Refraction

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Reflection of light is well understood refraction is a more difficult problem. Refraction has been treated as a classical property and recently it became apparent where this property finds its quantum origin. The Schrödinger equation is a non-relativistic truncation of a more general five term equation that is consistent with relativity in the laboratory frame (Wallace and Wallace, 2017). It is the solution of this five term equation that supplies the quantum nature of refraction. Three different components of the solar neutrino survival data supports a massless electron neutrino, ν_e, not processes where the electron-neutrino oscillates to different flavors. The neutrino's weak force interaction with matter is sufficient to produce a measurable refractive index for the neutrino. The ratio of refraction index between the neutrino passing through the earth and the photon in transparent materials reduced to the ratio of a weak force to the electromagnetic force.

CONTENTS

I. INTRODUCTION

Refraction is a well studied subject beginning with Newton's *Optick* (Newton, 1730) and extended to a non-relativistic quantum description of a field interacting with an atom (Heitler, 1954). The property is a dynamic polarization mediating the dispersion of radiation by transparent matter that falls in the gap between driving transitions and indifference.

Refraction of light by a dielectric has a well establish classical description modeled by of a bound electrons whose charges respond to the passing **E**-field. The model is compactly described by using Newton's second law, Hooke's law, and a loss term (Feynman *et al.*, 1964).

$$F = q_e\mathbf{E} = m(\ddot{x} + \gamma\dot{x} + \omega_o^2 x) \tag{1}$$

Where x is the electron's displacement, γ is the strength of the loss mechanism, and ω_o^2 captures the Hooke's law response. The right hand side of the equation describes the medium light is traveling through and the left hand side defines the field interaction with the electron's charge. The non-relativistic quantum description is an extension of this model where the energy of the electron is shared between two field terms. The bound state refraction model functions in two way: first to reduce the speed of light in the medium and secondly to provide a mechanism for absorption. The property that must be understood is the mechanism for slowing the speed of light without absorption.

II. RELATIVISTIC QUANTUM REFRACTION

When trying to derive the Schrödinger equation in a self-consistent manner using a relativistic starting point with the embedded self-reference frame scale definition of inertial mass, a more general field equation was found for the laboratory frame from which the Schrödinger equation constitutes only 3/5 of the terms of the full equation. The missing 2/5's from the Schrödinger equation is why only a few problems can actually be solved in quantum mechanics without modifying the Schrödinger equation. Bring relativitiy into the mix allows a cleaner approach to problem of refraction that neither excites a permanante transition nor is completely benign. A short derivation of the full five term equation is found in Appendix A (Wallace and Wallace, 2017).

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$$\frac{\hbar^2}{2m}\{\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}\} + i\hbar\frac{\partial\Phi}{\partial t} = (V + \frac{V^2}{2mc^2})\Phi \quad (2)$$

For a particle in vacuum there are no external potentials setting the two right hand terms to zero, $V + V^2/2mc^2 = 0$, yields two solutions $V = 0$ and $V = -2mc^2$. The quadratic potential term contains the mechanism for generating and array of different emergent phenomenon from a general mechanism first introduced as Dirac's hole theory. First it allows the creation of an excitation and its corresponding hole where the total energy is found in the denominator as $2mc^2$. This is the source of the statistical behavior of quantum particles because the two solutions are equally weight and the identity of the surviving particle after annihilation is masked in this on going process. The equation is actually more general and can operate when other lower energy excited states are available not just pair production from the original particle.

Another feature of the quadratic potential is that a fluctuating potential with a zero mean value, which will be found in any material that is encounter will always have $V^2 > 0$. This ensures the quadratic potential term does not vanish. That is essential for understanding persistent effects such as the refraction of light in a dielectric medium.

