The black hole binary gravitons and thermodynamics

Miroslav Pardy
Department of Physical Electronics
Masaryk University,
Kotlářská 2, 611 37 Brno, Czech Republic
e-mail:pamir@physics.muni.cz

October 19, 2018

Abstract

The energy spectrum of graviton emitted by the black hole binary is calculated in the first part of the chapter. Then, the total quantum loss of energy, is calculated in the Schwinger theory of gravity.

In the next part we determine the electromagnetic shift of energy levels of H-atom electrons by calculating an electron coupling to the black hole thermal bath. Energy shift of electrons in H-atom is determined in the framework of nonrelativistic quantum mechanics.

In the last section we determine the velocity of sound in the black hole atmosphere, which is here considered as the black hole photon sea. Derivation is based on the thermodynamic theory of the black hole photon gas.

Keywords Graviton, Schwinger source theory, spectrum of H-Atom, Coulomb potential, black hole spektrum, energy shift, sound.

1 The graviton spectrum of the black hole binary

In 1916, Schwarzschild published the solution of the Einstein field equations [1] that was later understood to describe a black hole [2,3] and in 1963 Kerr generalized the solution to rotating black holes [4]. The year 1970 was the starting point of the theoretical work leading to the understanding of black hole quasinormal modes [5, 6, 7], and in the 1990 higher-order post-Newtonian calculations [8] was performed and later the extensive analytical studies of relativistic two-body dynamics realized [9,10]. These advances, together with numerical relativity breakthroughs in the past decade [11, 12, 13], have enabled modeling of binary black hole mergers and accurate predictions of their gravitational waveforms. While numerous black hole candidates have now been identified through electromagnetic observations [14,15,16], black hole binary mergers have not been still observed. Nevertheles, the black hole binary and their rotation and mergers is open problem of the astrophysics and is is the integral part of the binary black hole and binary pulsar physics.

The binary pulsar system PSR B**1913+16** (also known as PSR J1915+1606) discovered by Hulse and Taylor [17] and subsequent observations of its energy loss by Taylor and Weisberg [18] demonstrated the existence of gravitational waves [19].

By the early 2000s, a set of initial detectors was completed, including TAMA 300 in Japan, GEO 600 in Germany, the Laser Interferometer Gravitational-Wave Observatory (LIGO) in the United States, and Virgo in Italy. In 2015, Advanced LIGO became the first of a significantly more sensitive network of advanced detectors (a second-generation interferometric gravitational wave detector) to begin observations [20].

Taylor and Hulse, working at the Arecibo radiotelescope, discovered the radiopulsar PSR B**1913+16** in a binary, in 1974, and this is now considered as the best general relativistic laboratory [21].

Pulsar PSR B**1913+16** is the massive body of the binary system where each of the rotating pairs is 1.4 times the mass of the Sun. These neutron stars rotate around each other in an orbit not much larger than the Sun's diameter, with a period 7.8 hours. Every 59 ms, the pulsar emits a short signal that is so clear that the arrival time of a 5-min string of a set of such signals can be resolved within 15 μs.

A pulsar model based on strongly magnetized, rapidly spinning neutron stars was soon established as consistent with most of the known facts [22] its electrodynamical properties were studied theoretically [23] and shown to be plausibly capable of generating broadband radio noise detectable over interstellar distances. The binary pulsar PSR B**1913+16** is now recognized as the harbinger of a new class of unusually short-period pulsars, with numerous important applications.

Because the velocities and gravitational energies in a high-mass binary pulsar system can be significantly relativistic, strong-field and radiative effects come into play. The binary pulsar PSR B**1913+16** provides significant tests of gravitation beyond the weak-field, slow-motion limit [24; 25].

We do not here repeat the derivation of the Einstein quadrupole formula in the Schwinger gravity theory [26]. We show that just in the framework of the Schwinger gravity theory it is easy to determine the spectral formula for emitted gravitons and the quantum energy-loss formula of the binary system. The energy-loss formula is general, including black hole binary and it involves arbitrarily strong gravity.

Since the measurement of the motion of the black hole binaries goes on, we hope that sooner or later the confirmation of our formula will be established.

1.1 The Schwinger approach the problem

Source methods by Schwinger are adequate for the solution of the calculation of the spectral formula of gravitons and energy loss of binary. Source theory [27–28] was initially constructed to describe the particle physics situations occurring in high-energy physics experiments. However, it was found that the original formulation simplifies the calculations in the electrodynamics and gravity, where the interactions are mediated by photon and graviton respectively. The source theory of gravity forms the analogue of quantum electrodynamics because, while in QED the interaction is mediated by the photon, the gravitational interaction is mediated by the graviton [29]. The basic formula in the source theory is the vacuum-to-vacuum amplitude [30]:

$$\langle 0_+|0_-\rangle = e^{\frac{i}{\hbar}W(S)}, \tag{1}$$

where the minus and plus tags on the vacuum symbol are causal labels, referring to any time before and after region of space-time, where sources are manipulated. The exponential form is introduced with regard to the existence of the physically independent experimental arrangements, which has the simple consequence that the associated probability amplitudes multiply and the corresponding W expressions add [27; 28].

In the flat space-time, the field of gravitons is described by the amplitude (1) with the action ($c = 1$ in the following text) [31]:

$$W(T) = 4\pi G \int (dx)(dx') \left[T^{\mu\nu}(x) D_+(x - x') T_{\mu\nu}(x') - \frac{1}{2} T(x) D_+(x - x') T(x') \right], \tag{2}$$

where the dimensionality of $W(T)$ is the same as the dimensionality of the Planck constant $\hbar$; $T_{\mu\nu}$ is the tensor of momentum and energy that, for a particle moving along the trajectory $\mathbf{x} = \mathbf{x}(t)$, is defined by the equation [32]:

$$T^{\mu\nu}(x) = \frac{p^\mu p^\nu}{E} \delta(\mathbf{x} - \mathbf{x}(t)), \tag{3}$$

where p^μ is the relativistic four-momentum of a particle with a rest mass m and

$$p^\mu = (E, \mathbf{p}) \tag{4}$$

$$p^\mu p_\mu = -m^2, \tag{5}$$

and the relativistic energy is defined by the known relation

$$E = \frac{m}{\sqrt{1 - \mathbf{v}^2}}, \tag{6}$$

where $\mathbf{v}$ is the three-velocity of the moving particle.

