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Some New Families of 4-Prime Cordial Graphs

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Abstract: Let G be a (p,q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a function. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-prime cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, $i, j \in \{1, 2, \ldots, k\}$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labeled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labeled with 1 and not labeled with 1. A graph with admits a k-prime cordial labeling is called a k-prime cordial graph. In this paper we investigate 4-prime cordial labeling behavior of shadow graph of a path, cycle, star, degree splitting graph of a bistar, jelly fish, splitting graph of a path and star.

Key Words: Cordial labeling, Smarandachely cordial labeling, cycle, star, bistar, splitting graph.

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§1. Introduction

In this paper graphs are finite, simple and undirected. Let G be a (p,q) graph where p is the number of vertices of G and q is the number of edge of G. In 1987, Cahit introduced the concept of cordial labeling of graphs [1]. Sundaram, Ponraj, Somasundaram [5] have been introduced the notion of prime cordial labeling and discussed the prime cordial labeling behavior of various graphs. Recently Ponraj et al. [7], introduced k-prime cordial labeling of graphs. A 2-prime cordial labeling is a product cordial labeling [6]. In [8, 9] Ponraj et al. studied the 4-prime cordial labeling behavior of complete graph, book, flower, mC_n, wheel, gear, double cone, helm, closed helm, butterfly graph, and friendship graph and some more graphs. Ponraj and Rajpal singh have studied about the 4-prime cordiality of union of two bipartite graphs, union of trees, durer graph, tietze graph, planar grid $P_m \times P_n$, subdivision of wheels and subdivision of helms, lotus inside a circle, sunflower graph and they have obtained some 4-prime cordial graphs from 4-prime cordial graphs [10, 11, 12]. Let x be any real number. In this paper we have studied about the 4-prime cordiality of shadow graph of a path, cycle, star, degree splitting graph of a

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bistar, jelly fish, splitting graph of a path and star. Let x be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for smallest integer greater than or equal to x. Terms not defined here follow from Harary [3] and Gallian [2].

§2. k-Prime Cordial Labeling

Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \cdots, k\}$ be a map. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-prime cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, $i, j \in \{1, 2, \cdots, k\}$ and $|e_f(0) - e_f(1)| \leq 1$, and conversely, if $|v_f(i) - v_f(j)| \geq 1$, $i, j \in \{1, 2, \cdots, k\}$ or $|e_f(0) - e_f(1)| \geq 1$, it is called a Smarandachely cordial labeling, where $v_f(x)$ denotes the number of vertices labeled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labeled with 1 and not labeled with 1. A graph with a k-prime cordial labeling is called a k-prime cordial graph.

First we investigate the 4-prime cordiality of shadow graph of a path, cycle and star. A shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G, G' and G'' and joining each vertex u' in G' to the neighbors of the corresponding vertex u'' in G''.

Theorem 2.1 *$D_2(P_n)$ is 4-prime cordial if and only if $n \neq 2$.*

Proof It is easy to see that $D_2(P_2)$ is not 4-prime cordial. Consider $n > 2$. Let $V(D_2(P_n)) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(D_2(P_n)) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_{i+1}, v_i u_{i+1} : 1 \leq i \leq n-1\}$. In a shadow graph of a path, $D_2(P_n)$, there are $2n$ vertices and $4n - 4$ edges.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_{2t}$ then assign 2 to the vertices $v_1, v_2, \cdots, v_{2t}$. For the vertices v_{2t+1}, v_{2t+2}, we assign $3, 1$ respectively. Put the label 1 to the vertices $v_{2t+3}, v_{2t+5}, \cdots, v_{4t-1}$. Now we assign the label 3 to the vertices $v_{2t+4}, v_{2t+6}, \cdots, v_{4t-2}$. Then assign the label 1 to the vertex v_{4t}. Next we consider the vertices $u_{2t+1}, u_{2t+2}, \cdots, u_{4t}$. Put $3, 3$ to the vertices u_{2t+1}, u_{2t+2}. Then fix the number 1 to the vertices $u_{2t+3}, u_{2t+5}, \cdots, u_{4t-1}$. Finally assign the label 3 to the vertices $u_{2t+4}, u_{2t+6}, \cdots, u_{4t}$.

Case 2. $n \equiv 1 \pmod 4$.