A. Emergent Phenomenon

Elementary processes such as pair production are not the only processes that can be described by the quadratic potential term of equation 2. It is not necessary to expend *1.024 MeV* to generate a positron-electron pair when only the lowest energy atomic states can be excited in the eV range or a collective excitation at even lower energies to make the V^2 term a significant contribution. To represent multiple states, ΔE_i with different energies the index, i, is added as a subscript. This appears to be the mechanism that supports a prompt process producing refraction within the wave front of the passing field.

$$\frac{\hbar^2}{2m}\{\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}\} + i\hbar\frac{\partial\Phi}{\partial t} = (V + \sum_i \frac{V_i^2}{\Delta E_i})\Phi \quad (3)$$

The strongest effect will be for the minimum available excitation ΔE. In any stable material such as glass for a photon there will be internal fluctuating potentials that have a mean value of zero value leaving only the quadratic term as a continuous contributor. Loss transitions dominate at higher energies coupling through the linear potential term, V, to produce absorption. For weak process interactions when a neutrino is passing through matter the quadratic term will be competing with the linear term in determining the behavior of the neutrino. The efficient penetrating ability of the neutrino through matter cannot completely eclipse contributions from the quadratic term through the weak force interaction.

$$\underbrace{\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}}_{Field} = \underbrace{\frac{2m}{\hbar^2}\{-i\hbar\frac{\partial\Phi}{\partial t} + \frac{V^2}{\Delta E}\Phi\}}_{Medium} \quad (4)$$

What is left is an interesting equation which differs from its classical and non-relativistic counter parts because the field half and the medium half of the equation can be separated. The two terms that have a mass as a factor represent the mediums response to the field. If we have a field like a photon or a neutrino in empty spaces there is no mass. However, for a photon there is an interaction with a medium with weakly excited oscillation of valence electrons that inhibits the fields motion reducing its velocity. The structure of the wave function Φ, which encompasses characteristics of both the field and medium, will be treated as product of the field portion, ϕ_{field} and the medium ϕ_{medium}.

$$\Phi = \phi_{field} \otimes \phi_{medium} \quad (5)$$

Those terms multiplied by mass are associated with the medium and the rest are associated with the field.

The obvious question to ask about equation 4 is for what value of field's velocity will the left hand side of the equation equal zero, so it represents a measurable non-dissipating propagating massless field. The result is a wave equation with a reduced velocity v.

$$\overbrace{\nabla^2\Phi - \frac{1}{v^2}\frac{\partial^2\Phi}{\partial t^2}}^{v<c} = \frac{2m}{\hbar^2}\{-i\hbar\frac{\partial\Phi}{\partial t} + \frac{V^2}{\Delta E}\Phi\} = 0 \quad (6)$$

Then the right hand side of the equation becomes a relation where mass is no longer a parameter. The expression takes a very simple well known form as the time dependence of the medium as a Hamiltonian local to the field.

$$i\hbar\frac{\partial\phi_{medium}}{\partial t} = \frac{V^2}{\Delta E}\phi_{medium} \quad (7)$$

This solely time dependent Hamiltonian for the medium, whose spatial dependence is only defined by the field front, which is now sharing some of its original energy with the medium without generating a transition. This interaction occurs within the field front whose dynamics also determines the field quantization and angular momentum (Wallace and Wallace, 2014) (Wallace and Wallace, 2017). It is not apparent if it is possible to further unravel the details within the light front. The product of the two wave functions isolates the medium's response as a prompt interaction with the passing field. What is a pleasant surprise is the simplicity of how the five term field equation accommodates both the field and the medium along with refraction and absorption.

$$i\hbar\frac{\partial\phi_{medium}}{\partial t} = E_{medium}\ \phi_{medium}$$

$$\nabla^2\phi_{field} - \frac{1}{v^2}\frac{\partial^2\phi_{field}}{\partial t^2} = 0 \tag{8}$$

These two equations have simple and well known solutions. The quantum computation of the second order potential term that generates refraction requires knowledge of the available states within the medium that will become active.

Energy is conserved in the partitioning which involves no transition only a quadratic local potential interaction with the easiest available excitable states of the medium without a transition. A dynamic response to a static potential should not be a surprise. The net result of this separation of effort produces a basic description of refraction, where the reduction in the velocity of the propagating field has part of its energy diverted to driving a potential oscillation of the medium. This is a model of a locally available ether in the spirit of Torricelli (Torricelli, 1715). Such behavior is also useful in understanding nonlinear optical materials where the quadratic term does not enter in as perturbation correction and can be used on strong fields generated by lasers.

