MAGIC SQUARES

Ankur Shukla and Anurag Singh

Gorakhpur, Uttar Pradesh, INDIA

shuklaankur123456789@gmail.com zehendreg@gmail.com

Abstract – This article provide unique methods of creating perfect magic squares of order 4 and using certain conditions the magic square can be made more precise and varied. The article contains rules which connects all the perfect squares of order 4 and bring them under one roof. The rules contained in this article deals with various methods of arranging a square maintaining the perfectness and bringing new appearance every time using symmetry. The main objective is to create a method for generation of perfect formula of magic square.

• Introduction-

Magic Square is a square matrix showing incredible properties of addition. Any ordered arrangement of the square yields the same sum.

ANU Structure

a	b	c	d
d + x	c – x	b - 3x	a + 3x
b - 2x	a + 2x	d + 2x	c - 2x
c + x	d - x	a + x	b - x

Provided that

(Ank Eqn) $a + d + 4x = b + c$.

a, b, c, d, x € R. . $S = a+b+c+d$ and $x \neq 0$.

This condition makes the square a perfect one .

Now looking at the special arrangement for understanding its property.

A11	A12	A13	A14
A21	A22	A23	A24
A31	A32	A33	A34
A41	A42	A43	A44

Now , As per a common magic square

A11 + A12 + A13 + A14 = S

A21 + A22 + A23 + A24 = S

A31 + A32 + A33 + A34 = S

A41 + A42 + A43 + A44 = S

A11 + A22 + A33 + A44 = S

A41 + A32 + A23 + A14 = S

Now, Due to Ank Eqn.

A11+ A12 + A21 + A22 = S.

A31+ A32 + A41 + A42 = S.

A33+ A34 + A43 + A44 = S.

A13+ A14 + A23 + A24 = S.

A12 + A13 +A22 + A23 = S.

$A22 + A23 + A32 + A33 = S$.

$A32 + A33 + A42 + A43 = S$.

$A21 + A22 + A31 + A32 = S$.

$A23 + A42 + A33 + A34 = S$.

$A11 + A13 + A31 + A33 = S$.

$A12 + A14 + A32 + A34 = S$.

$A21 + A23 + A41 + A43 = S$.

$A22 + A24 + A42 + A44 = S$.

$A11 + A14 + A41 + A44 = S$.

- Notations-

Where, x is the control factor and S is sum constant of Magic Square.

- Mathematical Ankurag Numeric Square Interpreter (MANSI rule)
  - RULE 1. Sum Constant Rule

    $a + b + c + d = S$.

    $a + d + 2x = S/2$.

    $b + c - 2x = S/2$.

- INFERENCE- Sum of Square depends on any two arbitrary value and x(known as control factor).

• RULE 2. Stereoisomer

a	b	c	d
d + x	c - x	b - 3x	a + 3x
b - 2x	a + 2x	d + 2x	c - 2x
c + x	d - x	a + x	b - x

This includes rearranging certain positions in order to get a new form of the magic square and with the same properties.

a).

• Mode 1.

• Step 1. *Fix certain positions.*

	[a	b]	c	d	
FIX	d + x	c - x	[b - 3x	a + 3x]	FIX
	b - 2x	a + 2x	[d + 2x	c - 2x]	
	[c + x	d - x]	a + x	b - x	
	LHS(Left hand side)		RHS (Right hand side)		

• Step 2. Considering the partitioned line as mirror, the 2^{nd} row elements of LHS are placed in the 4^{th} row RHS . And 3^{rd} row elements of LHS are placed in 1^{st} row RHS.

Step 3. Now for the fixed positions, Placing 1st row elements of LHS in place of 4 th row elements and vice versa. For 2nd row elements RHS exchange them with 3 rd row elements.

Now the steps are completed, our new square-

-

c + x	d - x	a + 2x	b - 2x
b - x	a + x	d + 2x	c - 2x
d	c	b - 3x	a + 3x
d	b	a + x	b - x

- **Mode 2.**

- **Step 1.** *Fix certain positions.*

	a	b	[c	d]	
FIX	[d + x	c - x]	b - 3x	a + 3x	FIX
	[b - 2x	a + 2x]	d + 2x	c - 2x	
	c + x	d - x	[a + x	b - x]	
	LHS(Left hand side)		RHS (Right hand side)		

- **Step 2.** Interchanging the location of 1st row of LHS with 3rd row RHS.