Take $n = 4t+1$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_{2t+1}$. Then assign the label 3 to the vertices $u_{2t+2}, u_{2t+4}, \cdots, u_{4t}$ and put the number 1 to the vertices $u_{2t+3}, u_{2t+5}, \cdots, u_{4t+1}$. Next we turn to the vertices $v_1, v_2, \cdots, v_{2t+1}$. Assign the label 2 to the vertices $v_1, v_2, \cdots, v_{2t+1}$. The remaining vertices v_i $(2t + 2 \leq i \leq 4t + 1)$ are labeled as in u_i $(2t + 2 \leq i \leq 4t + 1)$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4t + 2$. Assign the labels to the vertices u_i, v_i $(1 \leq i \leq 2t + 1)$ as in case 2. Now we consider the vertices $u_{2t+2}, u_{2t+3}, \cdots, u_{4t+2}$. Assign the labels $3, 1$ to the vertices u_{2t+2}, u_{2t+3} respectively. Then assign the label 1 to the vertices $u_{2t+4}, u_{2t+6}, \cdots, u_{4t+2}$. Put the integer 3 to the vertices $u_{2t+5}, u_{2t+7}, \cdots, u_{4t+1}$. Now we turn to the vertices $v_{2t+2}, v_{2t+3}, \cdots, v_{4t+2}$. Put the labels $3, 3, 1$ to the vertices $v_{2t+2}, v_{2t+3}, v_{2t+4}$ respectively. The remaining vertices

$v_i\,(2t+5 \le i \le 4t+2)$ are labeled as in $u_i\,(2t+5 \le i \le 4t+2)$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4t+3$. Assign the label 2 to the vertices $u_i\,(1 \le i \le 2t+2)$. Then put the number 3 to the vertices $u_{2t+3}, u_{2t+5}, \cdots, u_{4t+1}$. Then assign 1 to the vertices $u_{2t+4}, u_{2t+6}, \cdots, u_{4t+2}$ and u_{4t+3}. Now we turn to the vertices $v_1, v_2, \cdots, v_{4t+3}$. Assign the label 4 to the vertices $v_i\,(1 \le i \le 2t+2)$. The remaining vertices $v_i\,(2t+3 \le i \le 4t+3)$ are labeled as in $u_i\,(2t+3 \le i \le 4t+3)$. Then relabel the vertex v_{4t+3} by 3.

The vertex and edge conditions of the above labeling is given in Table 1.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$2n-2$	$2n-2$
$n \equiv 1 \pmod 2$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	$2n-2$	$2n-2$

Table 1

It follows that $D_2(P_n)$ is a 4-prime cordial graph for $n \neq 2$. □

Theorem 2.2 *$D_2(C_n)$ is 4-prime cordial if and only if $n \ge 7$.*

Proof Let $V(D_2(C_n)) = \{u_i, v_i : 1 \le i \le n\}$ and $E(D_2(C_n)) = \{u_iu_{i+1},\ v_iv_{i+1},\ u_iv_{i+1},\ v_iu_{i+1} : 1 \le i \le n-1\} \cup \{u_nv_1,\ v_nu_1,\ u_nu_1,\ v_nv_1\}$. Clearly $D_2(C_n)$ consists of $2n$ vertices and $4n$ edges. We consider the following cases.

Case 1. $n \equiv 0 \pmod 4$.

One can easily check that $D_2(C_4)$ can not have a 4-prime cordial labeling. Define a vertex labeling f from the vertices of $D_2(C_n)$ to the set of first four consecutive positive integers as given below.

$$\begin{aligned}
f(v_{2i}) &= f(u_{2i-1}) &&= 2, \quad 1 \le i \le \tfrac{n}{4}\\
f(v_{2i+1}) &= f(u_{2i}) &&= 4, \quad 1 \le i \le \tfrac{n}{4}\\
f(v_{\frac{n}{2}+2+2i}) &= f(u_{\frac{n}{2}+2+2i}) &&= 1, \quad 1 \le i \le \tfrac{n-4}{4}\\
f(v_{\frac{n}{2}+3+2i}) &= f(u_{\frac{n}{2}+3+2i}) &&= 3, \quad 1 \le i \le \tfrac{n-8}{4}
\end{aligned}$$

$f(u_{\frac{n}{2}+1}) = f(u_{\frac{n}{2}+2}) = f(u_{\frac{n}{2}+3}) = f(v_{\frac{n}{2}+2}) = 3$ and $f(v_1) = f(v_{\frac{n}{2}+3}) = 1$.