III. NEUTRINO REFRACTION

Analysis of a complex problem like neutrino behavior can be easily misdirected if poor assumptions are made about its basic properties. The apparent loss of 50% of the computed solar neutrino flux (Derbin and group, 2016) predicted from the standard solar model was dependent assumptions about the interaction cross section (Bahcall, 1989). The popular explanation was a theory based on an analogy to the behavior of the strange quark with the charge neutral Kaons that have multiple states (Pontecorvo, 1957). It was thought that electron-neutrino, ν_e may also move to different states so that would not be detected as a ν_e. To have that ability and conserve energy the ν_e would need to have a mass. Fortunately a good argument can be made to show it has no mass, however it has a reduced scattering cross section consistent experimental solar data (Wallace and Wallace, 2017). If the neutrino is massless it will be susceptible to refraction from weak force interactions that will produce an enhanced count by refracting the flux on passing through the earth.

TABLE I **Four regions of different composition and energies for the principal weak transition that would affect solar neutrinos in refraction. Energy in parentheis is in MeV. The four different regions have very different characteristics that should be illuminated with the collection of more solar neutrion spectroscopic through earth data.**

Atmosphere mass frac.	**Crust** mass frac.	**Mantle** mass frac.	**Core** mass frac.
N .78 (5.14)	**O .46** (1.1)	**O .45** (1.1)	**Fe .89** (3.7)
O .21 (1.11)	**Si .28** (4.64)	**Mg .23** (13.87)	Ni .058
Ar .01	Al .082	Si .22	S .045
	Fe .056	Fe .058	trace
	Ca .042	Ca .023	
	Na .025	Al .022	
	Mg .025	Na .003	
	K .02		
	Ti .006		

It is easy show that the missing solar ν_e are not missing only their scattering cross section is reduced. The total wave function of the ν_e in the self-reference frame is made of product of two parts $\phi(r,\tau) = u(r)g(\tau)$ the time dependence being of the form $e^{-i\omega\tau}$ becomes a factor of 1 in the probability density function. The particle density in the self-reference frame in three dimensions is given by the expression $u^*(r)u(r)r^2$. The core of density $u^*(r)u(r)$ in the case of a massive fermion is proportional to the static electric field and removes the $1/r^2$ singularity of the point electron at its center of symmetry (Wallace and Wallace, 2015). In the case of the massive boson the properties of weak charge result and the description is found in (Wallace and Wallace, 2014). For the massless fields the boson density is a constant as it is for the photon field. However, for the fermion field it has an oscillatory behavior as shown in Figure 1. It is the oscillatory character of the fermion field density function that reduces the particles interaction cross section by one half that is the measured loss in both the Borexino and Super-Kamiokande experiments.

$$u^*_{neutrino}(r)u_{neutrino}(r)r^2 \sim Sin^2\kappa r \tag{9}$$

The mean value of the Sin^2 term is exactly one half. This behavior in the spatial portion of the wave function is unique among particles and will lead to a reduction in detected sensitivity by exactly 50% in measured data whether from solar or reactor generated electron neutrinos.

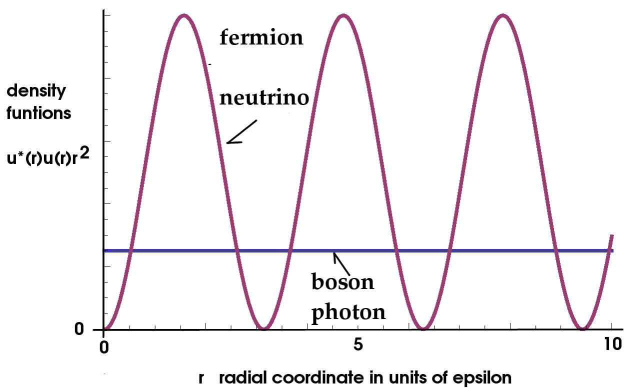

FIG. 1 **Density functions of two massless fields in the self-reference frame. The individual density scales are arbitrary so the functions separate. The reduced neutrino cross section is a result of the oscillatory behavior of the density function (Wallace and Wallace, 2017)**

The extensive literature on the neutrino-cross section as a function of energy are dynamic calculations at a level above of the density calculation for the neutrino in the self-reference frame (Formaggio and Zeller, 2012). The kinematic models do not involve the structure of the particles themselves, only their bulk properties and allowed interactions. It is not necessary to involve the specific mechanisms for the energy dependent calculation of cross-sections, because the correction being introduced will affect the neutrino across its entire energy range uniformly.

