Calculation of the atomic masses

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According to the generally accepted physical theory, the synthesis of the elements may happen at a very high temperature in supernova explosions. In consequence of nuclear fusion, the supernova stars emit a very strong electromagnetic (EM) radiation, predominantly in form of X-rays and gamma rays. The intensive EM radiation drastically decreases the masses of the exploding stars, directly causing mass defects of the resulting atoms. The description of black body EM radiation is based on the famous Planck's radiation theory, which supposes the existence of independent quantum oscillators inside the black body. In this paper, it is supposed that in exploding supernova stars, the EM radiating oscillators can be identified with the nascent elements losing their specific yields of their own rest masses in consequence of the radiation process. The final binding energy of the atoms (nuclei) is additionally determined by the strong neutrino radiation what also follows the Maxwell-Boltzmann distribution in the extremly high temperature. Extending Planck's radiation law for discrete radiation energies, a very simple formula is obtained for the theoretical determination of the atomic masses. In addition, the newly introduced theoretical model gives the fusion temperature what is necessary for the generation of the atoms of the Periodic Table.

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I. INTRODUCTION

The theoretical determination of the atomic masses dates back to 1935, when *C. F. von Weizsäcker* [1] published his famous *liquid-drop model* for the calculation of nuclear binding energy of the nuclei. This nuclear model provides a general overview of the atomic masses and related stability of their nuclei and assumes the nucleus behaves in a gross collective manner, similar to an electrically charged drop of liquid. The semi-empirical mass formula based on this phenomenological model was applied successfully mainly in the earlier period of nuclear physics. From the simple drop model one can easily calculate approximately the mass of any neutral atoms by adding the Z number of electron mass to the mass of *X(Z,A)* nuclei.

It is widely known that the majority of the elements in the periodic table are synthesized in the stars. The synthesis of the elements may happen only at very high temperature, for example, in supernova explosions. In consequence of the nuclear fusion at high temperature, the supernova stars emit very strong electromagnetic (EM) radiation, predominantly in form of gamma and X-rays. In addition, the EM radiation is combined with strong neutrino radiation. The different intensive energy radiations continuously decreases the masses of the stars, directly causing mass defects of the nascent atoms (nuclei), and at least the strong binding of nuclei. The individual atoms represent quantized black body oscillators; their frequencies are determined by their mass numbers A. From this simple physical model, one can conclude that the binding energy curve of the nuclei is in immediate connection to the Planck's radiation law in the very high temperature region. In our paper, we have fitted the Planck's radiation law to the binding energy curve of the nuclei, supposing that the radiation frequency f of an arbitrary atom is proportional to the root-square of its mass number.

II. EXTENSION OF PLANCK'S RADIATION LAW

According to the Planck's radiation law the energy density of the EM radiation in function of the radiation frequency is

$$dE_f = \frac{2h}{c^2}\frac{f^3}{\exp(hf/kT)-1}df, \tag{1}$$

where T is the absolute temperature, c is the speed of light, k is the Boltzmann's constant, h is the Planck's constant and f is the radiation frequency. The new model for explaining the origin of the elements requires discrete radiation frequencies of the stars depending on the mass numbers of the nuclei. In classical electrodynamics, the radiation energy density of a simple dipole antenna is proportional to the frequency on the fourth power

$$E_f \propto f^4. \tag{2}$$

From the analogy, the discrete energy emitted by the individual atom at absolute temperature T must be

$$E_{rad}(Z,A) \propto \frac{f^4(Z,A)}{\exp(hf/kT)-1}. \tag{3}$$

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One can suppose that this equation is a natural generalization of the Planck's radiation law for discrete radiation frequencies. The most important task was to determine the mathematical relation between the radiation frequency and the arbitrary atom. It was obviously supposed that the radiation frequency is mainly determined by the mass of the individual atom. The simplest assumption is that the square of the discrete radiation frequencies are proportional to the atomic masses

$$f^2(A) \propto AM_0, \tag{4}$$

where A is the mass number of the atom and M_0 is the average value of a single nucleon.