Case 2. $n \equiv 1 \pmod 4$.

It is easy to verify that $D_2(C_5)$ is not a prime graph. Now we construct a map $f : V(D_2(C_n)) \to \{1, 2, 3, 4\}$ as follows:

$$\begin{aligned}
f(u_{2i-1}) &= 2, \quad 1 \le i \le \tfrac{n+3}{4}\\
f(u_{2i}) &= 4, \quad 1 \le i \le \tfrac{n+3}{4}\\
f(v_{2i}) &= 2, \quad 1 \le i \le \tfrac{n-1}{4}\\
f(v_{2i+1}) &= 4, \quad 1 \le i \le \tfrac{n-1}{4}\\
f(v_{\frac{n+5}{2}+2i}) = f(u_{\frac{n+5}{2}+2i}) &= 1, \quad 1 \le i \le \tfrac{n-5}{4}\\
f(v_{\frac{n+7}{2}+2i}) = f(u_{\frac{n+7}{2}+2i}) &= 3, \quad 1 \le i \le \tfrac{n-9}{4}
\end{aligned}$$

$f(v_1) = f(v_{\frac{n+5}{2}}) = f(u_{\frac{n+5}{2}}) = f(u_{\frac{n+7}{2}}) = 3$ and $f(v_{\frac{n+7}{2}}) = f(v_{\frac{n+3}{2}}) = 1$.

Case 3. $n \equiv 2 \pmod 4$.

Obviously $D_2(C_6)$ does not permit a 4-prime cordial labeling. For $n \neq 6$, we define a function f from $V(D_2(C_n))$ to the set $\{1, 2, 3, 4\}$ by

$$\begin{array}{rcrcl} & & f(u_{2i-1}) & = & 2, \quad 1 \le i \le \frac{n+2}{4} \\ & & f(u_{2i}) & = & 4, \quad 1 \le i \le \frac{n+2}{4} \\ & & f(v_{2i}) & = & 2, \quad 1 \le i \le \frac{n-2}{4} \\ & & f(v_{2i+1}) & = & 4, \quad 1 \le i \le \frac{n-2}{4} \\ f(v_{\frac{n+6}{2}+2i}) & = & f(u_{\frac{n+6}{2}+2i}) & = & 1, \quad 1 \le i \le \frac{n-6}{4} \\ f(v_{\frac{n+8}{2}+2i}) & = & f(u_{\frac{n+8}{2}+2i}) & = & 3, \quad 1 \le i \le \frac{n-8}{4} \end{array}$$

and

$$f(v_1) = f(v_{\frac{n+6}{2}}) = f(u_{\frac{n+4}{2}}) = f(u_{\frac{n+6}{2}}) = f(u_{\frac{n+8}{2}}) = 3,$$
$$f(v_{\frac{n+2}{2}}) = f(v_{\frac{n+4}{2}}) = f(v_{\frac{n+8}{2}}) = 1.$$

Case 4. $n \equiv 3 \pmod 4$.

Clearly $D_2(C_3)$ is not a 4-prime cordial graph. Let $n \neq 3$. Define a map $f : V(D_2(C_n)) \rightarrow \{1, 2, 3, 4\}$ by $f(v_1) = 1$,

$$\begin{array}{rcrcl} f(v_{2i}) & = & f(u_{2i-1}) & = & 2, \quad 1 \le i \le \frac{n+1}{4} \\ f(v_{2i+1}) & = & f(u_{2i}) & = & 4, \quad 1 \le i \le \frac{n+1}{4} \\ f(v_{\frac{n+3}{2}+2i}) & = & f(u_{\frac{n+3}{2}+2i}) & = & 1, \quad 1 \le i \le \frac{n-3}{4} \\ f(v_{\frac{n+5}{2}+2i}) & = & f(u_{\frac{n+5}{2}+2i}) & = & 3, \quad 1 \le i \le \frac{n-5}{4} \end{array}$$

and $f(u_{\frac{n+3}{2}}) = f(u_{\frac{n+5}{2}}) = f(v_{\frac{n+5}{2}}) = 3$. The Table 2 gives the vertex and edge condition of f.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$n \equiv 0, 2 \pmod 4$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$2n$	$2n$
$n \equiv 1, 3 \pmod 4$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	$2n$	$2n$

Table 2

It follows that $D_2(C_n)$ is 4-prime cordial iff $n \geq 7$. □

Example 2.1 A 4-prime cordial labeling of $D_2(C_9)$ is given in Figure 1.

