Refraction

John P. Wallace

Casting Analysis Corp., 8379 Ursa Lane, Weyers Cave, Virginia 24486 USA *

Michael J. Wallace

Phoenix, AZ 85050 USA

Refraction is treated classically, which is not physically realistic. Unlike optical reflection that is well understood refraction, is a more difficult problem exposing a major missing piece of quantum mechanics. Refraction normally is treated either classically or as a non-relativistic perturbation response. Recently it became apparent where this property finds its quantum origin in a full relativistic quantum description. *draft 1 August 2019*

CONTENTS

I. Introduction 1

II. Relativistic Quantum Refraction 2
A. Quadratic Potential 2
B. Emergent Phenomenon 3

III. Collective Polarization 4

IV. Neutrino Mass and Refraction 4

V. Refraction in the Laboratory Frame 4
A. Neutrino Geo-Refraction 5
B. Design of Refraction Detector for ν_e 5

VI. Discussion 5

VII. Acknowledgments 6

A. Extended Wave Equation 6

References 6

I. INTRODUCTION

Refraction is a well studied subject beginning with Robert Hooke's papers and Newton's *Optick* (Newton, 1730) to disern the nature of color and light and extended to a non-relativistic quantum description of a field interacting with an atom (Heitler, 1954). The property's origin is in the dynamic polarization of charge neutral matter that mediates the dispersion of radiation by transparent matter driving neither a transition nor being totally unaffected. Polarization is a property poorly handled by quantum mechanics.

Refraction of light by a dielectric has a well establish classical description modeled by of a bound electrons whose charges respond to the passing **E**-field. The model is a combination of Newton's second law, Hooke's law, and a loss term.

$$F = q_e\mathbf{E} = m(\ddot{x} + \gamma\dot{x} + \omega_o^2 x) \quad (1)$$

Where x is the electron's displacement, γ is the strength of the loss mechanism, and ω_o^2 captures the polarization via Hooke's law. The right hand side of the equation describes the medium light is traveling through and the left hand side defines the field interaction with the electron.

A non-relativistic quantum description is an extension of this model where the energy of the electron is shared between the two field terms. The bound state refraction model functions in two ways: first to reduce the speed of light in the medium and secondly to provide a mechanism for absorption. The property that must be better understood is the mechanism for slowing the speed of light without absorption.

Our interests are not only in the refraction of light by the electromagnetic interaction, but the refraction of neutrinos by nuclear matter via the weak force interaction. This brings in the larger question of what is the basis for refraction of these two propagating fields. To resolve this problem and generate the necessary fields, relativistic energy conservation has to be recognized. To satisfy the relativistic conservation of energy for a particle, $E^2 = p^2c^2 + (mc^2)^2$, and a field, $E = \hbar\omega$, requires statistically independent spaces, one to define dynamics in the laboratory frame and the second to generate the individual properties of the particle/field (Wallace *et al.*, 2011). These secondary spaces are labeled as self-reference frames for individual particles and field. No linear set of energy conservation equations derived either as Hamiltonian or Lagrangian are accurate because they are not consistent with relativity. This eliminates effective field theories of high energy physics.

Energy conservation forces separate statistially independent spaces to generate particle and field structures that are independent of the laborary frame. The principal casulty of the analysis from energy conservation is a mathematically convention, the mathematical con-

* jpw@castinganalysis.com

tinuum. The mathematical continuum does not appear to be physically realistic for spaces of limited precision, which have neither point masses or point charges. A further problem with the contiuum is the ambiguous role played by dimensions. Dimension in the mathmatical continuum become arbitrary appendages and not physically realized (Cantor, 1878) (Dauben, 1979). In order to generate statistical independence between the laboratory frame and individual self-reference frames the spaces have limited precision. This is significant reduction in mathematical overhead, where singularities cannot be describe, and calculations using infinite sums in function spaces are probably not supported. By avoiding these problems a set of second order relativistic quantum field equations can be derived for both particles and fields. The long standing measurement problem of quantum mechanics is also resolved without an external observer as these dual spaces monitors their mutual existence.

What verified these necessary additions to quantum mechanics was a long term study of spin zero boson with mass, which was an exciton in well annealed iron with a scale of .14 m and a mass 10^{-9} of an electron (Wallace, 2009) (Wallace and Wallace, 2014b). Confirming the structure derived for the electron in the correction to the hydrogen 1S absolute energy level using the charge structure of electron rather than a point charge (Wallace and Wallace, 2015). A third test was using the derived fractional quantization of charge to generate quark wave functions to compute the bound state and the energy level of the excited nuclear state of deuterium that decays in what is commonly called a short range nuclear correlation (Wallace and Wallace, 2017) (Arrington *et al.*, 2012). Finally the computation of the electron-neutrino's wave function and its comparison to that of the photon showing it will have one half of its normally computed scattering cross section across the entire energy spectrum (Wallace and Wallace, 2017). The problem of refraction is different in that it requires the full solution of the relativistic wave equation in the laboratory frame, which replaces the Schrödinger equation.