Symbol $T(x)$ in formula (2) is defined as $T = g_{\mu\nu} T^{\mu\nu}$, and $D_+(x - x')$ is the graviton propagator whose explicit form will be determined later.

1.2 The power spectral formula in general

It may be easy to show that the probability of the persistence of vacuum is given by the following formula [27]:

$$|\langle 0_+|0_-\rangle|^2 = \exp\left\{ -\frac{2}{\hbar} \mathrm{Im} W \right\} \stackrel{d}{=} \exp\left\{ -\int dt d\omega \frac{1}{\hbar\omega} P(\omega, t) \right\}, \tag{7}$$

where the so-called power spectral function $P(\omega, t)$ has been introduced [27]. In order to extract this spectral function from Im W, it is necessary to know the explicit form of the graviton propagator $D_+(x - x')$. The physical contents of this propagator is analogous to the contents of the photon propagator. It involves the graviton property of spreading with velocity c. It means that its explicit form is just the same as that of the photon propagator. With regard to Schwinger et al. [33] the x-representation of $D(k)$ in eq. (2) is as follows:

$$D_+(x - x') = \int \frac{(dk)}{(2\pi)^4} e^{ik(x-x')} D(k), \tag{8}$$

where

$$D(k) = \frac{1}{|\mathbf{k}^2| - (k^0)^2 - i\epsilon}, \tag{9}$$

which gives

$$D_+(x - x') = \frac{i}{4\pi^2} \int_0^\infty d\omega \frac{\sin \omega |\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} e^{-i\omega|t-t'|}. \tag{10}$$

Now, using formulae (2), (7) and (10), we get the power spectral formula in the following form:

$$P(\omega, t) = 4\pi G\omega \int (d\mathbf{x})(d\mathbf{x}')dt' \frac{\sin \omega|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} \cos \omega(t - t') \times$$

$$\left[T^{\mu\nu}(\mathbf{x}, t)T_{\mu\nu}(\mathbf{x}', t') - \frac{1}{2} g_{\mu\nu}T^{\mu\nu}(\mathbf{x}, t)g_{\alpha\beta}T^{\alpha\beta}(\mathbf{x}', t')\right]. \tag{11}$$

1.3 The power spectral formula for the binary system

In the case of the binary system with masses m_1 and m_2, we suppose that they move in a uniform circular motion around their centre of gravity in the xy plane, with corresponding kinematical coordinates:

$$\mathbf{x}_1(t) = r_1(\mathbf{i} \cos(\omega_0 t) + \mathbf{j} \sin(\omega_0 t)) \tag{12}$$

$$\mathbf{x}_2(t) = r_2(\mathbf{i} \cos(\omega_0 t + \pi) + \mathbf{j} \sin(\omega_0 t + \pi)) \tag{13}$$

with

$$\mathbf{v}_i(t) = d\mathbf{x}_i/dt, \quad \omega_0 = v_i/r_i, \quad v_i = |\mathbf{v}_i| \quad (i = 1, 2). \tag{14}$$

For the tensor of energy and momentum of the binary we have:

$$T^{\mu\nu}(x) = \frac{p_1^\mu p_1^\nu}{E_1} \delta(\mathbf{x} - \mathbf{x}_1(t)) + \frac{p_2^\mu p_2^\nu}{E_2} \delta(\mathbf{x} - \mathbf{x}_2(t)), \tag{15}$$

where we have omitted the tensor $t_{\mu\nu}^G$, which is associated with the massless, gravitational field distributed all over space and proportional to the gravitational constant G [32]:

After insertion of eq. (15) into eq. (11), we get [33]:

$$P_{total}(\omega,t) = P_1(\omega,t) + P_{12}(\omega,t) + P_2(\omega,t), \tag{16}$$

where $(t' - t = \tau)$:

$$P_1(\omega,t) = \frac{G\omega}{r_1\pi}\int_{-\infty}^{\infty} d\tau \frac{\sin[2\omega r_1 \sin(\omega_0\tau/2)]}{\sin(\omega_0\tau/2)}\cos\omega\tau\times$$

$$\left(E_1^2(\omega_0^2 r_1^2\cos\omega_0\tau - 1)^2 - \frac{m_1^4}{2E_1^2}\right), \tag{17}$$

$$P_2(\omega,t) = \frac{G\omega}{r_2\pi}\int_{-\infty}^{\infty} d\tau \frac{\sin[2\omega r_2 \sin(\omega_0\tau/2)]}{\sin(\omega_0\tau/2)}\cos\omega\tau\times$$

$$\left(E_2^2(\omega_0^2 r_2^2\cos\omega_0\tau - 1)^2 - \frac{m_2^4}{2E_2^2}\right), \tag{18}$$

$$P_{12}(\omega,t) = \frac{4G\omega}{\pi}\int_{-\infty}^{\infty} d\tau \frac{\sin\omega[r_1^2 + r_2^2 + 2r_1r_2\cos(\omega_0\tau)]^{1/2}}{[r_1^2 + r_2^2 + 2r_1r_2\cos(\omega_0\tau)]^{1/2}}\cos\omega\tau\times$$

$$\left(E_1E_2(\omega_0^2 r_1 r_2\cos\omega_0\tau + 1)^2 - \frac{m_1^2m_2^2}{2E_1E_2}\right). \tag{19}$$

1.4 The quantum energy loss of the binary

Using the following relations

$$\omega_0\tau = \varphi + 2\pi l, \qquad \varphi\in(-\pi,\pi), \quad l = 0,\ \pm1,\ \pm2,\ ... \tag{20}$$

$$\sum_{l=-\infty}^{l=\infty}\cos 2\pi l\frac{\omega}{\omega_0} = \sum_{l=-\infty}^{\infty}\omega_0\delta(\omega-\omega_0 l), \tag{21}$$

we get for $P_i(\omega,t)$, with ω being restricted to positive:

$$P_i(\omega,t) = \sum_{l=1}^{\infty}\delta(\omega-\omega_0 l)P_{il}(\omega,t). \tag{22}$$

