New axioms in set theory

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Abstract

We propose new axioms in set theory bound to the measure theory of Hilbert spaces.

1 The ZFC axioms

The set theory [J] is based on the axioms of Zermelo-Frankel and the choice axioms. These axioms are the foundation of fast all the mathematics.

2 The new axioms in set theory

We consider ZF+AC. We study the real Hilbert spaces H. Such Hilbert spaces are classified by their cardinality; more precisely the cardinality of their topological basis.

We construct axioms of measure theory over these Hilbert spaces.

2.1 First axiom

The first axiom tells that we have a continuous linear form over the space of continuous functions over the Hilbert space with the topology of uniform convergence over the compact sets.

$$\int_{x\in H} f(x)(D^H x)$$

2.2 Second axiom

We have the Fubini property:

$$D^H x = (D^{H'} x).(D^{H''} x)$$

for:

$$H = H' \oplus H''$$

orthogonal direct sum of Hilbert spaces.

2.3 Last axiom

We have compatibility with Lebesgue measure of real sets:

$$D^{\mathbf{R}^n}x = dx_1.dx_2\dots dx_n$$

References

[E] H.-D. Ebbinghaus, & co, "Numbers", Springer-Verlag, Berlin, 1991.

[J] T.Jech, "Set Theory", Springer Verlag, Berlin, 2006.