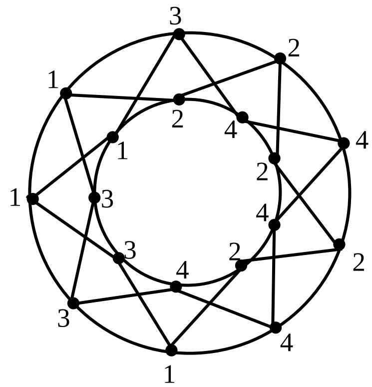

Figure 1

Theorem 2.3 *$D_2(K_{1,n})$ is 4-prime cordial if and only if* $n \equiv 0 \pmod 2$.

Proof It is clear that $D_2(K_{1,n})$ has $2n+2$ vertices and $4n$ edges. Let $V(D_2(K_{1,n})) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $E(D_2(K_{1,n})) = \{uu_i, vv_i, vu_i, uv_i : 1 \leq i \leq n\}$.

Case 1. $n \equiv 0 \pmod 2$.

Assign the label 2 to the vertices $u_1, u_2, \cdots, u_{\frac{n}{2}+1}$. Then assign 4 to the vertices $u_{\frac{n}{2}+2}, \cdots, u_n, u, v$. Now we move to the vertices v_i where $1 \leq i \leq n$. Assign the label 3 to the vertices $v_i\,(1 \leq i \leq \frac{n}{2})$ then the remaining vertices are labeled with 1. In this case $v_f(1) = v_f(3) = \frac{n}{2}$, $v_f(2) = v_f(4) = \frac{n}{2} + 1$ and $e_f(0) = e_f(1) = 2n$.

Case 2. $n \equiv 1 \pmod 2$.

Let $n = 2t+1$. Suppose there exists a 4-prime cordial labeling g, then $v_g(1) = v_g(2) = v_g(3) = v_g(4) = t+1$.

Subcase 2a. $g(u) = g(v) = 1$.

Here $e_g(0) = 0$, a contradiction.

Subcase 2b. $g(u) = g(v) = 2$.

In this case $e_g(0) \leq (t-1) + (t-1) + (t+1) + (t+1) = 4t$, a contradiction.

Subcase 2c. $g(u) = g(v) = 3$.

Then $e_g(0) \leq (t-1) + (t-1) = 2t-2$, a contradiction.

Subcase 2d. $g(u) = g(v) = 4$.

Similar to Subcase 2b.

Subcase 2e. $g(u) = 2$, $g(v) = 4$ or $g(v) = 2$, $g(u) = 4$.

Here $e_g(0) \leq t + t + t + t = 4t$, a contradiction.

Subcase 2f. $g(u) = 2$, $g(v) = 3$ or $g(v) = 2$, $g(u) = 3$.

Here $e_g(0) \leq (t+1) + t + t = 3t+1$, a contradiction.

Subcase 2g. $g(u) = 4$, $g(v) = 3$ or $g(v) = 4$, $g(u) = 3$.

Similar to Subcase 2f.

Subcase 2h. $g(u) = 2$, $g(v) = 1$ or $g(v) = 2$, $g(u) = 1$.

Similar to Subcase 2f.

Subcase 2i. $g(u) = 4$, $g(v) = 1$ or $g(v) = 4$, $g(u) = 1$.

Similar to Subcase 2h.

Subcase 2j. $g(u) = 3$, $g(v) = 1$ or $g(v) = 3$, $g(u) = 1$.

In this case $e_g(0) \leq t$, a contradiction.

Hence, if $n \equiv 1 \pmod 2$, $D_2(K_{1,n})$ is not a 4-prime cordial graph. □

The next investigation is about 4-prime cordial labeling behavior of splitting graph of a path, star. For a graph G, the splitting graph of G, $S'(G)$, is obtained from G by adding for each vertex v of G a new vertex v' so that v' is adjacent to every vertex that is adjacent to v. Note that if G is a (p, q) graph then $S'(G)$ is a $(2p, 3q)$ graph.