Using the definition of the Bessel function $J_{2l}(z)$

$$J_{2l}(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\varphi\cos\left(z\sin\frac{\varphi}{2}\right)\cos l\varphi, \tag{23}$$

from which the derivatives and their integrals follow, we get for P_{1l} and P_{2l} the following formulae:

$$P_{il} = \frac{2G\omega}{r_i}\Big((E_i^2(v_i^2-1) - \frac{m_i^4}{2E_i^2}\Big)\int_0^{2v_il} dx\, J_{2l}(x) +$$

$$4E_i^2(v_i^2-1)v_i^2 J'_{2l}(2v_i l) + 4E_i^2 v_i^4 J'''_{2l}(2v_i l)\Big), \quad i = 1,\, 2. \tag{24}$$

Using $r_2 = r_1 + \epsilon$, where ϵ is supposed to be small in comparison with radii r_1 and r_2, we obtain

$$[r_1^2 + r_2^2 + 2r_1 r_2 \cos\varphi]^{1/2} \approx 2a\cos\left(\frac{\varphi}{2}\right), \tag{25}$$

with

$$a = r_1\left(1 + \frac{\epsilon}{2r_1}\right). \tag{26}$$

So, instead of eq. (19) we get:

$$P_{12}(\omega, t) = \frac{2G\omega}{a\pi}\int_{-\infty}^{\infty} d\tau \frac{\sin[2\omega a\cos(\omega_0\tau/2)]}{\cos(\omega_0\tau)/2]}\cos\omega\tau\times$$

$$\left(E_1E_2(\omega_0^2 r_1 r_2\cos\omega_0\tau + 1)^2 - \frac{m_1^2 m_2^2}{2E_1E_2}\right). \tag{27}$$

Now, we can approach the evaluation of the energy-loss formula for the binary from the power spectral formulae (24) and (27). The energy loss is defined by the relation

$$-\frac{dU}{dt} = \int P(\omega)d\omega =$$

$$\int d\omega\sum_{i,l}\delta(\omega - \omega_0 l)P_{il} + \int P_{12}(\omega)d\omega = -\frac{d}{dt}(U_1 + U_2 + U_{12}). \tag{28}$$

From [34] we have Kapteyn's formula

$$\sum_{l=1}^{\infty}\frac{J_{2l}(2lv)}{l^2} = \frac{v^2}{2}. \tag{29}$$

After differentiating the last relation with respect to v, we have

$$\sum_{l=1}^{\infty} lJ'''_{2l}(2lv) = 0. \tag{30}$$

From [34] we learn other Kapteyn's formulae:

$$\sum_{l=1}^{\infty} 2lJ'_{2l}(2lv) = \frac{v}{(1-v^2)^2}, \tag{31}$$

and

$$\sum_{l=1}^{\infty} l\int_0^{2lv} J_{2l}(x)dx = \frac{v^3}{3(1-v^2)^3}. \tag{32}$$

So, after application of eqs. (30), (31) and (32) to eqs. (24) and (28), we get:

$$-\frac{dU_i}{dt} = \frac{Gm_i^2 v_i^3 \omega_0}{3r_i(1-v_i^2)^3}[13v_i^2 - 15]. \tag{33}$$

Instead of using Kapteyn's formulae for the interference term, we will perform a direct evaluation of the energy loss of the interference term by the ω-integration in (27) [35]. So, after some elementary modification in the ω-integral, we get:

$$-\frac{dU_{12}}{dt} = \int_0^\infty P(\omega)d\omega =$$

$$A\int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\omega\omega e^{-i\omega\tau}\sin[2\omega a\cos\omega_0\tau]\left[\frac{B(C\cos\omega_0\tau+1)^2 - D}{\cos(\omega_0\tau/2)}\right], \tag{34}$$

with

$$A = \frac{G}{a\pi}, \quad B = E_1E_2, \quad C = v_1v_2, \quad D = \frac{m_1^2 m_2^2}{2E_1E_2}. \tag{35}$$

Using the definition of the δ-function and its derivative, we have, instead of eq. (34), with $v = a\omega_0$:

$$-\frac{dU_{12}}{dt} = A\omega_0\pi\int_{-\infty}^{\infty} dx\,\frac{[B(C\cos x+1)^2 - D]}{\cos(x/2)}\times$$

$$\left[\delta'(x - 2v\cos(x/2)) - \delta'(x + 2v\cos(x/2))\right]. \tag{36}$$

According to the Schwinger article [36], we express the delta-fuction as follows:

$$\delta(x \pm 2v\cos(x/2)) = \sum_{n=0}^{\infty}\frac{(\pm 2v\cos(x/2))^n}{n!}\left(\frac{d}{dx}\right)^n \delta(x). \tag{37}$$

Then

$$\delta'(x \pm 2v\cos(x/2)) = \sum_{n=0}^{\infty}\frac{(\pm 2v\cos(x/2))^n}{n!}\left(\frac{d}{dx}\right)^{n+1} \delta(x) = \tag{38}$$

and it means that

$$\frac{[\delta'(x + 2v\cos(x/2)) - \delta'(x - 2v\cos(x/2))]}{\cos(x/2)} =$$

$$(-2)\sum_{n=1}^{\infty}\frac{(2v)^{2n-1}(\cos(x/2))^{2(n-1)}}{(2n-1)!}\left(\frac{d}{dx}\right)^{2n}\delta(x) \tag{39}$$

Now, we can write eq. (36) in tge following form after some elmentary operations

$$-\frac{dU_{12}}{dt} = A\omega_0\pi\int_{-\infty}^{\infty} dx\,\Big(B(C\cos x+1)^2 - D\Big)\times$$

$$(-2)\sum_{n=1}^{\infty}\frac{(2v)^{2n-1}(\cos(x/2))^{2(n-1)}}{(2n-1)!}\left(\frac{d}{dx}\right)^{2n}\delta(x), \tag{40}$$

where $(B(C\cos x+1)^2 - D)$ can be written as follows:

$$\Big(B(C\cos x + 1)^2 - D\Big) =$$

$$4BC^2(\cos^4(x/2) + [4CB - 4BC^2](\cos^2(x/2) + [BC^2 - 2CB + B - D]. \tag{41}$$

