The complex Clifford and the complex Dirac operators

A.Balan

October 1, 2018

Abstract

We introduce a complex Clifford algebra (mixed with a Heisenberg algebra) and we deduce from it two Dirac operators.

1 The complex Clifford algebra

We consider a complex vector spaces and define the $(1,0)$ and $(0,1)$ parts. Then the complex Clifford algebra is defined by the following relations:
1)
$$e^{1,0}\otimes f^{1,0}+f^{1,0}\otimes e^{1,0}=-2g(e,f)1$$
2)
$$e^{0,1}\otimes f^{0,1}+f^{0,1}\otimes e^{0,1}=-2g(e,f)1$$
3)
$$e^{1,0}\otimes f^{0,1}-f^{0,1}\otimes e^{1,0}=2w(e,f)1$$
with g a non-degenerated metric and w a symplectic form.

2 The two Dirac operators

We define from the complex Clifford algebra two Dirac operators:

$$\mathcal{D}^{1,0}=\sum_i e_i^{1,0}\nabla_{e_i}$$

$$\mathcal{D}^{0,1}=\sum_i e_i^{0,1}\nabla_{e_i}$$

They are bounded by the complex Schrödinger equations:

$$(\mathcal{D}^{1,0})^2=\Delta_g+\alpha$$

$$(\mathcal{D}^{0,1})^2=\Delta_g+\bar{\alpha}$$

$$\mathcal{D}^{1,0}\mathcal{D}^{0,1}-\mathcal{D}^{0,1}\mathcal{D}^{1,0}=\Delta_w+\beta$$

References

[F] T.Friedrich, ”Dirac operators in Riemannian Geometry”, Graduate Studies in Mathematics vol 25, AMS, 2000.

[GHL] S.Gallot, D.Hulin, J.Lafontaine, ”Riemannian Geometry”, Springer, 2004.

[J] J.Jost, ”Riemannian Geometry and Geometric Analysis”, Springer, 2008.