II. RELATIVISTIC QUANTUM REFRACTION

Neither the Schrödinger equation, Dirac equation, nor matrix mechanics are derived consistent with relativity. That is why only a few problems can actually be solved in quantum mechanics. Bring relativitiy into the mix allows a cleaner approach to the problem of refraction that neither excites a permanent transition nor is completely benign (Wallace and Wallace, 2017).

Within the relativistic conservation relation the potential is derived from the mass of the particle. The variation $m - m_o = \delta m$ represents the source of the potential interaction.

$$E^2 = p^2c^2 + (m_o + \delta m)^2c^4 \tag{2}$$

$$E^2 - (m_oc^2)^2 = p^2c^2 + (2\delta m m_o + \delta m^2)c^4 \tag{3}$$

δm^2 is small relative to $2\delta m m_o$ and is dropped. The potential is taken to be $V = \delta mc^2$

$$E^2 - (m_oc^2)^2 = p^2c^2 + 2Vm_oc^2 + V^2 \tag{4}$$

$$\frac{E^2 - (m_oc^2)^2}{2m_oc^2} = \frac{p^2}{2m_o} + V(1 + \frac{V}{2m_oc^2}) \tag{5}$$

It is simple to derive something functional to replace the Klein-Gordon equation that conserves energy and compatible with relativity as a second order wave equation in the laboratory frame (Wallace and Wallace, 2014a) (Wallace and Wallace, 2017). The energy operator, which is a first order time derivative, is taken as the total energy less the self-energy.

$$i\hbar\frac{\partial}{\partial t} \rightarrow E - mc^2 \tag{6}$$

Using the momentum operator and the correct energy operator equation 5 is converted into the resulting differential equation, which has two additional terms absent from the Schrödinger equation. The second order time dependent term embedded the propagating field equation more commonly found from electromagnetic theory of Maxwell. The second addition is a quadratic term in the potential, whose presence brings in the statistical basis of quantum mechanics naturally so it is no longer an ad hoc postulate (Wallace and Wallace, 2017).

$$\frac{\hbar^2}{2m}\{\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}\} + i\hbar\frac{\partial\Phi}{\partial t} = (V + \frac{V^2}{2mc^2})\Phi \tag{7}$$

The equation can be reduced to the Schrödinger equation. This comes at a cost of losing its compatibility with relativity. That reduction introduces errors that have been commonly corrected by perturbation techniques, which are not unique and cause significant confusion.

A. Quadratic Potential

For a particle in vacuum there are no external potentials setting the two right hand terms to zero, $V + V^2/2mc^2 = 0$, yields two solutions $V = 0$ and $V = -2mc^2$. The quadratic potential term contains the mechanism for generating an array of different emergent phenomenon from a general mechanism. First it allows the creation of an excitation and its corresponding antiparticle where the total energy is found in the denominator as $2mc^2$. This is the source of inertial mass and the statistical behavior of quantum particle/field. The two solutions are equally weighted and the identity of the surviving particle after annihilation is masked in this on going process. The equation is actually more general and can operate when other lower energy couplings are available such as collective material states and not just pair production from the original particle or field.

Another feature of the quadratic potential is that fluctuating potentials with a zero mean value will be found active in any material as $< V^2 >$ is always greater than zero, $V^2 > 0$. This ensures the quadratic potential term does not vanish. Having a non-vanishing potential available is an essential feature of dielectric supporting the persistent effect of refraction of light.

