A more exact G gravitational constant value established by dimensional analysis

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Abstract

A more exact G gravitational constant value was established by dimensional analysis. This more exact value is equal to $6.674092\underline{281} \cdot 10^{-11} m^3 kg^{-1} s^{-2}$. This result was obtained based on the values of the Planck constant, the Avogadro constant and the Boltzmann constant that will be recommended, in November 2018, by the 26th meeting of the General Conference on Weights and Measures (CGPM). These constants will be considered, by a resolution of the CGPM, to have exact values. On the base of these exact values, the gravitational constant *G* is established by calculus. At the same time, the dimensional analysis presented in this article highlights very interesting aspects with respect to the mass.

Introduction

It is well-known the adjustment of the fundamental constants is carried out by the Committee on Data for Science and Technology (CODATA), through its Task Group on Fundamental Constants (TGFC). Currently CODATA TGFC achieves a special adjustment of these constants applying a statistical method (least-squares adjustment - LSA), published by Mohr and Taylor in CODATA recommended values, 1998, Appendix E [1].

In the present article, a possibility for adjusting the value of the gravitational constant *G* by the dimensional analysis is shown. This analysis could be taken into account as an additional nonconventional instrument to assist existing mechanism applied to the adjustment of the fundamental constants. A question could be: why has the gravitational constant *G* been subjected to a such analysis? It is recognized the gravitational constant *G* value is established having a very high relative standard uncertainty ($4.7\text{x}10^{-5}$) [2], compared with other constants. Probably the measurements of *G* since 1980 have unrecognized large systematic errors and new measurements are needed [3]. There are articles suggesting a correlation between measurements of the gravitational constant, *G*, and the length of day [3,4]. The constant *G* is really very important physical quantity for the technical and scientific world.

In this article, the analysis is base on the adjusted values which will be taken into consideration by the 26th meeting of the General Conference on Weights and Measures (CGPM) which will be held from 13 to 16 November 2018 in Versailles, France. This meeting will adopt, among others, a resolution on the revision of the International System of Units (SI) [5]. One of the most important definition proposed for revision will be the kilogram, symbol *kg,* the SI unit of mass. The kilogram will be defined by taking the fixed numerical value of the Planck constant *h* to be $6.626\,070\,15 \times 10^{-34}$ when expressed in the unit *Js,* which is equal to $m^2\, kg\, s^{-1}$. The meter and the second are defined in terms of speed of light *c* and the unperturbed ground-state hyperfine transition frequency of the cesium 133 atom $\Delta \nu Cs$ [5]. A consequence of this change is that the new definition of the kilogram is dependent on the definitions of the second and the meter and is determined experimentally. The results of this adjustment are published, namely, the numerical values of the Planck constant $h = 6.62607015 \cdot 10^{-34} Js$, elementary charge $e = 1.602176634 \cdot 10^{-19}\ C$, Boltzmann constant $k = 1.380649 \cdot 10^{-23} JK^{-1}$ and Avogadro constant $N_A = 6.02214076 \cdot 10^{23}\ mol^{-1}$ [6]. All these four fundamental constants will be considered exact values in the revised SI [5,6].

Having in view these new proposals on the exact values for the revised SI, certain relationships between fundamental constants were established on the basis of dimensional analysis. The analysis is focused mainly on the relationship between the Planck constant and the Avogadro constant, whereas such a relationship establishes a very interesting "liaison" between two masses: one is the mass defined by the Planck constant in accordance with the new proposed definition (CGPM 2018) [5] and the second is the mass defined by the Avogadro constant as a number of mass entities existing in a molar volume V_m.

Starting from this special relationship between Planck constant h and Avogadro constant N_A, a value for gravitational constant G is established by calculus.

