Comment on “A Motion Paradox from Einstein´s Relativity of Simultaneity”

Rodrigo de Abreu
Departamento de Física, Instituto Superior Técnico
Universidade Técnica de Lisboa, 1049-001 Lisboa, Portugal

Abstract

In this paper we analyse “a new and potentially important paradox related to Einstein’s theories of special relativity and relativity of simultaneity” introduced by Espen Gaarder Haug in a very interesting recent paper. Based in our previous work we show that there is no paradox since it is impossible that “one reference frame will claim that the train is moving and that the other frame must claim that the train is standing still in the time window “between” two distant events”. The world indeed is not “bizarre” since there is an indeterminacy in the standard formulation of the theory that disappear if we consider a third frame where the one-way speed of light is isotropic.

Introduction

In a recent paper [1] Haug claim that there is a potentially paradox in Special Relativity (SR), the “Motion Paradox”. He also claim that two distant events that happen simultaneously in one reference frame cannot happen simultaneously as observed from another reference frame. We show that this affirmation is only true in relation to Einstein’s simultaneity. It is not true in relation to simultaneity as we have shown in our previous work in a broad approach of the theory [2-15]. Indeed Einstein’s simultaneity is defined with clocks Einstein synchronized, Lorentzian clocks. In this broad approach of SR only in one frame Lorentzian clocks are synchronized, only in the frame where the one-way speed of light is isotropic with the value c (the measured two-way speed of light in any frame in vacuum). In another frame moving in relation to this frame that we previously call Einstein’s frame (EF), the Lorentzian clocks are desynchronized. Einstein’s simultaneity maintain the criterium of simultaneity with desynchronized clocks distant apart, the reading of the clocks marking the same number. This criterium, naturally, originates a huge terminological confusion. With this criterium if we are not aware of the meaning of what we are doing or saying the world can be seen as a bizarre world where paradoxes happen everywhere. Like the Motion Paradox [1]. Or the Twin Paradox [12, 13]. All kind of paradoxes emerge (by the way this is not specific with SR since we can classically define with desynchronized clocks “simultaneity” with the reading of the clocks marking the same number as it happen in “Summer Time” change of hour, originating eventually a prejudicial confusion when we are not aware of the change of the hour). Following Feynman, a "paradox is a situation which gives one answer when analysed one way, and a different answer when analysed another way, so that we are left in somewhat of a quandary as to actually what would happen. Of course, in physics there are never any real paradoxes because there is only one correct answer; at least we believe that nature will act in only one way (and that is the right way, naturally). So in physics a paradox is only a confusion in our understanding " [16]. This is why is so important to debate a paradox, finding the correct answer if the theory permit so. Special Relativity can solve the “Motion Paradox”.

In Ia we consider two rods S and S´. S is at rest in EF and S´ is moving longitudinally, in the direction defined by the rods. We obtain eq. (1) of the article of Haug without any paradox. The events are simultaneous in EF and are also simultaneous in frame S´ [9, 11]. The time window referred by Haug is a result of the desynchronization of the two Lorentzian clocks in frame S´, located at the extremities of rod S´. Therefore there is no paradox since there is no movement associated to the Lorentzian time gap in S´.

In Ib we consider two rods S´ and S´´ moving in relation to EF with speed v_1 and v_2 respectively. As shown previously in a recent article [14] the events that Haug consider correspond to the coincidence of the extremities of the rods A´ and A´´ and B´ and B´´ satisfying the condition that in one of those frames the coincidence is Einstein simultaneous. In this generic case (v_1 and v_2 are different of zero) the events cannot be simultaneous whatever the frame considered, although in one frame the events are Einstein simultaneous. Therefore in this generic case we have indeed a time gap between the events in both frames (since the events are not simultaneous) and both frames moves in relation to EF and in relation of each other without any paradox.

We also interpret Ib with the principle of relativity in a restricted sense [5, 9, 11]. Indeed in this restricted sense with Lorentzian coordinates we have for Ib the same description that we observe in Ia. This situation can be considered bizarre and can originate the paradox if we consider the Principle of Relativity as standard formulation does affirming the equivalence of all frames. Since in the standard formulation allegedly EF does not exist, the gap in Ib is not considered [14] and therefore it seems that we have the "Motion Paradox" as pointed out by Haug [1].

I. The resolution of the "Motion Paradox"

Ia. One of the rods is at rest in the EF

Consider a rod S´ with proper length l_1 moving with speed v_1 in relation to EF where is located another bar S with proper length l_0 (Fig.1).