Interchanging the location of 4th row of LHS with 2 nd row of RHS. The interchanging will be done as per mirror imaging.

• **Step 3.** The fixed positions will now be interchanged as- 2^{nd} row LHS with 3^{rd} row LHS, 1^{st} row RHS with 4^{th} row RHS.

The structure formed will be-

c - 2x	d + 2x	a + x	b - x
b - 2x	a + 2x	d –x	c + x
d + x	c - x	b	a
a + 3x	b - 3x	c	d

• ***Significance of this rule*** - Using Mode_1 and Mode_2, one can create stereoisomers from a single Skelton square by changing the modes alternatively. This will go on until you have 8 structures in total. Then the pattern will start repeating itself.

b).i

Rows \ Columns	1	2	3	4
1	a	b	c	d
2	d + x	c - x	b - 3x	a + 3x
3	b - 2x	a + 2x	d +2x	c - 2x
4	c + x	d - x	a + x	b - x

Now, 2^{nd} row and 4^{th} row are exchanged. 1^{st} and 3^{rd} columns are exchanged. We get a new structure.

Ank Eqn.

$a + d + 4x = b + c.$

c	b	a	d
a + x	d-x	c + x	b-x
d+2x	a+2x	b-2x	c-2x
b-3x	c-x	d+x	a+3x

Above arrangement does not change the perfectness.

Similarly,

Rows		1	2	3	4	Columns
	1	a	b	c	d	
	2	d + x	c - x	b - 3x	a + 3x	
	3	b - 2x	a + 2x	d + 2x	c - 2x	
	4	c + x	d - x	a +x	b - x	

2nd and 4th row are exchanged then 1st and 3rd are exchanged.

The result,

b - 2x	c - 2x	d + 2x	a + 2x
d + x	a + 3x	b - 3x	c - x
a	d	c	b
c + x	b - x	a + x	d - x

Above arrangement does not change Ank eqn and sum constant.

- **RULE 3**. *Destroyer and Creator*

This Rule as the name suggests Destroys complete square and recreates a new one. Those newly created squares will also follow Ank Eqn.

Ank Avg. – Arithmetic mean of all the elements of a magic square.

Ank Avg. = [∑ { ∑ (i, j) }] / 16

Where ∑ extends from 1 to 4.

Simply putting, Ank Avg= (a+b+c+d)/4.

Mean Square(Ms.) = { (a + d) / 2 } + x

If each element of the magic square is subtracted from Ms, Then we will get a new magic square whose sum constant will be zero.

The new magic square created is known as ZEN (Zero Effect of Number)

Or SHUYA MAAN PARIMAAN .

Taking an example-

204					1
	16	64	40	84	
	85	39	61	19	
	62	18	86	38	
	41	83	17	63	

Ms. = (64 + 16) / 2 + 1

= 51

Now,

Ms. - { Each element of square }.

Now the new square,

0					-1
	35	-13	11	-33	
	-34	12	-10	32	
	- 11	33	-35	13	
	10	-32	34	-12	

• RULE 4. Fusion - Fission Numerology.

This Rule deals with the superimposition of one magic square on another.

Taking example,

FUSION-

86					2
	12	40	7	27	
	29	5	34	18	
	36	16	31	3	
	9	25	14	38	

FU

88					2
	15	42	6	25	
	27	4	36	21	
	38	19	29	2	
	8	23	11	40	

RESULT =

8688					202
	1215	4042	706	2725	
	2927	504	3436	1821	
	3638	1619	3129	302	
	908	2523	1417	3840	

Before application of MANSI rule 4. Following should be noted -

a). All single digit numerals must be converted to double digits by taking a digit 0 before them. Eg. 6 → 06 .

b).Note that <A> FU < B > $\neq$ <B > FU < A>.

Where <A> and <B > are two magic squares.

• FISSION

This method requires splitting of a magic square into two or more than two magic squares. For eg.