Method

The basic principles of dimensional analysis can be found in any serious chemical engineers' handbook [7]. Its applications include from astrophysics, aerodynamics, hydraulics, chemical engineering, to biology and even economics [8]. Initially the author of this article applied dimensional analysis to the mass transfer phenomena [9, 10, 11]. Subsequently he has been interested in applying this method for establishing new correlations between fundamental constants [12,13]. His results were obtained from the data published by Mohr *et al* in 2008 and 2012 [14,15]. The method used in this article was published by the author in 2014 [12]. On the basis of this method every fundamental constant was expressed using the powers of number *2* and a so-called "*characteristic length*", that is an unidimensional quantity having the order of magnitude in the Planck length range ($10^{-35}\ m$). In this way speed of light c or gravitational constant G can be written as: $c = 2.99792458 \cdot 10^{8}\ m\ s^{-1} = (1.344315875237 \cdot 10^{-35} m)(2^{144} s^{-1})$ and $G = 6.67408 \cdot 10^{-11} m^3 kg^{-1} s^{-2} = (1.25986571 \cdot 10^{-35} m)^3 (2^{314} kg^{-1} s^{-2})$, where $1.344315875237 \cdot 10^{-35}\ m$ and $1.25986571 \cdot 10^{-35}\ m$ are the „*characteristic lengths*". If characteristic lengths are related to the characteristic length of the speed of light, for every constant a normalized value noted X is obtained. Such as for c the normalized value is $X_c = 1.0$ and for G, the normalized value is $X_G = 0.93717982$.

Results and discussions

For this article the following fundamental constants have been taken into consideration: speed of light c, Planck constant h, gravitational constant G, Boltzmann constant k, gases constant R, constant product pV_m and Avogadro constant N_A.

Normalized values for these constants are presented in table 1. The normalized values are *universal dimensionless values* and they do not depend any way by the units of measurement, if space and mass are defined by speed of light c and frequency of the cesium 133 atom $\Delta \nu Cs$ [5, 12, 13].

Concerning the data in table 1, the following elements must be highlighted: the Planck constant h was expressed as Planck constant $\hbar$ (h "bar" or Dirac constant), where $\hbar = h/2\pi$ (table 1, row 3); the Planck constant $\hbar$ was expressed as the Planck constant $\hbar^*$, where 2π is considered a dimensional quantity similar to a circle having radius equal to $1m$ (table 1, row 4). In this second case Planck constant $\hbar^*$ has dimensions $ms^{-1}kg$ similarly to an impulse (velocity × mass) [12]. Having in view the universal gases law $pV_m = RT$, a normalized value equal to $X_T = X_{pVm} - X_R$ was defined for the absolute temperature $T = 273.15K$ (table 1, row 9).

Referring to the normalized value X_{NA} for Avogadro constant N_A (table 1, row 10), some clarifying elements must be added. Avogadro constant N_A is equal to $6.02214076 \cdot 10^{23}\ mol^{-1}$. Its signification is a constant number of physical entities existing in a molar volume V_m. If it is taken into account that relationships presented in this article are based on the powers of number 2, it is easy to notice that $N_A = 6.02214076 \cdot 10^{23}$ is very close to $2^{79} = 6.044629098\ ...\ 10^{23}$. Having in view this last value, we can consider that 2^{79} could be similar to an "ideal Avogadro constant" noted $N_{A0} = 2^{79}\ mol^{-1}$, containing 2^{79} elementary ideal physical entities. These elementary ideal physical entities could be identical with the elementary physical entities existing in light (photons). The volume containing 2^{79} elementary ideal physical entities is considered an ideal volume. In this context two normalized values could be defined:
$X_{NA0} = 2^{79}\ mol^{-1} / N_{A0}\ mol^{-1} = 1$ for "Avogadro constant N_{A0}" and $X_{NA} = N_A / N_{A0} = 6.02214076 \cdot 10^{23}\ mol^{-1} / 2^{79}\ mol^{-1} = 0.9962796165$ for Avogadro constant N_A.

But entities existing in both molar and ideal volumes, can be characterized not only by volume as above, but also by length (diameter of entity) or by mass (mass of entity having the same density in both volumes). In this context, the normalized value X_{NA} for Avogadro constant can be compared with normalized values obtained from characteristic lengths (see table 1, row 10) or with normalized value $X_{\hbar}^*$ for Planck constant $\hbar^*$ expressing the ratio of two masses (see explanation below).