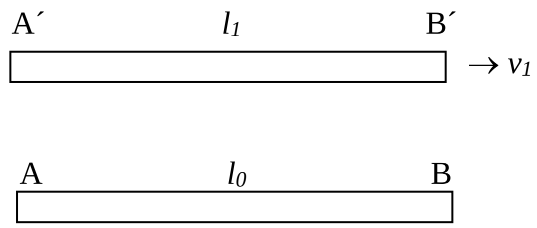

Figure 1. Rod S´ is moving with speed v_1 in relation to rod S at rest in EF. The extremities of the rods coincide simultaneously and therefore can synchronize clocks at A, A´ and B, B´

Rod S´ is moving with speed v_1. Since the bar S´ is Lorentz contracted (since rod S is at rest in the EF) we know l_1 when the extremities of the rods pass by each other simultaneously, when A´ coincide with A and B´ with B as represented in the figure 1. This is the most primitive notion of simultaneity that Special Relativity does not ruled out. However standard interpretation induce to think that it is impossible to synchronize clocks because it is not possible to send a signal from A´ to B´ with infinite speed and since the one-way speed of light was not known in frame S´ Einstein postulate that the one-way light speed is *c*. In this context this affirmation must be ruled out [14]. Indeed we have

$$l_1 = \frac{l_0}{\sqrt{1-\frac{v_1^2}{c^2}}} \quad (1)$$

This relation can also be obtained, naturally, from the Lorentz transformation

$$x' = \frac{x-v_1\, t}{\sqrt{1-\frac{v_1^2}{c^2}}} \quad (2)$$

for $x=l_0$ and $t=0$ the coordinates of the extremity B of rod S.

The Lorentz time at B´ is obtained from

$$t'_L = \frac{t-v_1\, x/c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} \quad (3)$$

for $x=l_0$ and $t=0$ the coordinates of the extremity B of rod S and from (1)

$$t'_L = -v_1\, l1/c^2 \quad (4)$$

Therefore the Lorentz clock at B´ is desynchronized in relation to the clock at A´ marking 0 and of course it is also desynchronized in relation to a synchronized clock at B´ marking also 0, $t'=0$ where t' is the synchronized time coordinate [2, 6, 7, 9, 11]. This is an intrinsic desynchronization of the Lorentzian clocks at S´ that standard relativity avoid to refer [2] inducing that the Lorentzian clocks at S´ are truly synchronized. If the Lorentzian clocks are consider synchronized the "Motion Paradox" seems to follow. A similar problem can be formulated with classic physics without relativistic effects as in the example of the F1 car referred in the comment of the article of Greaves et al. [17, 9]. Therefore exist a time gap $0 - t'_L = v_1 \frac{l1}{c^2}$ eq. (1) of Haug article. Because the clocks are desynchronized. No movement. No paradox.

Ib. Two rods moving in relation to EF

We introduce now a third rod S´´ with length l_2.

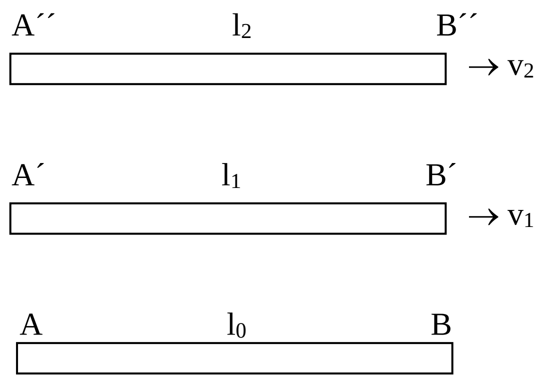

Figure 2. A third rod S´´ is moving with speed v_2 in relation to EF synchronizing the clock at B´ with A´, when A´´ pass by A´ and B´´ by B´

The rod S´´ is moving with speed v_2 in relation to EF. The rod have proper length l_2

$$l_2 = \frac{l_0}{\sqrt{(1-\frac{v_2^2}{c^2})}} \quad (5)$$

From (1) and (5) we have

$$l_2 = l_1 \frac{\sqrt{(1-\frac{v_1^2}{c^2})}}{\sqrt{(1-\frac{v_2^2}{c^2})}} \quad (6)$$

It is easy to obtain from (6)

$$l_2 = \frac{l_1}{\sqrt{\left(1-\frac{v_E'^2}{c^2}\right)}}\left(1+\frac{v_1\, v'_E}{c^2}\right) \quad (7)$$

since Einstein's speed [5, 6, 9] of rod S´´ in relation to S´ is given by

$$v_E' = \frac{v_2 - v_1}{1-\frac{v_1 v_2}{c^2}} \quad (8)$$

The correct evaluation of the distance l_2 is crucial as we pointed out in several previous works [2, 5, 12-14] (see also [9, 18]) and used to solve the Twin Paradox in a one-way trip [13] analysing the approach of Φ. Grϕn [18] and in the experimental determination of the one-way speed of light [14].

