P ≠ co-NP

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Abstract

We prove that class P ≠ class co-NP.

Using first order logic, create a finite set of arbitrary axioms in the following form (with duplicable negatable literals replacing �):

� → (� ∨ �) ∧ (� ∨ �)

Proving the axioms are consistent (true or false can be assigned to each literal) is class NP (not asymptotically harder than deciding the satisfiability of a Boolean formula).

Recreate the axioms but make sure that:

- All positive literals are used as conditions (literals left of the arrow) exactly once.
- Exactly 1 positive literal (the "entrance") is used a condition but not in a disjunction.
- No negative literals are used as conditions.

Definitions:

- Duplicate literals are unique "doors" of the same "color".
- Positive colors are "cold".
- Negative colors are "hot".
- Each disjunction is a "pair" of doors.
- Each conjunction of pairs is a "room".
- To apply an axiom is to "open" a door.
- A hot door is "on fire" (a contradiction) if a door of the opposite color is between the entrance and it (inclusive).
- A room is on fire if it has a pair of doors that are both on fire.
- A cold door is on fire if it opens to a room that's on fire.

Notice:

- Doors of the same color open to rooms with doors of the same color.
- To prove a cold door is on fire, it must first be proven that at best 2 and at worst superpolynomial (in terms of unique colors used for doors) hot doors doors are on fire (specifically, the smallest set of hot doors on fire needed to "cause" that cold door to be on fire).
- Because the entrance is a cold door, to prove that it is on fire, at worst you must first prove superpolynomial hot doors are on fire, which is not class P (a deterministic Turing machine can't do it in polynomial time).
- Proving the entrance is not on fire is the same as proving the axioms are consistent, which is class NP, so proving the entrance is on fire is class co-NP.

Proving the entrance is on fire is therefore in class co-NP but not class P, which implies class P ≠ class co-NP.

Note that class P ≠ class co-NP implies class P ≠ class NP.