Table 1 Normalized values for fundamental constants

	Symbol	**Constants expressed by the „*characteristic lengths*"**	**Normalized values**
0	*1*	*2*	*3*
1	c	$c = 2.99792458 \cdot 10^{8}\ ms^{-1} =$ $=(1.344315875 \cdot 10^{-35} m)(2^{144}\ s^{-1})$	$X_c = 1.0$
2	h	$h = 6.62607015 \cdot 10^{-34} m^2\ kg\ s^{-1} =$ $=(1.1163446084 \cdot 10^{-35}\ m)^2 (2^{122}\ kg\ s^{-1})$	$X_h = 0.83041837783$
3	$\hbar = h/2\pi$	$\hbar = 1.0545718176 \cdot 10^{-34}\ m^2\ kg\ s^{-1} =$ $=(1.259659999 \cdot 10^{-35}\ m)^2 (2^{119}\ kg\ s^{-1})$	$X_{\hbar} = 0.93702679751$
4	$\hbar^* = \hbar/1m = h/2\pi m$	$\hbar^* = 1.0545718176 \cdot 10^{-34}\ m\ kg\ s^{-1} =$ $=(1.318214772 \cdot 10^{-35}\ m)(2^{3}\ kg\ s^{-1})$	$X_{\hbar}^* = 0.98058409953$
5	G	$G = 6.67408 \cdot 10^{-11} m^3 kg^{-1} s^{-2} =$ $=(1.25986571 \cdot 10^{-35} m)^3 (2^{314} kg^{-1} s^{-2})$	$X_G = 0.93717982$
6	k	$k = 1.380649 \cdot 10^{-23}\ m^2 kgs^{-2} K^{-1} =$ $(1.229424828 \cdot 10^{-35}\ m)^2\ (2^{156}\ kg\ s^{-2}\ K^{-1})$	$X_k = 0.91453567659$
7	pV_m	$pV_m = 2271.095464\ m^2\ kg\ s^{-2} mol^{-1} =$ $=(1.2675737059 \cdot 10^{-35}\ m)^2 (2^{243} kgs^{-2} mol^{-1})$	$X_{pVm} = 0.9429135885$
8	R	$R = 8.314462618\ m^2 kgs^{-2} mol^{-1} K^{-1} =$ $=(1.2271357315 \cdot 10^{-35}\ m)^2\ (2^{235}\ kg\ s^{-2}\ mol^{-1} K^{-1})$	$X_R = 0.91283287965$
9	T	$pV_m = RT\ ;\quad X_T = X_{pVm} - X_R$	$X_T = 0.0300807088$
10	N_A	$X_{NA} = N_A / 2^{79}$	$X_{NA} = 0.9962796165$

Concerning the normalized values it must be underlined, they are universal dimensionless constants expressing the ratios between the characteristic length of every constant and the characteristic length of speed of light. But such universal dimensionless ratios can be obtained as well, if volumes or masses are taken into account. Having in view that units of both length and mass will be redefined by the CGPM 2018 [5], it results finally as all these universal dimensionless ratios can be based on speed of light.

If the Planck constant $\hbar^*$ expressed in ms^{-1}***kg*** is compared to the speed of light c expressed in ms^{-1}, it is easy to notice their ratio represents a mass. In this context the normalized value $X_\hbar^*=0.98058409953$ (table 1, row 4) could be associated with a mass. At the same time, the normalized value $X_{NA} =0.9962796165$ (table 1, row 10), for Avogadro constant N_A could be associated with a mass, as well (see explanation above).

In these circumstances we have two normalized values representing the mass: one is $X_\hbar^* = 0.98058409957$ and the other is $X_{NA} = 0.9962796165$. If $X_\hbar^*=0.98058409953$ is related to $X_{NA} =0.9962796165$ the following is obtained:

$$X_\hbar^* / X_{NA} =0.98058409957 / 0.9962796165 = 0.9842458712. \qquad (1)$$

If this result is divided by 2^{19} the result is:

$$(X_\hbar^* / X_{NA}) / 2^{19} = 0.9842458712 / 2^{19} =18773.000176\cdot 10^{-10}\ , \qquad (2)$$

that is a normalized value very close to $(\alpha^{-1})^2 10^{-10} = 18778.865051\cdot 10^{-10}$, where $(\alpha^{-1})^2$ is the square of the fine structure constant equal to $137.035999139^2 =18778.865051$.