204					1
	16	64	40	84	
	85	39	61	19	
	62	18	86	38	
	41	83	17	63	

Fisses into

19					0.25
	1	6	4	8	
	8.25	3.75	5.75	1.75	
	5.5	1.5	8.5	3.5	
	4.25	7.75	12.	57.5	

14					-1.5
	6	4	0	4	
	2.5	1.5	8.5	1.5	
	7	3	1	3	
	-1.5	5.5	4.5	5.5	

- **Rules for disintegration (fission) -**

1. Splitting the elements of 1st row by 0° unlocked (explained further).

2. Creating 2 independent magic squares by Anu structure.

3. The sum constants of the two magic squares when 1° locked (explained further) will yield the original sum constant.

As, 1 9

$$\begin{array}{r} 1\ 9 \\ +\ \ 1\ 4 \\ \hline 2\ 0\ 4. \\ \hline \end{array}$$

- Types of Fusion and Fission –

Numeric Interlocking

1. a) 0° Lock – In this type the element are fused simply.
 Eg. 16 and 64 if 0° locked will become 1664.
 10 and 2 if 0° locked will become 1002.
 102 and 8 if 0° locked will become 102008.
 In short, If a is n digit number and show 0° lock with b, then the fused number will be { a * (10^n) } + b .

b) 0° unlock - It is the reverse of lock.

Eg. 16 is 0° unlocked , it will become 1 and 6.

Like wise, 1664 can be 0 unlocked to

1 and 664.

16 and 64.

166 and 4.

Note: After 0 locking, during fusion, the sum constant also shows 0 lock. And control factor changes as

x° = { x * (10^n) } + y.

where, x° = control factor of fused square <C>.

x = control factor of <A>.

y = control factor of <B>.

n = number of maximum digits present in the square.

And <A> FU <B> = <C>.

2. a) 1° Lock
   The 1st digit of the latter numeral will be added to the former numeral and the rest will be written as it is.
   For eg. 16 1° lock 48 =

$$\begin{array}{r} 1\,6 \\ +\ \ 4\,8 \\ \hline 2\,0\,8 \end{array}$$

b) 1° Unlock

This involves simple splitting .

Likewise Locking and Unlocking can be done to any degree.

• RULE 5.

This section is the most important and special tool.

"All Magic squares in the world are either made by our formulas or they are not perfect."

Let us consider the given formulae,

ANU Structure

a	b	c	d
d + x	c - x	b - 3x	a + 3x
b - 2x	a + 2x	d + 2x	c - 2x
c + x	d - x	d + x	b - x

Provided that $a + d + 4x = b + c$. (Ank Eqn)

Note- here two types of ank eqn are described.
$a + d + 4x = b + c$.
$a + b + 4x = c + d$. Various others are also possible.

Step 1. interchange $b \leftrightarrow d$. In Ank Eqn. and also in structure of magic square.

Ank Eqn.

$a + b + 4x = c + d$.

a	d	c	b
b + x	c - x	d - 3x	a + 3x
d - 2x	a + 2x	b + 2 x	c - 2x
c + x	b - x	a + x	d - x

Step 2.

> Note- The substitution which do not alter the Ank Eqn will not change the perfectness of magic square.

$a = A - x.$ $b = B + x$ $c = C - x.$ $d = D + x.$

$a + b + 4x = c + d$. (Substituting the values)

$A + B + 4X = C + D$. (No change in Ank Eqn. so we can do the same in structure).

A - x	D + x	C - x	B + x
B + 2x	C - 2x	D – 2x	A + 2x
D - x	A + x	B + 3x	C - 3x
C	B	A	D

$A + B + 4x = C + D.$

Step 3. Apply property of reflection. (Replacing x by – x).

C	B	A	D
D + x	A - x	B - 3x	C +3x
B - 2x	C + 2x	D + 2x	A - 2x
A + x	D - x	C + x	B - x

A + B - 4x = C + D.

Step 4. Apply RULE 2.b)ii (see on page 3).

i.e. Column 1^{st} ↔ Column 3^{rd} correspondingly Row 2^{nd} ↔ Row 4^{th} .

A	B	C	D
C + x	D - x	A + x	B - x
D + 2x	C + 2x	B - 2x	A - 2x
B - 3x	A - x	D + x	C + 3x

A + B – 4x = C + D.

Now, this is another formula of magic square derived from the original one.

Similarly, (a + c + 4x = b + d. Ank Eqn.) Magic square can be created.

Now, replacing x by –x in the Ank Eqn. then same change in square structure.

Taking a = 7, b = 12, c = 1, d = 14 in the above formulae.