We see from eq. (7) that we can consider two lengths

$$l_2 = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}} \quad (9)$$

and

$$l_2 = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}\left(1+\frac{v_1\, v'_E}{c^2}\right) \quad (10)$$

When v_1=0 S´ is at rest in EF we have only for l_2 the value given by (9). However when $v_1 \neq 0$ and $v_2 \neq 0$ there are several values of l_2 between the values given by (9) and (10). We can define these several values by

$$l_2 = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}\left(1+\alpha\,\frac{v_1\, v'_E}{c^2}\right) \quad (11)$$

with $\alpha \in [0, 1]$.

Eq. (10) can also be obtained from Lorentz Transformation. We have

$$x'' = \frac{x' - v'_E\; t'_L}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}} \quad (12)$$

and for the passing of B´´ by B´, $x'=l_1$ and $t'_L = -(v_1/c^2)l_1$. Substituting these values into eq. (12) we obtain eq. (10).

Consider now Fig. 3. When $x'=l_1$ and $t'_L = 0$ we have the passing of extremity B´´ of rod S´´ with length $l_2 = l_1/\sqrt{1-\frac{v'^2_E}{c^2}}$ by B´ from eq. (12)

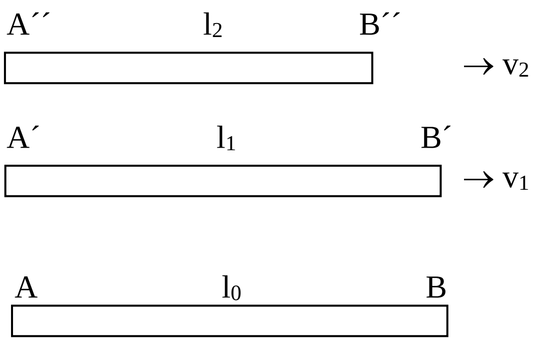

Figure 3. S´´ with the proper length l_2 that "synchronize" a Lorentzian clock located at B´ with the clock located at A´ when B´´ pass by B´ (eq. (10) for $\alpha = 0$)

Since we know $v_E^{\prime}$ we know l_2 given by (9). Therefore we can obtain eq. (4) from the Lorentz transformation (the following eq. (13)) for $x' = l_{1,}\ t'_L = 0$ when $v'_E = v_1$ and $l_1 = l_0$

$$t''_L = \frac{t'_L - v'_E\, x'/c^2}{\sqrt{1-\frac{v'^2_E}{c^2}}} \quad (13)$$

We obtain therefore the same time gap obtained previously at Ia. But the meaning now is obviously different. We have two events located at the extremities of the rod S´, A´ and B´ and two events at the extremities of rod S´´, A´´ and B´´. This events are defined by the coincidence of extremities A´´ with A´ and B´´ with B´. Since the relation between the proper lengths correspond formally to the Lorentz contraction of Ia we have the same description with Lorentz coordinates as in Ia as the Principle of Relativity prescribe [9]. The events in S in Ia are Einstein synchronous and also synchronous (since S is the EF) although are not Einstein synchronous in S´. The frames are not equivalent [9]. The rod S´´ is shorter then rod S´ (Fig. 3) and both rods moves in relation to each other consistently with SR. We do not have a paradox.

To clearly show that we do not have any inconsistency we can calculate the travelled distance of B´ in S´´ and of B´´ in S´ between the events (Fig. 3). Obviously the distance travelled by B´ in S´´ is

$$\frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}\left(1+\frac{v_1\, v'_E}{c^2}\right) - \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}} = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}\frac{v_1\, v'_E}{c^2} \quad (14)$$

And the distance travelled by B´´ is

$$l_1 - l_1^* = l_1 - \frac{l_1}{\left(1+\frac{v_1\, v'_E}{c^2}\right)} = \frac{l_1}{\left(1+\frac{v_1\, v'_E}{c^2}\right)}\frac{v_1\, v'_E}{c^2} \quad (15)$$

since l_1^* the coordinate of B´´ at S´ can be obtained from (12) for $x' = l_1^*$ and $t_L^{\prime} = -\frac{v_1}{c^2} l_1^*$

$$l_2 = \frac{l_1^*}{\sqrt{1-\frac{v'^2_E}{c^2}}}\left(1+\frac{v_1\, v'_E}{c^2}\right) = \frac{l_1}{\sqrt{1-\frac{v'^2_E}{c^2}}} \quad (16)$$

Therefore we obtain the Lorentzian travel times for the trips at S´´ and S´

$$\Delta t''_L = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}\frac{v_1}{c^2} \quad (17)$$

$$\Delta t'_L = \frac{l_1}{\left(1+\frac{v_1\, v'_E}{c^2}\right)}\frac{v_1}{c^2} \quad (18)$$

For v_1=0 the Lorentzian travel times are zero as they must be (Ia).