After application of the per partes method, we get from eq. (40) the following mathematical object:

$$-\frac{dU_{12}}{dt} = (-2)A[4BC^2]\omega_0\pi \int_{-\infty}^{\infty} dx\,\delta(x) \sum_{n=1}^{\infty} \left(\frac{d}{dx}\right)^{2n} (2v)^{2n-1} \frac{(\cos(x/2))^{2n+2}}{(2n-1)!} -$$

$$2A[4CB - 4BC^2]\omega_0\pi \int_{-\infty}^{\infty} dx\,\delta(x) \sum_{n=1}^{\infty} \left(\frac{d}{dx}\right)^{2n} (2v)^{2n-1} \frac{(\cos(x/2))^{2n}}{(2n-1)!} -$$

$$2A[BC^2 - 2CB + B - D]\omega_0\pi \int_{-\infty}^{\infty} dx\,\delta(x) \sum_{n=1}^{\infty} \left(\frac{d}{dx}\right)^{2n} (2v)^{2n-1} \frac{(\cos(x/2))^{2(n-1)}}{(2n-1)!}. \tag{42}$$

We get after some elementary operations $\int \delta f(x) = f(0)$

$$J_1 = \sum_{n=1}^{\infty} \left(\frac{d}{dx}\right)^{2n} (2v)^{2n-1} \frac{(\cos(x/2))^{2n+2}}{(2n-1)!}|_{x=0} = \sum_{n=0}^{\infty} f(n) v^{2n} = F(v^2), \tag{43}$$

$$J_2 = \sum_{n=1}^{\infty} \left(\frac{d}{dx}\right)^{2n} (2v)^{2n-1} \frac{(\cos(x/2))^{2n}}{(2n-1)!}|_{x=0} = \sum_{n=0}^{\infty} g(n) v^{2n} = G(v^2) \tag{44}$$

and

$$J_3 = \sum_{n=1}^{\infty} \left(\frac{d}{dx}\right)^{2n} (2v)^{2n-1} \frac{(\cos(x/2))^{2(n-1)}}{(2n-1)!}|_{x=0} = \sum_{n=0}^{\infty} h(n) v^{2n} = H(v^2) \tag{45}$$

where f, g, h, F, G, H are functions vhich must be determined

So we get instead of eq. (41) the following final form

$$-\frac{dU_{12}}{dt} = (-2)A[4BC^2]\omega_0\pi G(v^2) - 2A[4CB - 4BC^2]\omega_0\pi F(v^2) -$$

$$2A[-2CB + BC^2 + B - D]\omega_0\pi H(v^2) \tag{46}$$

Let us remark that we can use simple approximation in eq. (41) as follows: $(\cos(x/2))^{2n+2} \approx (\cos(x/2))^2, (\cos(x/2))^{2n} \approx (\cos(x/2))^2, (\cos(x/2))^{2(n-1)} \approx (\cos(x/2))^2$. Then, after using the well-known formula

$$\left(\frac{d}{dx}\right)^{2n} \cos^2(x/2) = \frac{1}{2}\cos(x + \pi n) \tag{47}$$

and

$$\frac{1}{2}\cos(x + \pi n)|_{x=0} = \frac{1}{2}(-1)^n. \tag{47}$$

So, instead of eq. (46) se have:

$$-\frac{dU_{12}}{dt} = A\omega_0\pi \Big\{2BC + BC^2 + B - D\Big\} \sum_{n=1}^{\infty} \frac{(2v)^{2n-1}(-1)^n}{(2n-1)!} \tag{48}$$

2 Energy shift of H-atom electrons due to the black hole thermal bath

We here determine the electromagnetic shift of energy levels of H-atom electrons by calculating an electron coupling to the black hole thermal bath. Energy shift of electrons in H-atom is determined in the framework of non-relativistic quantum mechanics.

The Gibbons-Hawking effect is the statement that a temperature can be associated to each solution of the Einstein field equations that contains a causal horizon. It is named after Gary Gibbons and Stephen William Hawking.

Schwarzschild spacetime contains an event horizon and so can be associated with temperature. In the case of Schwarzschild spacetime this is the temperature T of a black hole of mass M, satisfying T/M .

De Sitter space which contains an event horizon has the temperature T proportional to the Hubble parameter H. We consider here the influence of the heat bath of the Gibbons-Hawking photons on the energy shift of H-atom electrons.

The considered problem is not in the scientific isolation, because some analogical problems are solved in the scientific respected journals. At present time it is a general conviction that there is an important analogy between black hole and the hydrogen atom. The similarity between black hole and the hydrogen atom was considered for instance by Corda [37], who discussed the precise model of Hawking radiation from the tunnelling mechanism. In this article an elegant expression of the probability of emission is given in terms of the black hole quantum levels. So, the system composed of Hawking radiation and black hole quasi-normal modes introduced by Corda [38] is somewhat similar to the semiclassical Bohr model of the structure of a hydrogen atom.

The time dependent Schrödinger equation was derived for the system composed by Hawking radiation and black hole quasi-normal modes [39]. In this model, the physical state and the correspondent wave function are written in terms of an unitary evolution matrix instead of a density matrix. Thus, the final state is a pure quantum state instead of a mixed one and it means that there is no information loss. Black hole can be well defined as the quantum mechanical systems, having ordered, discrete quantum spectra, which respect 't Hooft's assumption that Schrödinger equations can be used universally for all dynamics in the universe.

Thermal photons by Gibbons and Hawking form so called blackbody, which has the distribution law of photons derived in 1900 by Planck [40, 41, 42]. The derivation was based on the investigation of the statistics of the system of oscillators inside of the blackbody. Later Einstein in 1917 [43] derived the Planck formula from the Bohr model of atom where electrons have the discrete energies and the energy of the emitted photons are given by the Bohr formula

$\hbar\omega = E_i - E_f$, E_i, E_f are the initial and final energies of electrons.

Now, let us calculate the modified Coulomb potential due to blackbody. The starting point of the determination of the energy shift in the H-atom is the potential $V_0(\mathbf{x})$, which is generated by nucleus of the H-atom. The potential at point $V_0(\mathbf{x} + \delta\mathbf{x})$, evidently is [44, 45]:

$$V_0(\mathbf{x} + \delta\mathbf{x}) = \left\{1 + \delta\mathbf{x}\nabla + \frac{1}{2}(\delta\mathbf{x}\nabla)^2 + ...\right\} V_0(\mathbf{x}). \tag{1}$$

If we average the last equation in space, we can eliminate so called the effective potential in the form

$$V(\mathbf{x}) = \left\{1 + \frac{1}{6}(\delta\mathbf{x})_T^2\Delta + ...\right\} V_0(\mathbf{x}), \tag{2}$$

where $(\delta\mathbf{x})_T^2$ is the average value of te square coordinate shift caused by the thermal photon fluctuations. The potential shift follows from eq. (2):

$$\delta V(\mathbf{x}) = \frac{1}{6}(\delta\mathbf{x})_T^2\Delta V_0(\mathbf{x}). \tag{3}$$