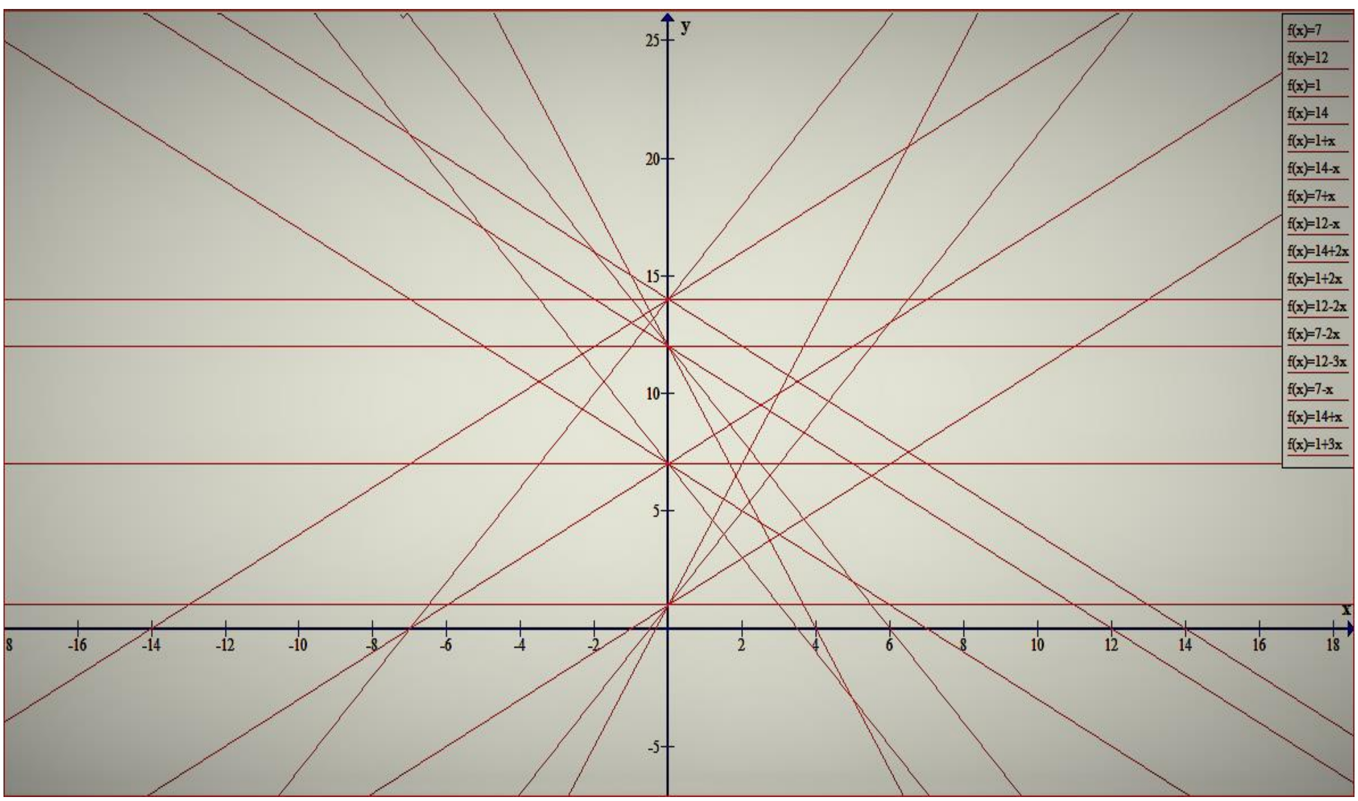
7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

FACT - This Square was inscribed on the walls of Parshvannath temple in 12^{th} century AD, located in Khajuraho in Madhya Pradesh, INDIA.

FAMILY OF SQUARES

The elements of a magic square can be considered as separate linear equations of straight lines of a particular structure. Using the graph of the 16 elements one can find all the elements of a square whose 1st row elements are known at any particular value of x (control factor). For doing so one has to take a desired value of x, draw its graph and note down its intersections to each line sequentially. Now, Square from any 4 values using a particular structure at any x is created.

Here, diagrammatic representation of square mentioned just above has been given. That will yield the sum constant of 34.