III. NEW MASS-FORMULA FOR THE NEUTRAL ATOMS

The formula finally found for the calculation of atomic masses is the next

$$\begin{gathered} M(Z,A) = AM_0 + M_{rad}(A) + \\ + M_{as}(Z,A) + M_p(Z,A), \end{gathered} \tag{5}$$

where *M(Z,A)* is the calculated atomic mass, AM_0 is the atomic mass before the nuclear synthesis, $M_{rad}(A)$ is the mass defect caused by EM and neutrino radiation. The last two terms depend on the atomic number Z (*number of protons*): $M_{as}(Z,A)$ is the "asymmetry" mass (energy) and $M_p(Z,A)$ is the "pairing" mass (energy).

The mass defect of the atoms caused by the radiation can be written into the next form

$$\begin{aligned} M_{rad}(A) &= -C_{rad} f^4(A)/(B^f - 1) = \\ &= -C_{rad} A^2 M_0^2 / R(A) \end{aligned} \tag{6}$$

where the radiation frequency is determined by Eq. (4) and *R(A)* is a symbolic atomic radius associated to the atom having mass number A

$$R(A) = B^f - 1 = B^{\sqrt{AM_0}} - 1. \tag{7}$$

The asymmetry mass (energy) is related to the *Pauli extension principle* what is given by

$$M_{as}(Z,A) = C_{as} M_0^2 \left(\frac{A - 2Z}{A + 3} \right)^2, \tag{8}$$

where C_{as} is fitting parameter. The last term in the new atomic mass formula is the pairing mass (energy) what is the consequence of the spin-coupling of the nuclei

$$M_p(Z,A) = -\frac{1}{2} C_p M_0^2 \frac{(-1)^Z + (-1)^{A-Z}}{R(A)}. \tag{9}$$

This term connects to observation that the nuclei having even number of protons and even number of neutrons (even-Z, even-N), or, in short even-even nuclei, are most abundant and more stable. The odd-odd nuclei are the least stable, while even-odd and odd-even nuclei are intermediate in stability. Due to the Pauli exclusion principle the nucleus would have a lower energy if the number of protons with spin up were equal to the number of protons with spin down. This is also true for neutrons.

IV. NUMERICAL RESULTS

In the calculations of the atomic masses with the new atomic mass model were used the unified mass unit "dalton" or "Da". The advantage of this solution is that all the mass parameters are dimensionless numbers. From this reason all the variables occurring in the new atomic mass formula are also dimensionless.

During the fitting procedure a better expression has found for the radiation term given by Eq. (6)

$$\begin{gathered} M_{rad}(A) = -C_{rad}(A - 1.5)^2 M_0^2 / R(A), \\ R(A) = B^{\sqrt{(A-1.5)M_0}} - 1, \ (A \geq 2). \end{gathered} \tag{10}$$

The new mass formula was fitted to nearly 2000 measured neutral atomic masses obtained from the publication of G. Audi and A. H. Wapstra [2]. The result of the fitting procedure is

$$\begin{gathered} M_0 = 1.003258...\ Da; \\ Q = 2/9 = 0.222....\ \text{(dimensionless)}. \end{gathered} \tag{11}$$

The mass formula contains five fitting parameters, what are dimensionless

$$M_0,\ C_{rad},\ \ C_{as},\ C_p,\ B. \tag{12}$$

The intensive theoretical study has shown that these parameters are not independent of each other. Surprisingly, the last four parameters depend only on the single variable Q

$$\begin{gathered} C_{rad} = Q^5/2,\ C_{as} = Q, \\ C_p = Q^4/2,\ B = 1 + Q. \end{gathered} \tag{13}$$

The accuracy of the atomic mass formula was determined by the relative standard deviation (n = number of the involved isotopes)

$$\begin{gathered} \sigma = \sqrt{\tfrac{1}{n-1} \sum_i \left(\tfrac{M_{i,calc} - M_{i,\exp}}{M_{i,\exp}} \right)^2} = \\ = 1.55... \times 10^{-4}. \end{gathered} \tag{14}$$

The obtained nucleon mass M_0 is less about $5\ MeV$ than the known rest mass of the neutron. Physical explanation of this fact that at a very high fusion temperature, the masses of neutrons decreases by this average value. The missing part of the neutron mass appears in the energy of the thermal radiation field. Taking this into account,