Theorem 2.4 *$S'(P_n)$ is 4-prime cordial for all n.*

Proof Let $V(S'(P_n)) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(S'(P_n)) = \{u_iu_{i+1}, u_iv_{i+1}, v_iu_{i+1}: 1 \leq i \leq n-1\}$. Clearly $S'(P_n)$ has $2n$ vertices and $3n-3$ edges. Figure 2 shows that $S'(P_2)$, $S'(P_3)$ are 4-prime cordial.

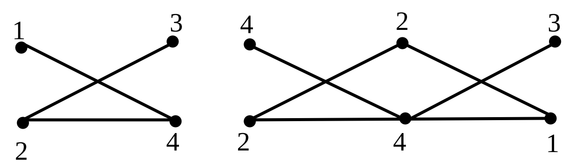

Figure 2

For $n > 3$, we consider the following cases.

Case 1. $n \equiv 0 \pmod 4$.

We define a function f from the vertices of $S'(P_n)$ to the set $\{1, 2, 3, 4\}$ by

$$\begin{aligned} f(v_{2i}) &= f(u_{2i-1}) &= 2, \quad 1 \leq i \leq \tfrac{n}{4} \\ f(v_{2i+1}) &= f(u_{2i}) &= 4, \quad 1 \leq i \leq \tfrac{n}{4} \\ f(v_{\frac{n+2}{2}+2i}) &= f(u_{\frac{n+2}{2}+2i}) &= 1, \quad 1 \leq i \leq \tfrac{n-4}{4} \\ f(v_{\frac{n+4}{2}+2i}) &= f(u_{\frac{n+4}{2}+2i}) &= 3, \quad 1 \leq i \leq \tfrac{n-4}{4} \end{aligned}$$

and $f(u_{\frac{n+2}{2}}) = f(u_{\frac{n+4}{2}}) = 3$, $f(v_1) = f(v_{\frac{n+4}{2}}) = 1$.

In this case $v_f(1) = v_f(2) = v_f(3) = v_f(4) = \frac{n}{2}$, and $e_f(0) = \frac{3n-4}{2}$, $e_f(1) = \frac{3n-2}{2}$.

Case 2. $n \equiv 1 \pmod 4$.

We define a map $f : V(S'(P_n)) \to \{1, 2, 3, 4\}$ by

$$\begin{array}{rcrcl} & & f(u_{2i-1}) & = & 2, \quad 1 \leq i \leq \frac{n+3}{4} \\ & & f(u_{2i}) & = & 4, \quad 1 \leq i \leq \frac{n-1}{4} \\ & & f(v_{2i-1}) & = & 4, \quad 1 \leq i \leq \frac{n+3}{4} \\ & & f(v_{2i}) & = & 2, \quad 1 \leq i \leq \frac{n-1}{4} \\ f(v_{\frac{n-1}{2}+2i}) & = & f(u_{\frac{n-1}{2}+2i}) & = & 3, \quad 1 \leq i \leq \frac{n-1}{4} \\ f(v_{\frac{n+1}{2}+2i}) & = & f(u_{\frac{n+1}{2}+2i}) & = & 1, \quad 1 \leq i \leq \frac{n-1}{4} \end{array}$$

Here $v_f(1) = v_f(3) = \frac{n-1}{2}$, $v_f(2) = v_f(4) = \frac{n+1}{2}$, and $e_f(0) = e_f(1) = \frac{3n-3}{2}$.

Case 3. $n \equiv 2 \pmod 4$.

Define a vertex labeling $f : V(S'(P_n)) \to \{1, 2, 3, 4\}$ by $f(v_1) = 3$, $f(v_{\frac{n}{2}+1}) = 1$,

$$\begin{array}{rcrcl} & & f(u_{2i-1}) & = & 2, \quad 1 \leq i \leq \frac{n+2}{4} \\ & & f(u_{2i}) & = & 4, \quad 1 \leq i \leq \frac{n+2}{4} \\ & & f(v_{2i}) & = & 2, \quad 1 \leq i \leq \frac{n-2}{4} \\ & & f(v_{2i+1}) & = & 4, \quad 1 \leq i \leq \frac{n-2}{4} \\ f(v_{\frac{n}{2}+2i}) & = & f(u_{\frac{n}{2}+2i}) & = & 3, \quad 1 \leq i \leq \frac{n-2}{4} \\ f(v_{\frac{n+2}{2}+2i}) & = & f(u_{\frac{n+2}{2}+2i}) & = & 1, \quad 1 \leq i \leq \frac{n-2}{4} \end{array}$$

Here $v_f(1) = v_f(2) = v_f(3) = v_f(4) = \frac{n}{2}$, and $e_f(0) = \frac{3n-4}{2}$, $e_f(1) = \frac{3n-2}{2}$.

