a scalar field in each point of S^3. The important property of this local S^3 is that it is isometric to the global space, which is also assumed as S^3. Thus, one can substitute the local space with the global space preserving the geodesic condition. Then, the extradimensional spatial scalar curvature can now be described by some scalar field in S^3 manifold.

Next, one can use the stereographic projection and translate S^3 into $\mathbf{RP}^3$ providing the scalar field description in the ordinary space $\mathbf{R}^3$. However, the scalar field depends on the inner parameter (curvature) of the 4D space containing undetectable extra dimension, which cannot be described with a real field due to the undetectability condition. Therefore, one needs to implement a complex scalar field. Ideally, S^3 should be translated into a complex manifold that 1) accounts for the periodicity (circle symmetry) of the extra dimension, 2) preserves the geodesic condition, and 3) is isometrically embeddable in $\mathbf{R}^3$. That can be done, if one replaces S^3 with a unit sphere $S(\mathbf{C}^2)$ in the complex coordinate space $\mathbf{C}^2$ and uses the principal Hopf bundle [6] over the complex projective space: $U(1) \to S(\mathbf{C}^2) \to \mathbf{CP}^1$. The projection map: $S(\mathbf{C}^2) \to \mathbf{CP}^1$ gives a Riemannian submersion with totally geodesic fibers isometric to U(1). This Hopf bundle is a generalization of the geometrical fibration: $S^1 \to S^3 \to S^2$. As the Hopf fibration is known to assign a great circle of S^3 to each point on S^2, it maps all the geodesics of S^3 onto S^2, which in turn is projected to $\mathbf{CP}^1$, and thus preserving the geodesic condition. Thus, $\mathbf{CP}^1$ is likely the simplest possible type of the scalar field describing the extradimensional curvature (S^4 original scalar curvature) in $\mathbf{R}^3$ (flat 3D space).

A similar construction can be used with the hyperbolic pseudosphere H^4, in case the elementary charge is assumed inducing a negatively curved local 4D space. With similar reasoning, the H^4 curvature can be described by a scalar field in $\mathbf{CP}^1$, but having an opposite sign. One can take the intersection of H^4 and $\mathbf{R}^4$, which again gives S^3. The H^4 original curvature lost in the transition can be described by a scalar field in each point of S^3. Assuming the negative extradimensional curvature counteracts the positive curvature, the second scalar field should have an opposite sign compared to the case above. The Hopf fibration: $U(1) \to S(\mathbf{C}^2) \to \mathbf{CP}^1$ again translates S^3 (replaced by $S(\mathbf{C}^2)$) into the complex projective space preserving the geodesic condition. Finally, one obtains the second component of the complex scalar field (with an opposite sign) describing the opposite elementary charge action in the ordinary 3D space.

Thus, the extradimensional curvature, which is assumed governing the electromagnetic field, can be described with a complex scalar field having two counteracting components, the “positive” and the “negative” fields. From this point, the derivation of the particle’s equations of motion is rather trivial. The complex scalar field has a Fubini-Study metric (which is an Einsteinian metric), and an action:

$$S = \int d^4x (\partial\mu\phi^*)(\partial^\mu\phi) - V(|\phi|), \qquad (1)$$

where ϕ acts as the “positively” charged field, ϕ^* acts as the “negatively” charged field, and $V(|\phi|)$ is the complex scalar field potential. The remaining derivation is rather trivial. By the construction, this action has a global symmetry under the group U(1), i.e. $\phi \to e^{i\alpha}\,\phi$, which can be translated to a local symmetry by introducing a gauge field with the gauge covariant derivative $D\mu\phi \to e^{i\alpha(x)}D\mu\phi$ (e is the elementary electric charge). One can find the gauge transformation-invariant form of the above-stated action, add the gauge field kinetic term defined by the transformation group U(1), $F_{\mu\nu}F^{\mu\nu}$ (where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$) and write the equations of motion with respect to the gauge field. Thus, the scalar field dynamics can be given by the Lagrangian density:

$$L = (D\mu\phi^*)(D^\mu\phi) - V(|\phi|) - 1/4\, F_{\mu\nu}F^{\mu\nu} \qquad (2)$$

This equation is similar to the QED Lagrangian, except it describes spinless charged scalar fields, not spin ½ leptons. The spin introduction is explained below.