We can check the consistency of both equations (17) and (18) with the proper times that gives the time travel for both trips

$$\tau'' = \Delta t'_L \sqrt{1 - \frac{v'^2_E}{c^2}} = \frac{l_1}{\left(1+\frac{v_1 v'_E}{c^2}\right)} \frac{v_1}{c^2} \sqrt{1 - \frac{v'^2_E}{c^2}} \qquad (18)$$

$$\tau' = \Delta t''_L \sqrt{1 - \frac{v'^2_E}{c^2}} = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}} \frac{v_1}{c^2} \sqrt{1 - \frac{v'^2_E}{c^2}} \qquad (19)$$

Therefore from (18) and (19)

$$\tau'' = \frac{\tau'}{\left(1+\frac{v_1 v'_E}{c^2}\right)} \sqrt{1 - \frac{v'^2_E}{c^2}} \qquad (20)$$

Eq. (20) is the equation of proper times obtained previously [2, 5, 11] and used to solve the twin paradox [12, 13].

We can obtain eq. (20) from Larmor's time dilation [21]. Indeed in relation to EF the proper time between two events are given by

$$\tau'' = \Delta t \sqrt{1 - \frac{v_2^2}{c^2}} \qquad (21)$$

$$\tau' = \Delta t \sqrt{1 - \frac{v_1^2}{c^2}} \qquad (22)$$

Since $v'_E = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}}$ we obtain as previously

$$\tau'' = \tau' \frac{\sqrt{1-\frac{v_2^2}{c^2}}}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{\tau'}{\left(1+\frac{v_1 v'_E}{c^2}\right)} \sqrt{1 - \frac{v'^2_E}{c^2}} \qquad (23)$$

Conclusion

It is our firm belief that physics should assume itself as the heir of natural philosophy. And thus question, with no fear nor prejudice, the postulates or hypothesis at the origin of each theory. Only in this way is it possible to claim that to understand a physical theory goes much beyond the simple knowledge of how to perform the calculations. Unfortunately, special relativity is presented in most textbooks by passing too swiftly over the discussion of its postulates [9, 19].

In this paper we analyse "a new and potentially important paradox related to Einstein's theories of special relativity and relativity of simultaneity" introduced by Espen Gaarder Haug in a very interesting recent paper formulating "The Motion Paradox" [1] . Based in our previous work we show that there is no paradox since it is impossible that "one reference frame will claim that the train is moving and that the other frame must claim that the train is standing still in the time window "between" two distant events"

In section I we consider several configurations of three rods designated by S, S´ and S´´ moving relatively to each other longitudinally. The idea is that rods moving in relation to each other can revealed the movement in relation to the frame where the one-way speed of light is isotropic with value *c*, the frame that we designate by Einstein Frame (EF) because the movement of the rods in relation to EF can affect differently each rod and this effect can be observable [14]. A similar idea has been defended by Espen Haug [20] with several pertinent questions in relation to the difficulty to conceive absolute simultaneity and also in this more recent article formulating the "Motion Paradox" [1].

In section Ia we consider a rod S´ moving with speed v_1 in relation to EF where rod S is at rest. Since S´ is Lorentz-Fitzgerald contracted it is easy to show that the "Motion Paradox" does not exist. Although exist a time gap in S´ because we are considering Lorentzian clocks the coincidence of the extremities of the rods are simultaneous and therefore there is no movement. The time gap in this case is a result only of the desynchronization of the clocks considered. With synchronized clocks in S´ there is no time gap consistent with the simultaneity revealed in S where the Lorentz clocks are also synchronized since in EF the one-way speed of light is c. In section Ib we consider a third rod moving with speed v_2 in relation to EF and we obtain the relation between the proper lengths of rods that reveals a gap of possible "synchronizations" that preserve the value *c* for the two-way speed of light [14] and reveals that in this case exist movement without paradox. We also confront our analysis with the Principle of Relativity [9].

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