But the square of the fine structure constant $(\alpha^{-1})^2$ is equal to the ratio between Bohr radius a_o and classical electron radius r_e [16]:

$$a_o / r_e = (\alpha^{-1})^2 =18778.8650599. \qquad (3)$$

The above relationships, between the normalized values $X_\hbar^*$ for Planck constant $\hbar^*$, X_{NA} for the Avogadro constant, the square of the fine structure constant $(\alpha^{-1})^2$ and the ratio Bohr radius / classical electron radius a_o / r_e , are taken in account for establishing by calculus a more exact value for gravitational constant G.

At this stage of analysis, the following aspect must be emphasized: the values resulting from calculus are not absolute values, they are dimensionless ratios only. In this context we can compare one ratio with another ratio though their magnitudes differ with the powers of number 2 or number 10. Regarding the powers of number 2 and 10, it is important to underline, there are published data showing that, between normalized values of the fundamental constants, powers of number 2, number 10 and irrational numbers π, direct relationships can be established. These relationships are based on speed of light and elementary geometrical correspondences [12,13].

Taking into account this aspect, we can say the following about the above results: the ratio between the normalized value of the mass defined by Planck constant $\hbar^*$ and the normalized value of the mass defined by Avogadro constant, respective $X_\hbar^* 10^{10} / X_{NA}$ (see rel. 2), is very close to the ratio between Bohr radius and classical electron radius a_o / r_e (see rel. 3). It is easy to noticed that 18773.00017 is close to 18778.8650599 but they are not equal. For obtaining an exact value equal to 18778.8650599, in rel. 2, the normalized value $X_{NA} = 0.9962796165$ for Avogadro constant N_A , must be smaller. Instead of 0.9962796165 it should be 0.9959684655. This last result noted $X_{NAs} = 0.9959684658$ is considered a normalized value of a physical entities mass smaller that Avogadro constant N_A. This new entities physical mass is noted N_{As} and is equal to:

$$N_{As} = 0.9959684655 \times 2^{79} = 6.020259969\cdot 10^{23}. \qquad (4)$$

The question is: what is the significance of the difference between normalized value for Avogadro constant N_A and normalized value for N_{As} respective, $X_{NA} - X_{NAs}$? Calculus show that:

$$X_{NA} - X_{NAs} = 0.9962796165 - 0.9959684655 = 3.111510425 \cdot 10^{-4}\,. \qquad (5)$$

If this result equal to $3.111510425 \cdot 10^{-4}$ is multiplied by 2^{201} is obtained:

$$3.111510425 \cdot 10^{-4} \times 2^{201} = 0.10000008955 \cdot 10^{58} \approx 1 \cdot 10^{57}\,, \qquad (6)$$

a value very close to 1 multiplied by 10^{57}. Referring to these results, the following elements must be highlighted: the exponent 201 of number 2 (see rel. 6) is equal to the sum of the exponents $122+79=201$, where 122 is exponent for the Planck constant h (see table 1, row 2, col. 2) and 79 is exponent for the Avogadro constant N_A (table 1, row 10). Regarding the exponent of 10^{57}, that is very close to the difference between magnitude of the Avogadro constant N_A (10^{23}) and the magnitude of the Planck length (10^{-35}), equal to 10^{58}.

These relationships are presented schematically in figs. 1 and 2.