A point-like test particle's movement along the special direction in the 4D space appears periodic in the 3D space due to the small size of the fifth dimension. Hence, the 5D spacetime dynamics must have certain attributes of a wave, which is indeed a well-registered experimental fact. It is a common assumption that the elementary particles are in a constant movement. Thus, the small projection hypersphere S^3 modeled an elementary spacetime deformation constantly moves in the 4D space. This movement can be separated into the movement in the ordinary 3D space and the movement along the special extradimensional direction. For the observer, the latter appears as a constant spin. The 3D space dynamics of a test particle have been described above; however, its spin requires one to introduce an additional correction for the scalar field description. Although the spin can have infinite possible directions in the 4D space, all those are reduced to just two, clockwise and counterclockwise, after the translation from the global S^3 to the planar $\mathbf{CP^1}$. As there are two parts of the complex scalar field and two possible spin directions, one must introduce certain corrections to equation (2) accounting for the proper spin direction and making the right commutation. The proper corrections are made by the Dirac matrices and the switch from the field ϕ to the bispinor field ψ. Then, the Lagrangian (2) takes the form:

$$L = \bar{\psi}(i\gamma^{\mu}D_{\mu})\psi - V(|\phi|) - 1/4\, F_{\mu\nu}F^{\mu\nu}, \tag{3}$$

where ψ is a bispinor field, i.e. electron-positron field; $\bar{\psi} \equiv \psi^{\dagger}\gamma^{0}$ is the Dirac adjoint; and γ^{μ} are the Dirac matrices. The gauge field potential minimum $V(|\phi|)_{\min}$ occurs at $|\phi| \neq 0$ in the presence of electric charge due to the charge being the 4D-space curvature origin. Hence, the gauge field behaves as a massive field, and its mass is proportional to the lepton's ground state mass-energy m, i.e. conventional electron's mass. Thus, the potential can be expressed as $V(|\phi|) = \bar{\psi}m\psi$. After the proper replacement, equation (3) takes the final form of the QED Lagrangian:

$$L = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi - 1/4\, F_{\mu\nu}F^{\mu\nu} \tag{4}$$

Discussion

The above-presented reasoning shows that the 5D spacetime geometry hypothesized by Kaluza in 1921 [1] is sufficient to construct a group U(1) abelian gauge theory identical to the QED. The theory can be directly quantized in a same way as the QED (e.g. using the S-matrix technique). Unlike in the KK approach, the quantum nature of the resulting theory is predetermined by the special properties of extradimensional geometry (microscopic size, closeness, and simple connectedness of the fifth dimension) and does not require any *ad hoc* conditions. However, due to the undetectability condition, parameters related to the extra dimensional geometry, i.e. the electric charge of the bispinor field, the Compton wavelength, and consequently the Planck constant, cannot be determined by the theory and should be added in accordance with the experimental values.

The unification with gravity comes quite naturally with the $V(|\phi|)$, the complex scalar field potential presently known as the Higg's field. The latter can be interpreted as the 4D spacetime curvature, a geometrical part of the 5D spacetime curvature, which is induced directly by the electric charges. Considering these facts, it seems reasonable to expect that the original Kaluza hypothesis (in the Klein's interpretation and absence of the cylinder condition) is fully sufficient to explain the electromagnetism. In general, this means that the 5D spacetime geometry may be sufficient to describe the electromagnetism, as the 4D spacetime geometry sufficiently explains the gravity. However, the presented Kaluza hypothesis-based explanation of the QED is not a formal proof and may need further detailed mathematical description.

Moreover, in case the Kaluza spacetime model is extended with three additional more compact extra dimensions, it seems possible that with a similar approach, the nuclear forces can be described solely with the spacetime geometry as well.

Aknowlegments

This work was in part inspired by the induced matter theory created by Dr. Paul Wesson and other members of the Space–Time–Matter Consortium. Although author realizes the general limitations of the KK approach, he greatly appreciates the valuable contributions made by Dr. Paul Wesson and would like to dedicate this short article to his memory.

References

1. Kaluza, Th. On the Unification Problem in Physics. *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, **54**, 966-972 (1921).
2. Klein, O. The atomicity of electricity as a quantum theory law. *Nature*, **118**, 516–520 (1926).
3. Overduin, J. M. & Wesson, P. S. Kaluza-Klein gravity. *Physics Reports*, **283**, 303–378 (1997).
4. Hensen, B., Bernien, H., Dréau, A. E., Reiserer, A., Kalb, N., Blok, M. S., Ruitenberg, J., Vermeulen, R. F. L., Schouten, R. N., Abellán, C., Amaya, W., Pruneri, V., Mitchell, M. W., Markham, M., Twitchen, D. J., Elkouss, D., Wehner, S., Taminiau, T. H. & Hanson, R. Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometers. *Nature*, **526**, 682-686 (2015).
5. Bell, J. S. On the Einstein–Podolsky–Rosen paradox. *Physics*, **1**, 195-290 (1964).
6. Hopf, H. Über die Abbildungen der dreidimensionalen Sphäre auf die Kugelfläche. *Mathematische Annalen*, **104**, 637-665 (1931).