A hidden circle in the paths of a family of projectiles

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Abstract

It is already known that an ellipse passes through the apexes of a family of the parabolic paths of projectiles shot from a point with constant speed but different angles of projection. In this article we describe a method to show that a circle passes through the focii of such a family of projectile paths whose center is the point of projection. In this method, we don't use vectors, or calculus or Newton's laws of motion.

Key words: Projectile family, constant speed projectiles, circle from foci of projectiles

Introduction

Recently we showed that an ellipse passes through the apexes of a family of the parabolic paths of projectiles[1]. This family of the parabolic paths is different from another family of the parabolic paths of projectiles considered by Chapou and coworkers[2]. Our method took into consideration paths of particles released from points on the first quadrant of a circle to fall freely (accelerate uniformly) along the vertical direction and reflected by mirrors located at points, midway between the points of release and the horizontal diameter of a circle in the vertical plane. After hitting the mirrors and getting reflected, the particles move along a parabolic path. The motion is a combination of two simultaneous motions – one, a uniform motion of constant speed along the horizontal direction and the second, a uniform accelerated motion along the vertical direction. The position of the mirrors are the apexes of the parabolic paths and lie on an ellipse.

The above method of arriving at the hidden ellipse in the path of a family of the parabolic paths is different from the method of arriving at that ellipse given by Chapou et.al[2]. Their method considers a family of projectiles shot from a point with constant speed but different angles of projection. The apexes of such parabolic paths lie on an ellipse. This ellipse is the same as the ellipse that we got.

In this note we show some more hidden properties of a family of paths of projectiles shot from points in the third quarter of a circle in the vertical plane. In this method, we don’t use vectors, or calculus or Newton’s laws of motion.

Description of the method and analysis

Let us consider an arbitrary circle in the vertical plane centered at point, O (see Fig. 1). Let its vertical diameter be AB. Draw the horizontal through the top end A, of the vertical diameter. We locate mirrors at points m_i on the circle in the third quadrant, inclined to the horizontal at an adjustable angle as desired. From points A_i on the horizontal through A, drop particles on to the mirrors on the circle. Adjust the inclination of the mirrors so as to reflect the particles to pass through A. As the particles leave the mirror after reflection they are subjected to a uniform accelerated motion in the vertically downward direction.

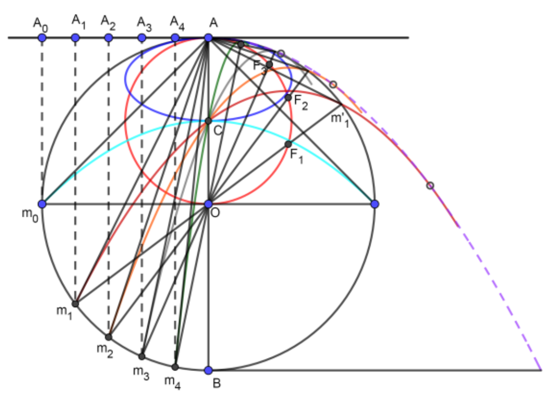

Fig. 1 F_i are the focii of the parabolic paths of the projectiles launched from points m_i on the circle in the third quadrant. The circle with center C and CF_i as radius passes through all the focii of the parabolas. The unfilled circles the points on the 'envelop parabola.

As a result of the initial speed in the direction of the chord m_iA and the uniformly accelerated motion in the vertically downward direction, the particles follow a parabolic path. We note, that the initial speed is not in the direction perpendicular to the vertically downward direction of the acceleration. The chords m_iA of the circle correspond to the tangents of these parabolic paths of the particles. It can be seen from Fig. 1 that all the projectile paths intersect at a point C on the diameter AB of the circle. The projectiles land on the circle at $m'_{i,}$ the opposite end of the diameter through m_i. The chord m'_iA of the circle forms the tangent to the projectile path at m'_i. Thus the two tangents to the projectile path at m_i, and m'_i, pass through A. Therefore, the triangle $m_i\, m'_i\, A$, is a right angled triangle with the right angle at A, $m_im'_i$ being the diameter of the circle. From A, draw a perpendicular on to $m_im'_i$ to meet it at F_i. With F_i as focus and the horizontal through A as the directrix we draw a parabola. This gives the parabolic path of the projectile shot from m_i[3]. Again, $m_im'_i$ forms the focal chord of the parabolic path and it is also a diameter of the circle. Thus, all the focal chords of the parabolic paths pass through O, the center of the circle.

With C as center and CF_i as radius we draw a circle. We find it passes through each of the points F_i (see Fig. 1). At C, all the projectiles have a common value of speed and different directions of motion. We can therefore, consider these paths from the point C onwards, as if they correspond to projectiles launched form a common point with a common value of speed at different angles of projection. We draw a parabola with O as focus, and the horizontal through the reflection of point O in A, as the directrix. This is the ‘envelope parabola’ of the family of parabolic projectile paths; it passes through the points of maximum range (shown in open circles in Fig. 1) of the projectile paths described above.

Thus we see that: 1. the focii of a family of projectile paths shot from a point C with a common value of speed and different angles of projection, lie on a circle whose center is the point of projection. 2. The apexes of these parabolic paths lie on an ellipse as was shown in earlier works[1,2]. 3. The points of maximum range lie on the parabola generally known as ‘envelope parabola’.

We didn’t use either vectors or calculus in our method. We used geometrical methods. The traditional method[2,4], on the other, hand involves a combination of two component motions that are not necessarily in orthogonal directions. While the component motion in the vertical direction is of constant acceleration, the other component motion could be along any direction. The two component motions are combined using vector addition. Not only are the methods of combining the component motions, but also the concept of time are different in the two methods.

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