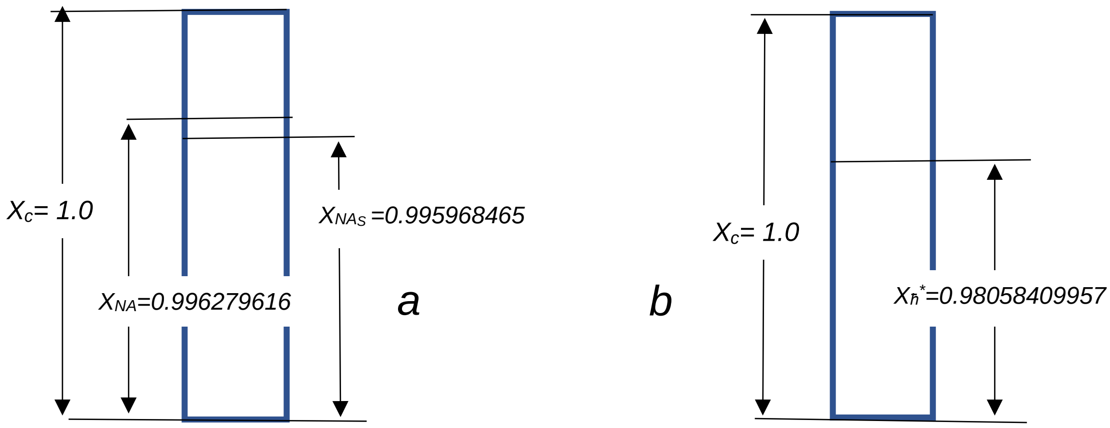

Fig. 1 Schematic relationships between normalized values X_{NA} for Avogadro constant N_A (a), and normalized value $X_\hbar^*$ for Planck constant $\hbar*$ (b)

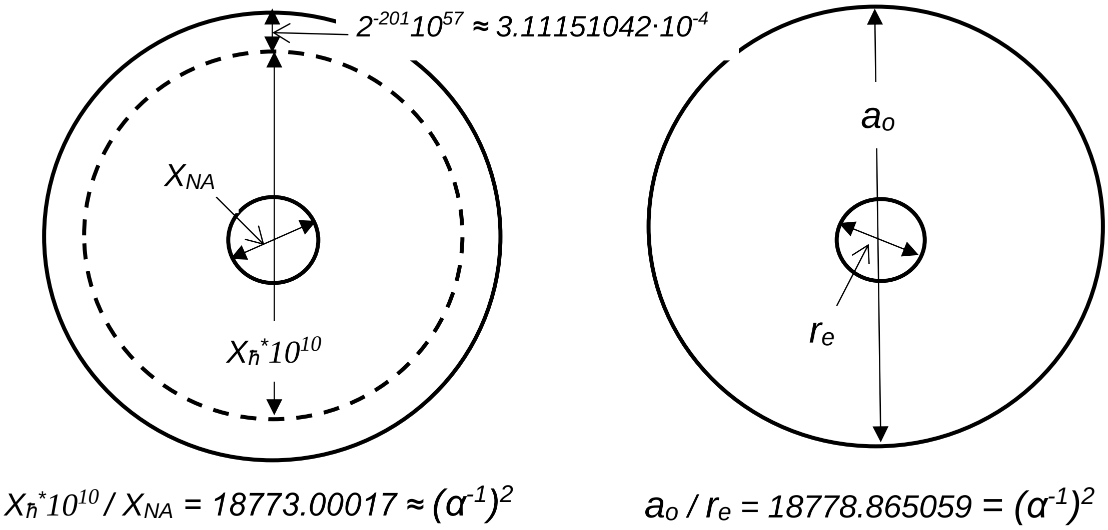

Fig. 2 A comparison between ratio $X_\hbar^* 10^{10} / X_{NA}$ and Bohr radius / electron radius (a_o / r_e)

The above relationships are taken into account for establishing by calculus a more exact value for the gravitational constant G. Why is it considered a more exact value? It has been shown the constant G was measured having a relative standard uncertainty (rsu) about 10^{-5} [17].

In this article the gravitational constant G is calculated on the base of the fine structure constant α^{-1} (rsu 10^{-10}), Planck constant h (rsu 10^{-8}), Avogadro constant N_A (rsu 10^{-8}) and Boltzmann constant k (rsu 10^{-7}). In the revised SI, these three constants will be considered exact values.

The calculus of the gravitational constant G is based on the following considerations:

-the normalized values of the fundamental constants are dimensionless ratios established on the base of the powers of number *2*;

- being dimensionless ratios, we can compere one ratio with another ratio though their magnitudes differ with the power of number *10;*

-whereas the normalized values are the ratios of some characteristic lengths, there are distinctive cases when these ratios have specific values such as: close to number π, close to ratio of two radius, close to number 1 or others. These individual cases have been taken into account for highlighting new relationships between natural phenomena.