IV. REFRACTION IN THE LABORATORY FRAME

The experimental data both from Borexino (Ludhova and et. al., 2012) (Derbin and group, 2016) and the Super-Kamiokande experiment (Abe and et. al., 2016) not only show a loss of flux close to 50% they have very different response to the day-night variation with the night enhancement found only by the Super-Kamiokande experiment. The Super-Kamiokande data shows more solar ν_e are detected when traveling through the earth by 3.6% $\pm$1.6% greater than when detected sourced from the zenith (Abe and et. al., 2016). The analog to such problems is found in classical physics where light is refracting in a transparent media. The earth is acting a lens for the detector. The same enhancement is not found in the Borexino data.

A. Neutrino Geo-Refraction

This analysis of refraction driven by a weak force, rather than electric charge, coupling to the field of the neutrino must be considered for both the solar electron neutrino and anti-electron neutrino sourced by reactors. In both measurement cases nuclear matter using a weak force interaction will refract the fields motion. In the earth's crust ^{28}Si which is abundant along with many other isotopes that have weak force transitions in the low MeV range that could produce a significant interaction. Currently there are numerous experiments ongoing trying to tease out information about the neutrino. There are a number of weak transitions that can supply the potential for refracting both electron and anti-electron neutrinos. For refracting anti-electron neutrinos reactor flux requires only water containing protons and oxygen which should be active. What is actually required from experiment is a measurement of refractive index as a function of energy, a dispersion relation. This complicated by the chemical sensitivity to such a relationship and the ability to be able to control this variable over such large scales. This maybe more easily achieved with reactor experiments where there can be both water and earth paths to provide data. Using the earth as a whole does have some advantages as the crust, mantle, and core should provide some contrast difference as a function of neutrino energy.

The analysis of neutrino data as a function of the angle of the sun to the detector orientation is complicated by the detector geometry. The ideal structure would be a right circular cylinder whose axis was aimed at the sun. In the Super-Kamiokande detector this nearly happens twice a day, noon and midnight at the summer solstice. At other times the detector will be less sensitive to refraction effects in a complicated fashion. The 20 meter radius of the detector is effectively enlarged by a factor of 1.018 to produce the total 3.6% enhancement. That is an effective radius increase of 18 cm that can be project to the earth to the far side to allow the angular deflection to be computed by dividing by the earths diameter to yield 1.41×10^{-8} radians for the deflection of the solar neutrino flux. This produces a mean refractive index for the weak force refraction of 1.0000000141 that is between seven and eight orders of magnitude less than produced for optical refraction in some glasses. The densities in the two cases are not the same as the earth's density at the core is significantly greater. The ratio is a practical measure of the ratio of the electromagnetic force to the weak force.

B. Design of Refraction Detector for ν_e

The detector design is important in isolating a measure for neutrino refraction as a function of the neutrino path through the full motion of earth as it rotates. Neutrino detectors being large stationary masses are not ideal astronomical instruments when trying to follow the sun. The refraction effects are modest when compared to the scale of the neutrino detectors as the detector should present both a long and uniform cross section when observing. Spherical detectors have a geometric restrictions on their sensitivity to refraction because they present only a small active region at their outer band at any one time for neutrino detection, which has a vanishingly small optical depth at its outer diameter. This is supported by the Borexino experiment's inability to detect a day-night variation (Ludhova and et. al., 2012). A smaller diameter steerable right circular cylinder would be a better detector, however unless it is long it will suffer from a low count rate.

V. DISCUSSION

Neutrino refraction generated dispersion curves would be of interest for geophysics. This data would add to understanding of how the neutrino moves through the earth dependent on density and composition. Knowledge of the dispersion curve for the neutrino should yield information about the distribution of isotopes in regions of the earth that are now only accessed by seismology.

More importantly at present there are now three pieces of experimental data that support the electron-neutrino being a massless spin one half fermion field rather than possessing mass: 50% reduced interaction cross section, refraction from the day-night flux difference for the Super-Kamiokande detector, and the inability of a spherical Borexino detector to pick up the day-night refraction signal.