B. Emergent Phenomenon

It is not necessary to expend *1.024 MeV* to generate a positron-electron pair when only the lowest energy atomic states can be excited in the eV range or a collective excitation at even lower energies to make the V^2 term a significant contribution. To represent multiple states, ΔE_i with different energies the index, i, is added as a subscript. This appears to be the mechanism that supports a prompt polarization process producing refraction within the wave front of the passing field.

$$\frac{\hbar^2}{2m}\{\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}\} + i\hbar\frac{\partial\Phi}{\partial t} = (V + \sum_i \frac{V_i^2}{2\Delta E_i})\Phi \tag{8}$$

The strongest effect are for the minimum available excitation ΔE. In any stable material such as glass for a photon there will be internal fluctuating potentials that have a mean value of zero value leaving only the quadratic term as a continuous contributor. Loss transitions dominate at higher energies couple through the linear potential term, V, to produce absorption. For weak process interactions, when a neutrino is passing through matter, the quadratic term will not vanish

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = \frac{2m}{\hbar^2}\{-i\hbar\frac{\partial\Phi}{\partial t} + \frac{V^2}{\Delta E}\Phi\} \tag{9}$$

$$Field \qquad\qquad\qquad Medium$$

Just by regrouping the five term equation, its coupled behavior is obvious. What remains, differs from its classical and non-relativistic counter part because the field half and the medium half of the equation now can be separated. The right side of equation 9 has mass as a factor represent the medium's response to the field. A field, like a photon, or an electron-neutrino as derived in free spaces have no mass. Even though there is a great deal of financial support for the electron-neutrino having a mass, the experimental data does not definitively support that conclusion. The apparent loss of solar neutrinos can be explained by a reduction in scattering cross section (Wallace and Wallace, 2017) However, for a photon there is an interaction with a medium with weakly excited oscillation of the lattice through the V^2 term that modifies the fields motion by reducing its velocity. The structure of the wave function Φ, which encompasses characteristics of both the field and medium, will be treated as product of the field portion, ϕ_{field} and the medium ϕ_{medium}.

$$\Phi = \phi_{field} \otimes \phi_{medium} \tag{10}$$

The question to ask about equation 9 is: for what value of a field's velocity will the left hand side of the equation equal zero, representing a measurable non-dissipating propagating massless field? The result is a wave equation with a reduced velocity v.

$$v < c$$
$$\nabla^2\Phi - \frac{1}{v^2}\frac{\partial^2\Phi}{\partial t^2} = \frac{2m}{\hbar^2}\{-i\hbar\frac{\partial\Phi}{\partial t} + \frac{V^2}{2\Delta E}\Phi\} = 0 \tag{11}$$

The equations have well know solution once it is realized that the mass, m, is nonzero and is representing the medium.

$$i\hbar\frac{\partial\phi_{medium}}{\partial t} = < E_{medium} > \ \phi_{medium}$$
$$\nabla^2\phi_{field} - \frac{1}{v^2}\frac{\partial^2\phi_{field}}{\partial t^2} = 0 \tag{12}$$

This solely time dependent equation for the medium, whose spatial dependence is defined by the field front, now shares some of its original energy with the medium. The product of the two wave functions isolates the medium's response to a prompt interaction with the passing field. What is a pleasant surprise is the simplicity of how the five term field equation accommodates both the field and the medium while separating refraction and absorption. The classical models fail at this task.

Energy is conserved in the partitioning, which involves no transition, only a quadratic local potential interaction with the medium. The net result of this separation of effort produces a basic description of refraction, where the reduction in the velocity of the propagating field has part of its energy diverted to driving a polarization oscillation of the medium as a whole. The analysis can be extended to understanding nonlinear optical materials where quadratic potential term enters not as a perturbation correction, but directly, and can be used in describing strong field behavior generated by lasers fields.

III. COLLECTIVE POLARIZATION

The basis of optical refraction appears to be strongly connected to a model where polarization can be treated much like elastic scattering a process which absorbs and delivers back any energy collected. This is opposed to a model where exchange of quanta are employed to describe the interaction. With a description of particles where potentials can be derived from their structure, this form of bulk interaction appears to operating not only for photons in glass but also for neutrinos with nuclear matter. These high speed interaction actually suppress the slower exchange of quanta that result in absorption losses.

A massless field is limited in its interactions as it has no decay mechanism built into its time dependence as do particles with mass (Wallace and Wallace, 2017). Limited in this way its fields and self-energy changes are constrained to conservative processes. A field passing through a medium can either be completely absorbed or polarize the medium and recover that energy on leaving the medium.

IV. NEUTRINO MASS AND REFRACTION

The history of the formerly massless electron-neutrino was complicated by an errant assumption in a paper by John Bahcall in trying to explain an apparent loss in the number of solar neutrinos in transit by using an elementary nuclear model of beta decay to estimate the scattering cross section (Bahcall, 1964). The mistake was assuming absorption and emission rates for the electron-neutrino were the same using beta decay data. This assumption fails because the electron-neutrino's density function is very different from that of the photon. Even in optical transitions the assumption that emission and absorption rates are equal can only be made on average for large populations in equalibrium (Wallace and Wallace, 2017). The technical mistake was then further compounded because the electron-neutrino was grouped with the other two high energy versions of the neutrino by analogy to high energy theory of kaon behavior (Pontecorvo, 1957). This is an assumption that really has no foundation for the low energy processes that affect the electron-neutrino which are leptons and not mesons.