The corresponding shift of the energy levels is given by the standard quantum mechanical formula [44]

$$\delta E_n = \frac{1}{6}(\delta\mathbf{x})_T^2(\psi_n\Delta V_0\psi_n). \tag{4}$$

In case of the Coulomb potential, which is the case of the H-atom, we have

$$V_0 = -\frac{e^2}{4\pi|\mathbf{x}|}. \tag{5}$$

Then for the H-atom we can write

$$\delta E_n = \frac{2\pi}{3}(\delta\mathbf{x})_T^2\frac{e^2}{4\pi}|\psi_n(0)|^2, \tag{6}$$

where we used the following equation for the Coulomb potential

$$\Delta\frac{1}{|\mathbf{x}|} = -4\pi\delta(\mathbf{x}). \tag{7}$$

Motion of electron in electric field is evidently described by elementary equation

$$\delta\ddot{\mathbf{x}} = \frac{e}{m}\mathbf{E}_T, \tag{8}$$

which can be transformed by the Fourier transformation into the folowing equation

$$|\delta\mathbf{x}_{T\omega}|^2 = \frac{1}{2}\left(\frac{e^2}{m^2\omega^4}\right)\mathbf{E}_{T\omega}^2, \tag{9}$$

where the index ω concerns the Fourier component of above functions.

On the basis of the Bethe idea [46] of the influence of vacuum fluctuations on the energy shift of electron, the following elementary relations was used by Welton [45], Akhiezer et al. [44] and Berestetzkii et al. [47]:

$$\frac{1}{2}\mathbf{E}_\omega^2 = \frac{\hbar\omega}{2} \tag{10}$$

and in case of the thermal bath of the blackbody, the last equation is of the following form [48]:

$$\mathbf{E}_{T\omega}^2 = \varrho(\omega) = \left(\frac{\hbar\omega^3}{\pi^2c^3}\right)\frac{1}{e^{\frac{\hbar\omega}{kT}}-1}, \tag{11}$$

because the Planck law in (11) was written as

$$\varrho(\omega) = G(\omega) < E_\omega >= \left(\frac{\omega^2}{\pi^2c^3}\right)\frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}}-1}, \tag{12}$$

where the term

$$< E_\omega >= \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}}-1} \tag{13}$$

is the average energy of photons in the blackbody and

$$G(\omega) = \frac{\omega^2}{\pi^2c^3} \tag{14}$$

is the number of electromagnetic modes in the interval $\omega, \omega + d\omega$.

Then,

$$(\delta\mathbf{x}_{T\omega})^2 = \frac{1}{2}\left(\frac{e^2}{m^2\omega^4}\right)\left(\frac{\hbar\omega^3}{\pi^2c^3}\right)\frac{1}{e^{\frac{\hbar\omega}{kT}}-1}, \tag{15}$$

where $(\delta\mathbf{x}_{T\omega})^2$ involves the number of frequences in the interval $(\omega, \omega + d\omega)$.

So, after some integration, we get

$$(\delta\mathbf{x})_T^2 = \int_{\omega_1}^{\omega_2}\frac{1}{2}\left(\frac{e^2}{m^2\omega^4}\right)\left(\frac{\hbar\omega^3}{\pi^2c^3}\right)\frac{d\omega}{e^{\frac{\hbar\omega}{kT}}-1} = \frac{1}{2}\left(\frac{e^2}{m^2}\right)\left(\frac{\hbar}{\pi^2c^3}\right)F(\omega_2-\omega_1), \tag{16}$$

where $F(\omega)$ is the primitive function of the omega-integral

$$J = \frac{1}{\omega}\frac{1}{e^{\frac{\hbar\omega}{kT}}-1}, \tag{17}$$

which cannot be calculated by the elementary integral methods and it is not involved in the tables of integrals.

Frequencies ω_1 and ω_2 will be determined with regard to the existence of the fluctuation field of thermal photons. It was determined in case of the Lamb shift [47; 44] by means of the physical analysis of the interaction of the Coulombic atom with the surrounding fluctuation field. We suppose here that the Bethe and Welton arguments are valid and so we take the frequencies in the Bethe-Welton form. In other words, electron cannot respond to the fluctuating field if the frequency which is much less than the atom binding energy given by the Rydberg constant [49] $E_{Rydberg} = \alpha^2mc^2/2$. So, the lower frequency limit is

$$\omega_1 = E_{Rydberg}/\hbar = \frac{\alpha^2mc^2}{2\hbar}, \tag{18}$$

where $\alpha \approx 1/137$ is so called the fine structure constant.

The specific form of the second frequency follows from the elementary argument, that we expect the effective cutoff, since we must neglect the relativistic effect in our non-relativistic theory. So, we write

$$\omega_2 = \frac{mc^2}{\hbar}. \tag{19}$$

If we take the thermal function of the form of the geometric series

$$\frac{1}{e^{\frac{\hbar\omega}{kT}} - 1} = q(1 + q^2 + q^3 + ...); \quad q = e^{-\frac{\hbar\omega}{kT}}, \tag{20}$$

$$\int_{\omega_1}^{\omega_2} q(1 + q^2 + q^3 + ...)\frac{1}{\omega}d\omega = \ln|\omega| + \sum_{k=1}^{\infty} \frac{(-\frac{\hbar\omega}{kT})^k}{k!k} + ...; \quad q = e^{-\frac{\hbar\omega}{kT}} \tag{21}$$

and the first thermal contribution is

$$Thermal\ contribution = \ln\frac{\omega_2}{\omega_1} - \frac{\hbar}{kT}(\omega_2 - \omega_1), \tag{22}$$

Then, with eq. (6)

$$\delta E_n \approx \frac{2\pi}{3}\left(\frac{e^2}{m^2}\right)\left(\frac{\hbar}{\pi^2 c^3}\right)\left(\ln\frac{\omega_2}{\omega_1} - \frac{\hbar}{kT}(\omega_2 - \omega_1)\right)|\psi_n(0)|^2, \tag{23}$$

where according to Sokolov et al., [50]

$$|\psi_n(0)|^2 = \frac{1}{\pi n^2 a_0^2} \tag{24}$$

with

$$a_0 = \frac{\hbar^2}{me^2}. \tag{25}$$

Let us only remark that the numerical form of eq. (23) has deep experimental astrophysical meaning.