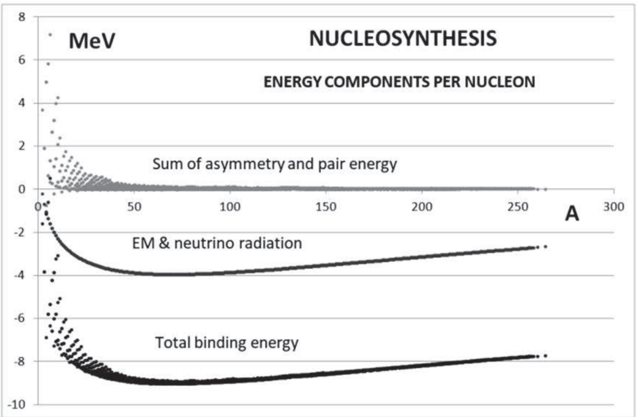

FIG. 1: The energy components per nucleon of the new nuclear radiation model vs. atomic mass.

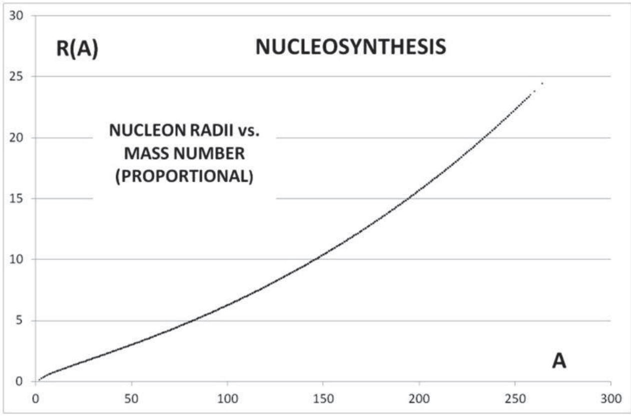

FIG. 2: The calculated atomic radii (proportional) by the new nuclear model vs. atomic mass.

we introduced the concept of *total binding energy* of the nuclei

$$E_B(Z,A) = A\,(M_0 - M_N) + M_{rad}(A) + \\ + M_{as}(Z,A) + M_p(Z,A) < 0, \tag{15}$$

where M_N is the mass of neutron. Interesting to mention that the value of M_0 is

$$M_0 \approx (1 - Q^3/2) M_N. \tag{16}$$

FIG. 1 and FIG. 2 demonstrate the results of the new atomic mass calculation model introduced in our paper.

V. FUSION TEMPERATURE

From this atomic generation model we can easily determine the fusion temperature for the all elements (isotopes) of the Periodic Table. From Eq. (6) and Eq. (12) follows the next relation

$$B = 1 + Q = \exp(1/kT), \tag{17}$$

where T is the fusion temperature, what is valid for the all generated atoms of the Periodic Table. From this

$$kT = 1/\ln(1+Q) = 4.983\ Da \Rightarrow \\ \Rightarrow 4.641...\ GeV, \tag{18}$$

what leads to fusion temperature

$$T = 5.386... \times 10^{13}\ K. \tag{19}$$

VI. CONCLUSION

Based on our new successful atomic mass formula, we have concluded that at extreme high temperature nuclear synthesis can be physically described exclusively following the generalized Planck's radiation law for discrete frequencies. By the new atomic generation model the nuclear fusion temperature of the Periodic Table's atoms has been determined. In Planck's model it is supposed that the *black body oscillators* are independent of each other, and have a Maxwell-Boltzmann energy distribution. Already in earlier nuclear physics, there were some experiences showed that all the nuclei inside the atom are weakly bound. This experimental fact is also proved theoretically in the present work.

The accuracy of the here-introduced atomic mass formula is comparable with the accuracy of the nuclear liquid drop model by *von Weizsäcker*. As is known, the nuclear drop model has five independent fit parameters, in contrast of only two independent fit parameters of the present model.

[1] C. F. von Weizsäcker, **Z. Phys.** 96, p. 431, (1935).
[2] G. Audi, A. H. Wapstra, Nuclear Physics, A595, vol. 4, p. 409-480, (1995).