The extensive literature on the neutrino-cross section

TABLE I **Four regions of different composition and energies for the principal weak transition that would affect solar neutrinos in refraction. Energy in parentheis is in MeV for the weak absorption threshold. The four different regions have very different characteristics that should be isolated with the collection of more solar neutrion spectroscopic through earth data.**

Atmosphere mass frac.	**Crust** mass frac.	**Mantle** mass frac.	**Core** mass frac.
N .78 (5.14)	**O .46** (1.1)	**O .45** (1.1)	**Fe .89** (3.7)
O .21 (1.11)	**Si .28** (4.64)	**Mg .23** (13.87)	Ni .058
Ar .01	Al .082	Si .22	S .045
	Fe .056	Fe .058	trace
	Ca .042	Ca .023	
	Na .025	Al .022	
	Mg .025	Na .003	
	K .02		
	Ti .006		

as a function of energy are dynamic calculations at a level above of the density calculation for the neutrino in the self-reference frame (Formaggio and Zeller, 2012). These kinematic models do not involve the structure of the fields themselves, only their bulk properties and allowed interactions. It is not necessary to involve the specific mechanisms for the energy dependent calculation of cross-sections, because the correction being introduced will affect the neutrino across its entire energy range uniformly.

V. REFRACTION IN THE LABORATORY FRAME

The experimental data both from Borexino (Ludhova and et. al., 2012) (Derbin and group, 2016) and the Super-Kamiokande experiment (Abe and et. al., 2016) not only show a loss of flux close to 50% they have very different response to the day-night variation with the night enhancement found only by the Super-Kamiokande experiment. The Super-Kamiokande data shows more solar ν_e are detected when traveling through the earth by 3.6% $\pm 1.6\%$ greater than when detected sourced from the zenith (Abe and et. al., 2016). The analog to such problems is found in classical physics where light is refracting in a transparent media. The earth is acting as a lens for the detector. The same enhancement is not found in the Borexino data because there is a geometrical design difference between the detectors.

A. Neutrino Geo-Refraction

This analysis of refraction driven by a weak force, rather than electric charge, coupling to the field of the neutrino must be considered for both the solar electron neutrino and anti-electron neutrino sourced by reactors. In both measurement cases nuclear matter using a weak force interaction will refract the fields motion. In the earth's crust ^{28}Si which is abundant along with many other isotopes that have weak force transitions in the low MeV range can produce a significant interaction. Currently, there are numerous experiments ongoing trying to tease out information about the neutrino. There are a number of weak transitions that can supply the potential for refracting both electron and anti-electron neutrinos. For refracting anti-electron neutrinos reactor flux requires only water containing protons and oxygen. What is actually required from experiment is a measurement of refractive index as a function of energy, a dispersion relation. This is complicated by the chemical sensitivity to such a relationship and the ability to be able to control this variable over such large scales. This maybe more easily achieved with reactor experiments where there can be both water and earth paths to provide data. Using the earth as a whole does have some advantages since the crust, mantle, and core should provide some contrast difference as a function of neutrino energy.

The analysis of neutrino data as a function of the angle of the sun to the detector orientation is complicated by the detector geometry. The ideal structure would be a right circular cylinder whose axis was aimed at the sun. In the Super-Kamiokande detector this nearly happens twice a day, noon and midnight at the summer solstice. At other times the detector will be less sensitive to refraction effects in a complicated fashion. The 20 meter radius of the detector is effectively enlarged by a factor of 1.018 to produce the total 3.6% enhancement. That is an effective radius increase of 18 cm that can be projected from the far side of the earth to allow the angular deflection to be computed by dividing by the earth's diameter to yield 1.41×10^{-8} radians for the deflection of the solar neutrino flux. This produces a mean refractive index for the weak force refraction of 1.0000000141 that is between seven and eight orders of magnitude less than produced for optical refraction in some glasses. The density difference in the two cases affect the ratio as the earth's density at the core is significantly greater. The ratio is a practical measure of the ratio of the electromagnetic force to the weak force.