It has been shown above the difference between the normalized value X_{NA} for Avogadro constant N_A and the normalized value X_{NAs} for N_{As} , respectively, $X_{NA} - X_{NAs}$ is equal to $3.111510425 \cdot 10^{-4}$ (see rel. 5). If this normalized value $3.111510425 \cdot 10^{-4}$ is related to the product: $(\alpha^{-1})^2 \times X_T \times X_G$, where: $(\alpha^{-1})^2$ is square of the fine structure constant, $X_T = X_{pVm} - X_R$ is the normalized value for temperature (see table 1, row 9) and X_G is the normalized value for gravitational constant G (see table 1, row 5) the following relationship results:

$$(X_{NA} - X_{NAs}) / [(\alpha^{-1})^2 \times X_T \times X_G] = \qquad (7)$$
$$=3.111510425 \cdot 10^{-4} /(18778.65059 \times 0.03008070885 \times 0.93717982)=5.877476814 \cdot 10^{-7}.$$

If this result is multiplied by 2^{127} the following is obtained:

$$5.877476814 \cdot 10^{-7} \times 2^{127} = 1.00000086 \cdot 10^{32} \approx 1 \cdot 10^{32}, \qquad (8)$$

a value very close to *1* multiplied by 10^{32}.

If is taken into consideration the Planck length $l_P=1.616229 \cdot 10^{-35} m$ [17] and this value is related to *1m*, a normalized value (dimensionless) equal to $X_{lP} =1.616229 \cdot 10^{-35}$ is obtained. If this normalized value is divided by product $(\alpha^{-1})^2 \times X_G$ and result is multiplied by 2^{133} is obtained:

$$[X_{lP} /(\alpha^{-1})^2 \times X_G] 2^{133} = \qquad (9)$$
$$=[1.616229 \cdot 10^{-35}/(18778.865059 \times 0.93717982)] 2^{133}=1.000000112 \cdot 10,$$

a value very close to *1* multiplied by *10*.

In this context there are three relations between normalized values such as: rel. 6, rel. 8 and rel. 9, each of which has the value very close to 1. It is considered that these three relations have not the values close to 1 but <u>exactly 1</u>. This is a particular situation when the normalized values are in a perfect "equilibrium". In this distinctive case, results from these three relations, a calculated normalized value for gravitational constant $(X_G)_c=0.937180394$. Having in view this calculated value $(X_G)_c =0.937180394$ and relation presented in table 1 row 5, results for the gravitational constant a value equal to $G= 6.6740922\underline{81} \cdot 10^{-11} m^3 kg^{-1} s^{-2}$.

The calculated value of $G= 6.6740922\underline{81} \cdot 10^{-11} m^3 kg^{-1} s^{-2}$ is compared with the most precise G measurements published in the last 35 years [3]. Taking into account the G

measurements presented in Table II of ref. [3], the normalized values of every G were established. On the base of these normalized values, the ratio $(X_{NA} - X_{NAs}) / [(\alpha^{-1})^2 \times X_{T\_} \times X_G]$ (see rel. 7) was calculated, for every measurement. If these ratios are multiplied by 2^{127}, values very close to number *1* are obtained (see rel. 8). These values are presented in fig. 3.