Serge Haroche [51] and his research group in the Paris microwave laboratory used a small cavity for the long life-time of photon quantum experiments performed with the Rydberg atoms. We considered here the thermal gas corresponding to the Gibbons-Hawking theory of space-time of black hole (at temperature T) as the preamble for new experiments for the determination of the energy shift of H-atom electrons interacting with the Gibbons-Hawkingon thermal gas. It is not excluded, that the observations performed by the well educated astro-experts will be the Nobelian ones.

3 Velocity of sound in the black hole photon gas

With regard ti the previous section, black hole can be comsidered as the black body and it means we can determine the velocity of sound in the Gibbons Hawking black hole thermal bath. So, we determine the velocity of sound in the blackbody gas of photons. Derivation is based on the thermodynamic theory of the photon gas and the Einstein relation between energy and mass. The spectral form for the n-dimensional blackbody is not derived.

In order to understand the the derivation of speed of sound in gas and in the relic photon sea, we start with the derivation of the speed of sound in the real elastic rod.

Let A be the cross-section of the element Adx of a rod, where dx is the linear infinitesimal length on the abscissa x. The $\varphi(x,t)$ let be deflection of the element Adx at point x at time t. The shift of he element Adx at point $x+dx$ is evidently

$$\varphi + \frac{\partial \varphi}{\partial x}dx. \tag{1}$$

The relative prolongation is evidently $\partial\varphi(x,t)/\partial x$. The differential equation of motion of the rod can be derived by the following obligate way. We suppose that the force tension $F(x,t)$ acting on the element Adx of the rod is given by the Hook law:

$$F(x,t) = EA\frac{\partial \varphi}{\partial x}, \tag{2}$$

where E is the Young modulus of elasticity. We easily derive that

$$F(x+dx) - F(x) \approx EA\frac{\partial^2 \varphi}{\partial x^2}dx \tag{3}$$

The mass of the element Adx is ϱAdx, where ϱ is the mass density of the rod and the dynamical equilibrium is expressed by the Newton law of force:

$$\varrho Adx\varphi_{tt} = EA\varphi_{xx}dx \tag{4}$$

or,

$$\varphi_{tt} - v^2\varphi_{xx} = 0, \tag{5}$$

where

$$v = \left(\frac{E}{\varrho}\right)^{1/2} \tag{6}$$

is the velocity of sound in the rod.

The complete solution of eq. (5) includes the initial and boundary conditions. We suppose that the velocity law (6) involving modulus of elasticity and mass density is valid also for gas intercalated in the rigid cylinder tube. According to the definition of the Young modulus of elasticity where $(\Delta L/L)$ is the relative prolongation of a rod, we have as an analogue for the tube of gas $\Delta V/V$, $F \to \Delta p$, where V is the volume of a gas and p is pressure of a gas. Then, the modulus of elasticity is defined as the analogue of eq. (2). Or,

$$E = -\frac{dp}{dV}V. \tag{7}$$

The process of the sound spreading in ideal gas is the adiabatic thermodynamic process with no heat exchange. We use it later as a model of the sound spreading in the gas of blackbody photons. Such process is described by the thermodynamical equation

$$pV^{\kappa} = const, \tag{8}$$

where κ is the Poisson constant defined as $\kappa = c_p/c_v$, with c_p, c_v being the specific heat under constant pressure and under constant volume.

After differentiation of eq. (8) we get the following equation

$$dpV^{\kappa} + \kappa V^{\kappa-1}dV = 0, \tag{9}$$

or,

$$\frac{dp}{dV} = -\kappa\frac{p}{V}. \tag{10}$$

After inserting of eq. (10) into eq. (7), we get from eq. (6) for the velocity of sound in gas the so called Newton-Laplace formula:

$$v = \sqrt{\kappa\frac{p}{\varrho}}, \tag{11}$$

where ϱ is the mass density of gas.

The density of the equilibrium radiation is given by the Stefan-Boltzmann formula

$$u = aT^4, ; \quad a = 7,5657.10^{-16}\frac{\mathrm{J}}{\mathrm{K^4m^3}}. \tag{12}$$

.

Then, with regard to the thermodynamic definition of the specific heat

$$c_v = \left(\frac{\partial u}{\partial T}\right)_V = 4aT^3. \tag{13}$$

Similarly, with regard to the general thermodynamic theory

$$c_p = c_v + \left[\left(\frac{\partial u}{\partial V}\right)_T + p\right]\left(\frac{\partial V}{\partial T}\right)_p = c_v, \tag{14}$$

because $\left(\frac{\partial V}{\partial T}\right)_T = 0$ for photon gas and in such a way, $\kappa = 1$ for photon gas. According to the theory of relativity, there is simple equivalence between mass and energy. Namely, $m = E/c^2$. At the same time, there is relation between pressure and the internal energy of the blackbody gas following from the electromagnetic theory of light $p = u/3$. So, in our case

$$\varrho = u/c^2 = \frac{aT^4}{c^2}; \quad p = \frac{u}{3}. \tag{15}$$

So, after insertion of formulas in equation (14) in to eq. (11), we get the final formula for the velocity of sound in three photon sea of the blackbody is as follows:

$$v = c\sqrt{\frac{\kappa}{3}} = \frac{c}{3}\sqrt{3}, \tag{16}$$

which is the result derived by Partovi [52] using the QED theory applied to the photon gas. No energy signal can move with velocity greater than the speed of light. And we correctly derived $v/c < 1$.

So, we have seen in this section, that using the classical thermodynamical model of sound in the classical gas we can easily derive some properties of the black body gas, namely the velocity of sound in it and in the relic photon sea. It is not excluded that the relic sound can be detected by the special microphones of Bell laboratories. Let us still remark that if we use van der Waals equation of state, or, the Kamerlingh Onnes virial equation of state, the obtained results will be modified with regard to the basic results.

Our derivation of the light velocity in the blackbody photon gas was based on the classical thermodynamical model with the adiabatic process ($\delta Q = 0$), controlling the spreading of sound in the gas. The problem was not solved by Einstein, because only QED, elaborated many years later was able to give motivation for the formulation of such problem. In other words, Einstein was not motivated for such activity. Partovi [52] derived additional radiation corrections to the Planck distribution formula and the additional correction to the speed of sound in the relic photon sea. His formula is of the form:

$$v_{sound} = \left[1 - \frac{88\pi^2\alpha^2}{2025}\left(\frac{T}{T_e}\right)^4\right]\frac{c}{\sqrt{3}}, \tag{17}$$

where α is the fine structure constant and $T_e = 5.9$ G Kelvin. We see that our formula is the first approximation in the Partovi expression.