B. Design of Refraction Detector for ν_e

The detector design is important in isolating a measure for neutrino refraction as a function of the neutrino path through the full motion of earth as it rotates. Neutrino detectors being large stationary masses are not ideal astronomical instruments when trying to follow the sun. The refraction effects are modest when compared to the scale of the neutrino detectors as the detector should present both a long and uniform cross section when observing. Spherical detectors have a geometric restriction on their sensitivity to refraction because they present only a small active region at their outer band at any one time for refracted neutrino detection, which has a vanishingly small optical depth at its outer diameter. This is supported by the Borexino experiment's inability to detect a day-night variation (Ludhova and et. al., 2012). A smaller diameter steerable right circular cylinder would be a better detector, however unless it is long it will suffer from a low count rate. The only efficient way to support the detector and structure is to have the detector submerged reducing the load of the detector mass so that the entire structure can be made to follow the sun. Then it will also be possible to scan the sun during the day to image the neutrino source distribution.

VI. DISCUSSION

The understand of refraction will improve our understanding of polarization by the various force: EM, weak, and strong, particularly in the limit of scattering not from mediums containing many particles but only a few. Compton scattering marks the end point of the scattering problem for a photon scattering from an electron where momentum can be exchanged. Whereas, the refraction of light imparts no momentum to a lens composed of many atoms it passes through. Constraints on matter determine the type of interactions. This is also an important problem for nuclear matter in the understanding of fusion, particularly at low energies.

Neutrino refraction generated dispersion curves would be of interest for geophysics. This data would add to understanding of how the neutrino moves through the earth dependent on density and composition. Knowledge of the dispersion curve for the neutrino should yield information about the elemental distribution of matter in regions of the earth that are now only accessed by seismology.

More importantly at present there are now three pieces of experimental data that support the electron-neutrino being a massless spin one half fermion field rather than possessing mass: 50% reduced interaction cross section, refraction from the day-night flux difference for the Super-Kamiokande detector, and the inability of a spherical Borexino detector to pick up the day-night refraction signal.

VII. ACKNOWLEDGMENTS

Discussions with Glenn Westphal about the scope of the paper were helpful.

REFERENCES

Abe, K., and et. al. (2016), "Solar neutrino measurements in super-kamiokande-iv , arxiv:1606.07538v1 [hep-ph]," Super-Kamiokande Collaboration.

Arrington, J., W. Higinbotham, G. Rosner, and M. Sargsian (2012), "Hard probes of short-range nucleon-nucleon correlations arxiv:1104.1196v3 [nucl-ex],".

Bahcall, J. (1964), Phys. Rev. **135** (1B), B136.

Bahcall, J. N. (1989), Phys Rev D **41** (10), 2964.

Cantor, G. (1878), Journal für die reine und angewandte Mathematik **84**, 242.

Dauben, J. W. (1979), *George Cantor His Mathematics and Philosophy of the Infinite* (Harvard Univ. Press, Cambridge, MA).

Derbin, A., and B. group (2016), "The main results of the borexino experiment, arxiv:1605.06795v1 [hep-ex],".

Formaggio, J., and G. Zeller (2012), Rev. Mod. Phys. **84**, 1307, arXiv: 1305.7513v1.

Heitler, W. (1954), *The Quantum Theory of Radiation*, 3rd ed. (Oxford Univ. Press, London).

Ludhova, L., and et. al. (2012), "Solar neutrino physics with borexino i, arxiv:1205.2989v1 [hep-ph]," Borixino Collaboration.

Newton, I. (1730), *Opticks or A Treatice of the Reflections, Refractions, Inflections, and the Color of Light*, 4th ed. (Dover Press, NYC) reprint 1952.

Pontecorvo, B. (1957), Journal of Experimental and Theoretical Physics, english trans. **6**, 429.

Wallace, J. (2009), "Electrodynamics in iron and steel, arxiv:0901.1631v2 [physics.gen-ph],".

Wallace, J., G. Myneni, and R. Pike (2011), in *SSTIN10 AIP Conference Proceedings 1352*, edited by G. Myneni and et. al. (AIP, Melville, NY) pp. 38–46.

Wallace, J., and M. Wallace (2014a), *The Principles of Matter amending quantum mechanics*, Vol. 1 (Casting Analysis Corp., Weyers Cave, VA).

Wallace, J., and M. Wallace (2014b), viXra:1405.0015v1.

Wallace, J., and M. Wallace (2015), in *Science and Technology of Ingot Niobium for Superconducting Radio Frequency Applications*, Vol. 1687, edited by G. Myneni (AIP, Melville, NY) pp. 040004–1–14, *Electrostatics*.

Wallace, J., and M. Wallace (2017), *"yes Virginia, Quantum Mechanics can be Understood"* (Casting Analysis Corp., Weyers Cave, VA).

©Casting Analysis Corp. 2019