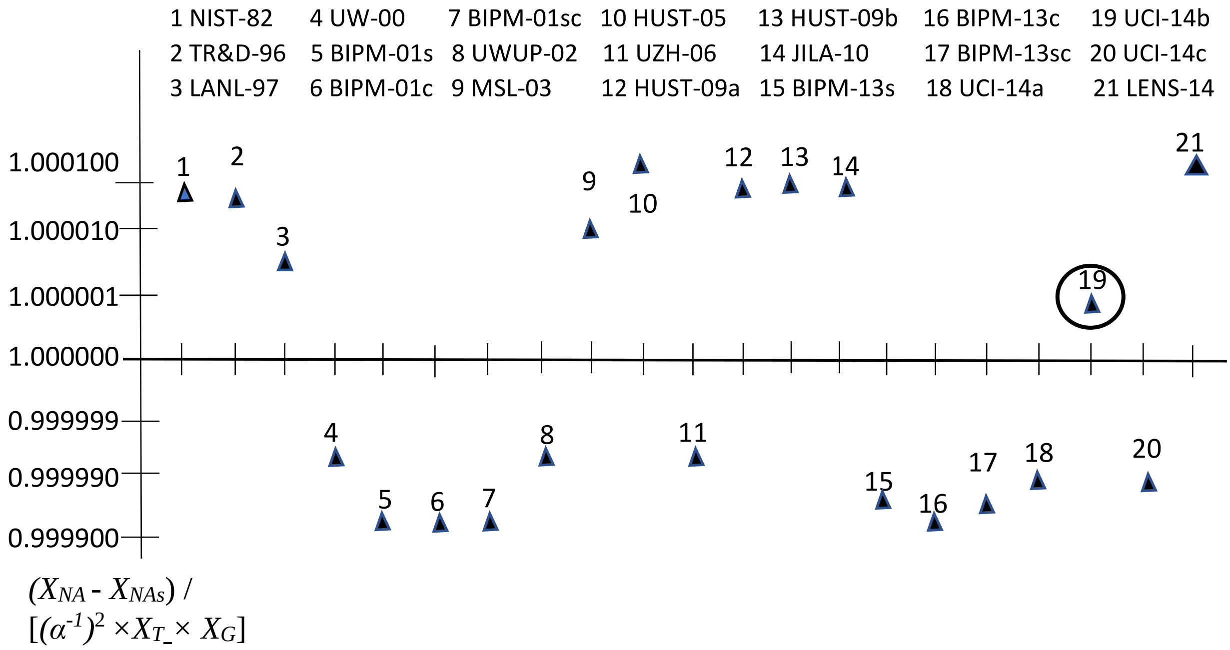

Fig. 3 Ratios $(X_{NA} - X_{NAs})/[(\alpha^{-1})^2 \times X_{T\_} \times X_G]$ for G measurements published in [3]

The most precise G measurement is considered the closest result to the value 1. This is UCI 14b (position 19) corresponding to the measurement made in 2002 by the University of California, Irvine (UCI) near Hanford, Washington [3]. This result corresponds to the $G = 6.67408 \cdot 10^{-11}\ m^3 kg^{-1} s^{-2}$ that is CODATA 2014 recommended value for G [17]. Close results to the value 1 are obtained by the same University of California, Irvine in 2000 (UCI 14a) and 2006 (UCI 14c) (fig 3), but not only (see fig. 3). Identifiers in fig 3 are similar to Table II published in [3].

Conclusions

In the proposed dimensional analysis, some fundamental constants can be rewritten using a power-law model by taking number *2* as base, thus a so-called "characteristic length" being highlighted every time. If the "characteristic length" for a fundamental constant is related to the "characteristic length" of the speed of light, every fundamental constant could be thus expressed as a normalized value. These normalized values could be used to explain in an nonconventional way, for instance some "liaisons" between fundamental constants and finally between fundamental phenomena.

The article presents the relationships between Planck constant and Avogadro constant established on the base of these normalized values. These relationships provide a nonconventional instrument for obtaining by calculus an adjusted value for gravitational constant G. This calculated value is equal to $6.674092\underline{281} \cdot 10^{-11} m^3 kg^{-1} s^{-2}$. This value is assumed having seven exact decimals corresponding to the largest relative standard uncertainty (rsu) for the Boltzmann constant (about 10^{-7}).

The method presented in the article is very important not only for establishing the value of G by calculus but in the case when the next more accurate measurements are different from those calculated, the difference could highlight more subtle relationship existing between natural phenomena. The proposed method could reveal such subtle "liaisons".

The method highlights a very interesting aspect concerning the mass. In accordance with the new definition of the kilogram [5], the mass is defined by time (second) and speed of light, whereas space (meter) is defined by time and speed of light, as well. In the article it has been shown that ratio of two mases -one established by Planck constant the other one by Avogadro constant- is in a direct relation with the ratio of two fundamental lengths: Bohr radius / classical electron radius and obviously with the square of fine structure constant.

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