P ≠ co-NP

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Abstract

We prove that class P ≠ class co-NP.

Using first order logic, create a finite set of arbitrary axioms in the following form (with duplicable negatable literals replacing �):

� → (� ∨ �) ∧ (� ∨ �)

Proving the axioms are consistent (true or false can be assigned to each literal) is class NP (not asymptotically harder than deciding the satisfiability of an arbitrary Boolean formula).

Recreate the axioms but make sure that:

- All positive literals are used as conditions (literals left of the arrow) exactly once.
- Exactly 1 positive literal (the "entrance") is used in a condition but not in a disjunction.
- No negative literals are used in conditions.

From the axioms, draw a tree with each literal as a node and each conjunction of disjunctions as 4 edges. Note that the tree has uncountably infinite nodes.

Definitions:

- Unique nodes with the same literal are unique "doors" of the same "color".
- Positive colors are "cold".
- Negative colors are "hot".
- Each disjunction is a "pair" of doors.
- Each conjunction of pairs is a "room".
- To apply an axiom is to "open" a door.
- A hot door is "on fire" (a contradiction) if a door of the opposite color is between the entrance and it.
- A room is on fire if it has a pair of doors that are both on fire.
- A cold door is on fire if it opens to a room that's on fire.

Notice:

- Doors of the same color open to rooms with doors of the same color.
- To prove a cold door is on fire, a set of hot doors on fire needed to "cause" (imply) that cold door to be on fire must be proven to be on fire first, the smallest of which is superpolynomial (in terms of unique colors used for doors) in size.
- Because the entrance is a cold door, to prove that it is on fire, it must first be proven superpolynomial hot doors are on fire, which is not class P (a deterministic Turing machine can't do it in polynomial time).
- Proving the entrance is not on fire is the same as proving the axioms are consistent, which is class NP, so proving the entrance is on fire is class co-NP.

Proving the entrance is on fire is therefore class co-NP but not class P, which implies class P ≠ class co-NP. Note that class P ≠ class co-NP implies class P ≠ class NP.

The key statement here is: to prove a cold door is on fire, a set of hot doors on fire needed to "cause" (imply) that cold door to be on fire must be proven to be on fire first, the smallest of which is superpolynomial (in terms of unique colors used for doors) in size.

If a cold door is proven to be on fire, it must also be proven why (the fire starters must be found, because you can't have "turtles all the way down" so to speak), and only hot doors can start fires (by being implied by their negatives). If cold door A is catches fire from cold doors B and C, then A's fire must have at least 4 hot door fire starters. If B and C both catch fire from cold doors, then A's fire must have at least 8 hot door fire starters… meaning the smallest set of hot door fire starters causing A to catch fire is superpolynomial.