There is rigorous statistical theory of transport of sound energy in gas based on the Boltzmann equation [53]. After application of Boltzmann equation to the photon gas, or, relic photon gas we can expect the rigorous results with regard to fact that the cross-section of the photon-photon interaction is very small. Namely, [47]:

$$\sigma_{\gamma\gamma} = 4,7\alpha^4\left(\frac{c}{\omega}\right)^2; \quad \hbar\omega \ll mc^2, \tag{18}$$

and

$$\sigma_{\gamma\gamma} = \frac{973}{10125\pi}\alpha^2 r_e^2\left(\frac{\hbar\omega}{mc^2}\right)^6; \quad \hbar\omega \gg mc^2, \tag{19}$$

where $r_e = e^2/mc^2 = 2,818 \times 10^{-13}$ cm is the classical radius of electron and $\alpha = e^2/\hbar c$ is the fine structure constant with numerical value $1/\alpha = 137,04$.

No doubt, the solution of the Boltzmann equation gives the existence of sound waves in the statistical system of particles.

4 Summary and perspectives

We have derived the spectral density of gravitons and the total quantum loss of energy of the black hole binary. The energy loss is caused by the emission of gravitons during the motion of the two black hole binary binary around each other under their gravitational interaction. The energy-loss formulae of the production of gravitons are derived here by the Schwinger method. Because the general relativity and theory of gravity do not necessarily contain the last valid words to be written about the nature of gravity and it is not, of course, a quantum theory [21], they cannot give the answer on the production of gravitons and the quantum energy loss, respectively. So, this article is the original text that discusses the quantum energy loss caused by the production of gravitons by the black hole binary system. It is evident that the production of gravitons by the binary system forms a specific physical situation, where a general relativity can be seriously confronted with the source theory of gravity.

This article is an extended version of an older article by the present author [33], in which only the spectral formulae were derived. Here we have derived the quantum energy-loss formulae, with no specific assumption concerning the strength of the gravitational field. We hope that future astrophysical observations will confirm the quantum version of the energy loss of the binary black hole.

In the next part of the chapter, the electromagnetic shift of energy levels of H-atom electrons was determined by calculating an electron coupling to the Gibbons-Hawking ectromagnetic field thermal bath of the black hole. Energy shift of electrons in H-atom is determined in the framework of nonrelativistic quantum mechanics.

In the last we have determined the velocity of sound in the blackbody gas of photons inside of black hole. Derivation was based on the thermodynamic theory of the photon gas and the Einstein relation between energy and mass. The spectral form for the n-dimensional blackbody was not here considered. With regard to the Russell philosophy of mathematics, there is no possibility to prove the dimensionality of space, or, space-time, by means of pure mathematics, because the statements of mathematics are non-existential. The existence of the external world cannot be also proved by pure mathematics. However, if there is an axiomatic system related adequately to the external world and reflecting correctly the external world, then, it is possible to do many predictions on the external world by pure logic. This is the substance of exact sciences. We know for instance that the success of special theory of relativity and quantum mechanics is based on the adequate axiomatic system and on logic. The text is based mainly on the author articles published in the international journals of physics [33], [54], [55].

There is the fundamental problem concerning of the maximal mass of the black hole. We do not solve this problem here but, let us inform on the possible solution of the problem. The theory of spece time with maximal acceleration constant was derived by author [56]. In this theory the maximal accelaration constant is the analogue of the maximal velocity in spcial theory relativity. The theory with the existence of the maximal acceleration, after confirmation, determine the black hole mass where the mass of the black hole is restricted by maximal acceleration of a body falling in the gravity field of black hole.

References

[1] Schwarzschild, K. (1916). Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie, *Sitzungsber. K. Preuss. Akad. Wiss.* 1, 189; English translation: On the gravitational field of a mass point according to Einsteins theory, *Gen. Relativ. Gravit.* 35, 951 (2003); ibid. [physics/9905030].

[2] Finkelstein, D. (1958). Past-future asymmetry of the gravitational field of a Point Particle, *Phys. Rev.* 110, 965.

[3 Kruskal, M. D. (1960). Maximal extension of Schwarzschild metric, *Phys. Rev.* 119, 1743.

[4] Kerr, R. P. (1963). Gravitational field of a spinning mass as an example of algebraically special metrics, *Phys. Rev. Lett.* 11, 237.

[5] Vishveshwara, C. V. (1970). Scattering of gravitational radiation by a Schwarzschild black-hole, *Nature (London)* 227, 936.

[6] Press, W. H. (1971). Long wave-trains of gravitational waves from a vibrating black hole, *Astrophys. J.* 170, L105.

[7] Chandrasekhar, S. and Detweiler, S. L. (1975). The quasi-normal modes of the Schwarzschild black hole, *Proc. R. Soc.* A 344, 441.

[8] Blanchet, L.; Damour, L. T.; Iyer, B. R.; Will, C.M. and Wiseman, A. G. (1995). Gravitational-radiation damping of compact binary systems to second-post-Newtonian order, *Phys. Rev. Lett.* 74, 3515.

[9] Blanchet, L. (2014). Gravitational radiation from post-Newtonian sources and inspiralling compact binaries, *Living Rev. Relativity* 17, 2.

[10] Buonanno, A. and Damour, T. (1999). Effective one-body approach to general relativistic two-body dynamics, *Phys. Rev.* D 59, 084006.

[11] Pretorius, F. (2005). Evolution of binary black hole spacetimes, Phys. Rev. Lett. 95, 121101.

[12] Campanelli, M.; Lousto, C. O.; Marronetti, P. and Zlochower, Y. (2006). Accurate evolutions of orbiting black-hole binaries without excision, *Phys. Rev. Lett.* 96, 111101.

[13] Baker, J. G.; Centrella, J.; Choi,D. I.; Koppitz, M. and van Meter, J. (2006). Gravitational-wave extraction from an inspiraling configuration of merging black holes, *Phys. Rev. Lett.* 96, 111102.