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Appendix A: Extended Wave Equation

Within the relativistic conservation relation the potential is derived from the mass of the particle. The variation $m - m_o = \delta m$ represents a potential interaction.

$$E^2 = p^2c^2 + (m_o + \delta m)^2 c^4 \tag{A1}$$

$$E^2 - (m_o c^2)^2 = p^2c^2 + (2\delta m m_o + \delta m^2)c^4 \tag{A2}$$

δm^2 is small relative to m^2 and dropped. The potential is taken to be $V = \delta mc^2$

$$E^2 - (m_o c^2)^2 = p^2c^2 + 2Vm_oc^2 + V^2 \tag{A3}$$

$$\frac{E^2 - (m_oc^2)^2}{2m_oc^2} = \frac{p^2}{2m_o} + V(1 + \frac{V}{2m_oc^2}) \tag{A4}$$

It is simple to derive something functional to replace the Klein-Gordon equation that conserves energy and compatible with relativity as a second order wave equation in the laboratory frame (Wallace and Wallace, 2014) (Wallace and Wallace, 2017). The energy operator, which is a first order time derivative, is taken as the total energy less the self-energy. This compatible both with the Schrödinger equation and the Dirac equation and does not violate the quadratic relativistic conservation of energy condition (Fermi, 1961).

$$i\hbar\frac{\partial}{\partial t} \rightarrow E - mc^2 \tag{A5}$$

Using the momentum operator and the correct energy operator equation A4 is converted into the resulting differential equation, which has two additional terms absent in the Schrödinger equation. The second order time dependent term embedded the propagating field equation more commonly found from electromagnetic theory of Maxwell. The second addition is a quadratic term in the potential, whose presence brings in the mechanics of the virtual field and pair-production naturally that is no longer an ad hoc postulate (Wallace and Wallace, 2017).

$$\frac{\hbar^2}{2m}\{\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}\} + i\hbar\frac{\partial\Phi}{\partial t} = (V + \frac{V^2}{2mc^2})\Phi \tag{A6}$$

The above equation can be reduced to the standard Schrödinger equation for some bound state and free propagation problems. However, it loses its compatibility with relativity. That reduction introduces errors which have been commonly corrected by perturbation techniques.

REFERENCES

Abe, K., and et. al. (2016), "Solar neutrino measurements in super-kamiokande-iv , arxiv:1606.07538v1 [hep-ph]," Super-Kamiokande Collaboration.

Bahcall, J. N. (1989), Phys Rev D **41** (10), 2964.

Derbin, A., and B. group (2016), "The main results of the borexino experiment, arxiv:1605.06795v1 [hep-ex],".

Fermi, E. (1961), *notes on Quantum Mechanics* (University of Chicago Press, Chicago, Ill.).

Feynman, R. P., R. B. Leighton, and M. Sands (1964), *The Feynman Lectures on Physics*, Vol. II (Addison-Wesley, Reading, MA).

Formaggio, J., and G. Zeller (2012), Rev. Mod. Phys. **84**, 1307, arXiv: 1305.7513v1.

Heitler, W. (1954), *The Quantum Theory of Radiation*, 3rd ed. (Oxford Univ. Press, London).

Ludhova, L., and et. al. (2012), "Solar neutrino physics with borexino i, arxiv:1205.2989v1 [hep-ph]," Borixino Collaboration.

Newton, I. (1730), *Opticks or A Treatice of the Reflections, Refractions, Inflections, and the Color of Light*, 4th ed. (Dover Press, NYC) reprint 1952.

Pontecorvo, B. (1957), Journal of Experimental and Theoretical Physics, english trans. **6**, 429.

Torricelli, E. (1715), "Lezioni accademiche, firenze, p. 25,".

Wallace, J., and M. Wallace (2014), *The Principles of Matter amending quantum mechanics*, Vol. 1 (Casting Analysis Corp., Weyers Cave, VA).

Wallace, J., and M. Wallace (2015), in *Science and Tech-*

nology of Ingot Niobium for Superconducting Radio Frequency Applications, Vol. 1687, edited by G. Myneni (AIP, Melville, NY) pp. 040004–1–14, *Electrostatics*.

Wallace, J., and M. Wallace (2017), *"yes Virginia, Quantum Mechanics can be Understood"* (Casting Analysis Corp., Weyers Cave, VA).