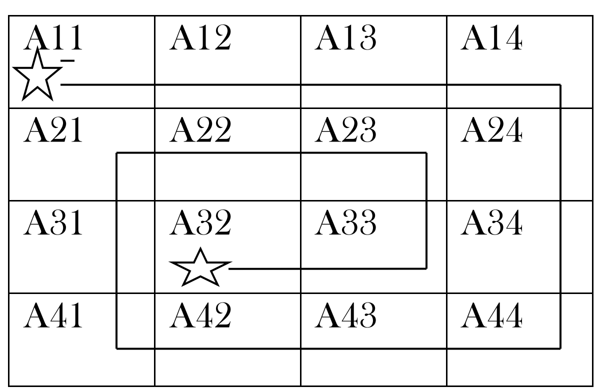
PROPERTIES

1. Any two or more magic squares of the same order can be simply added as matrix addition. And the sum constant of new square also gets added up.
2. Any two squares can be subtracted to get a new magic square under a condition that x (control factor) of the two squares must be same. And the sum constant of new square gets subtracted in the same manner.
3. If a single numerical value gets added or subtracted to the magic square that implies it is added or subtract from each element of the square and the sum constant will increase or decrease by 4 times the value added or subtracted.
4. The magic square will yield the same sum constant taking any arrangement of vertices of square within the magic square.
5. Now the below figure represents a unique property of this square.

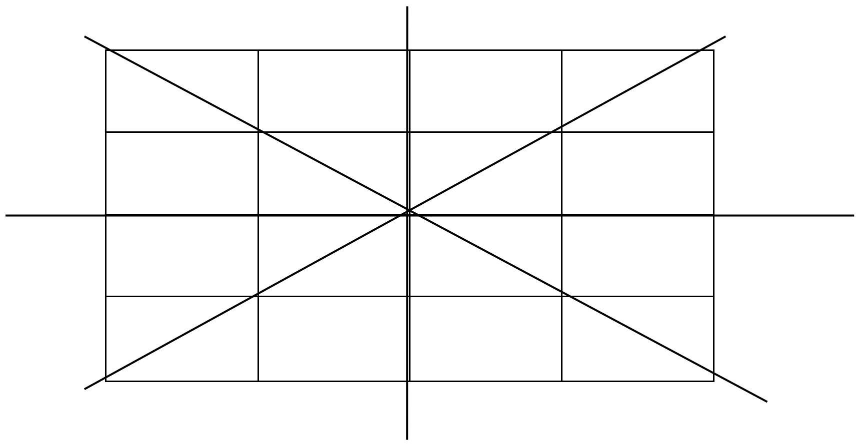
   Starting from A11 the line follows a spiral path and ends at A32. Now the interesting point- Taking the sum of the numbers alternatively starting from A11, A13, A24, A44, A42, A31, A22, A33.

   And likewise A12, A14, A34, A43, A41, A21, A23, A32.

   Now the difference of these two sums will be zero.

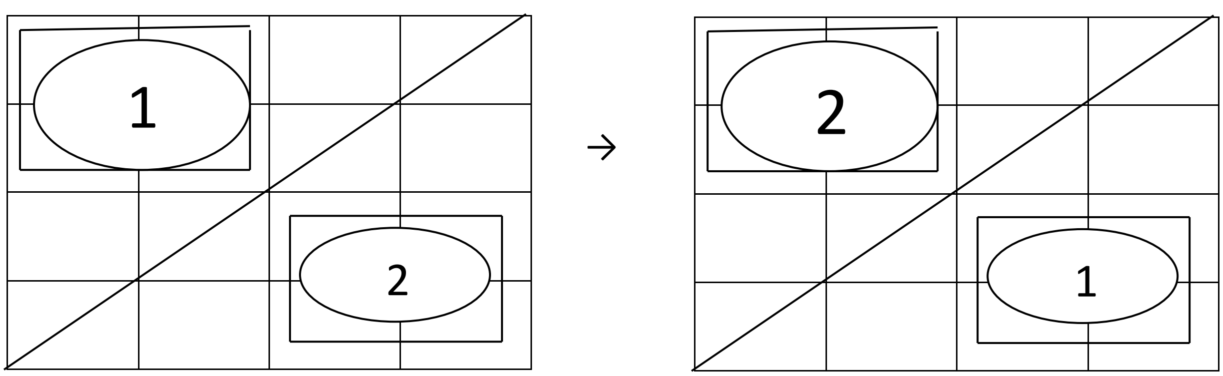


6. Line of symmetry



The Magic Square has 4 set of symmetries. 2 diagonals, a vertical and a horizontal. If line symmetry is considered as mirror and elements are inter reflected, then it will give new stereo-square of which sum constant remains same.

For e.g.



THE END