[14] Webster, B. L. and Murdin, P. (1972). Is Cygnus X-1-a spectroscopic binary with a heavy companion ?, *Nature (London)* 235, 37.

[15] Bolton, C. T. (1972). Is Cyg X-1 a neutron star?, *Nature (London)* 240, 124.

[16] Casares, J. and Jonker, P. G. (2014). Mass measurements of stellar and intermediate-mass black holes, *Space Sci. Rev.* 183, 223.

[17] Hulse, R. A. and Taylor, J. H. (1975). Discovery of a pulsar in a binary system, *Astrophys. J.* 195, L51.

[18] Taylor, J. H. and. Weisberg, J. M. (1982). A new test of general relativity: Gravitational radiation and the binary pulsar PSR B**1913+16**, *Astrophys. J.* 253, 908.

[19] Aasi, J. et al., (2015). Advanced Virgo: a second-generation interferometric gravitational wave detector, *Classical Quantum Gravity* 32, 074001.

[20] Acernese, et al., F. (2015). Advanced Virgo: a 2nd generation interferometric gravitational wave detector, *Class. Quantum Grav.* 32, 024001.

[21] Taylor, Jr., J. H. Binary pulsars and relativistic gravity (Nobel Lecture, 1993).

[22] Gold, T. (1968). Rotating neutron stars as the origin of the pulsating radio sources nature, *Nature 218,* 731.

[23] Goldreich, P. and Julian, W. H. (1969). Pulsar electrodynamics, Astrophys. J. 157, 869.

[24] Damour, T. and Taylor, J. H. (1992). Strong field tests of relativistic gravity and binary pulsars, *Phys. Rev. D* 15 45(6) 1840-1868.

[25] Taylor, J. H.; Wolszczan, A.; Damour, T. and Weisberg, J.M. (1992). *Nature* 355 132.

[26] Manoukian, E. B.(1990). A quantum viewpoint of gravitational radiationGen, *Rel. Grav.* 22 501.

[27] Schwinger, J.; Tsai W. Y. and T. Erber, (1976). Classical and quantum theory of synergic synchrotron-Cherenkov radiation, *Ann. Phys. (NY)* 96, 303.

[28] Dittrich, W. (1978). Source methods in quantum field theory,
Fortschr. Phys. 26, 289.

[29] Schwinger, J. (1976). Gravitons and photons: The methodological unification of source theory, *Gen. Rel. Grav.* 7, 251.

[30] Schwinger, J. Particles, sources and fields, Vol. 1 (Addison-Wesley Publ. Comp., Reading, Mass., 1970).

[31] Weinberg, S. Gravitation and cosmology, (John Wiley and Sons, Inc., New York, 1972).

[32] Cho, C. F. and Dass, N. D. Harri. (1976). Gravitational two-body problems, a source theory viewpoint, *Ann. Phys. (NY)* 90, 406.

[33] Pardy, M. (1983). The Synchrotron production of gravitons by the binary system, *Gen. Rel. Grav.* 15 No. 11, 1027.

[34] Prudnikov, A. P.; Bryčkov, Ju. A. and Maričev, O. I.
Integrals and series, – Special functions (Moscow, Nauka, 1983) (in Russian).

[35] Sokolov, A. A. andTernov, I. M. The relativistic electron (Moscow, Nauka, 1983) (in Russian).

[36] Schwinger, J. (1998). On radiation by Electrons in a Betatron, LNBL-39088, CBP Note-179, UC-414, Berkeley, CA 94720, January 13.

[37] Corda, Ch. (2015a). Precise model of Hawking radiation from the tunneling mechanism, *Class. and Quantum Gravity* 32, 195007.

[38] Corda, Ch. (2015b). Quasi-normal modes: the "electrons" of black holes as "gravitational atoms"? Implications for the black hole information puzzle. *Advances in High Energy Physics*, 867601.

[39] Corda, Ch. (2015c). Time dependent Schrödinger equation for black hole evaporation: no information loss, *Annals of Physics* 353, 71.

[40] Planck, M. (1900). Zur Theorie des Gesetzes der Energieverteilung im Normalspektrum, *Verhandlungen deutsch phys. Ges.* 2, 237;

[41] Planck, M. (1901). Ueber irreversible Strahlungsvorgänge *Ann. Phys.* 4, 553.

[42] Schöpf, H-G. Theorie der Wärmestrahlung in historisch-kritischer Darstellung, (Alademie/Verlag, Berlin, 1978).

[43] Einstein, A. (1917). Zur Quantentheorie der Strahlung, *Physikalische Zeitschrift* 18, 121.

[44] Akhiezer, A.I. and Berestetzkii, V.B. (1953) Quantum electrodynamics. GITTL, Moscow.

[45] Welton, Th. (1948). Some observable effects of the quantum-mechanical fluctuations of the electromagnetic field, *Phys. Rev.* 74, 1157.

[46] Bethe, H. A. (1947). The electromagnetic shift of energy levels, *Phys. Rev.* 2, 339.

[47] Berestetzkii, V. B.; Lifshitz, E. M. and Pitaevskii, L. P. Quantum electrodynamics, (Butterworth-Heinemann, Oxford, 1999).

[48] Isihara, A. Statistical mechanics, (Academic Press, New York,London, 1971).

[49] Rohlf, J. W. Modern physics from α *to* Z^0, (John Wiley & Sons LTD., London - New York, 1994).

[50] Sokolov, A. A.; Loskutov, Yu. M. and Ternov, I. M. Quantum mechanics, (State Pedagogical Edition, Moscow, 1962). (in Russian).

[51] Haroche S. (2012). The secrets of my prizewinning research, *Nature* 490, 311.

[52] Partovi, M. H. (1994). QED corrections to Plancks radiation law and photon thermodynamics, *Phys. Rev. D* 50, No. 2, 1118-1124.

[53] Uhlenbeck G. E. and Ford, G. W., Lectures in statistical physics, (American mathematical society, Providence, Rhode Island, 1963).

[54] Pardy, M. (2016). Energy shift of H-atom electrons due to Gibbons-Hawking thermal bath,

Journal of High Energy Physics, Gravitation and Cosmology, 2, 472-477.

[55] Pardy, M. (2013). Velocity of sound in the black-body, *Results in Physics* 3, 7073.

[56] Pardy, M. (2018). Thomas precession by uniform acceleration and gravity, Journal of High Energy Physics, Gravitation and Cosmology, 2018, 4, 132-142.