Case 4. $n \equiv 3 \pmod 4$.

Construct a vertex labeling f from the vertices of $S'(P_n)$ to the set $\{1, 2, 3, 4\}$ by $f(u_n) = 1$, $f(v_n) = 3$,

$$\begin{array}{rcrcl} f(v_{2i}) & = & f(u_{2i-1}) & = & 2, \quad 1 \leq i \leq \frac{n+1}{4} \\ f(v_{2i-1}) & = & f(u_{2i}) & = & 4, \quad 1 \leq i \leq \frac{n+1}{4} \\ f(v_{\frac{n-1}{2}+2i}) & = & f(u_{\frac{n-1}{2}+2i}) & = & 3, \quad 1 \leq i \leq \frac{n-3}{4} \\ f(v_{\frac{n+1}{2}+2i}) & = & f(u_{\frac{n+1}{2}+2i}) & = & 1, \quad 1 \leq i \leq \frac{n-3}{4} \end{array}$$

In this case $v_f(1) = v_f(3) = \frac{n-1}{2}$, $v_f(2) = v_f(4) = \frac{n+1}{2}$, and $e_f(0) = e_f(1) = \frac{3n-3}{2}$.

Hence $S'(P_n)$ is 4-prime cordial for all n. □

Theorem 2.5 *$S'(K_{1,n})$ is 4-prime cordial for all n.*

Proof Let $V(S'(K_{1,n})) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $E(S'(K_{1,n})) = \{uu_i, vu_i, uv_i : 1 \leq i \leq n\}$. Clearly $S'(K_{1,n})$ has $2n + 2$ vertices and $3n$ edges. The Figure 3 shows that $S'(K_{1,2})$ is a 4-prime cordial graph.

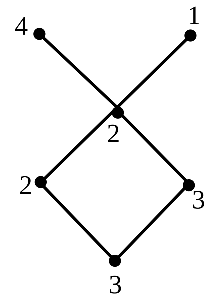

Figure 3

Now for $n > 2$, we define a map $f : V(S'(K_{1,n})) \to \{1, 2, 3, 4\}$ by $f(u) = 2$, $f(v) = 3$, $f(u_n) = 1$,

$$\begin{array}{lcl} f(u_i) & = & 2, \quad 1 \le i \le \lfloor \frac{n}{2} \rfloor \\ f(u_{\lfloor \frac{n}{2} \rfloor + i}) & = & 3, \quad 1 \le i \le \lfloor \frac{n-1}{2} \rfloor \\ f(v_i) & = & 4, \quad 1 \le i \le \lceil \frac{n+1}{2} \rceil \\ f(v_{\lceil \frac{n+1}{2} \rceil + i}) & = & 1, \quad 1 \le i \le \lfloor \frac{n-1}{2} \rfloor \end{array}$$

The Table 3 shows that f is a 4-prime cordial labeling of $S'(K_{1,n})$. □

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$n \equiv 0 \text{mod } 2$	$\frac{n}{2}$	$\frac{n}{2} + 1$	$\frac{n}{2}$	$\frac{n}{2} + 1$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1 \text{mod } 2$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$

Table 3

Next we investigate the 4-prime cordial behavior of degree splitting graph of a star. Let $G = (V, E)$ be a graph with $V = S_1 \cup S_2 \cup \cdots \cup S_t \cup T$ where each S_i is a set of vertices having at least two vertices and having the same degree and $T = V - \bigcup_{i=1}^{t} S_i$. The degree splitting graph of G denoted by $DS(G)$ is obtained from G by adding vertices $w_1, w_2 \cdots, w_t$ and joining w_i to each vertex of S_i $(1 \le i \le t)$.

Theorem 2.6 *$DS(B_{n,n})$ is 4-prime cordial if* $n \equiv 1, 3 \pmod 4$.

Proof Let $V(B_{n,n}) = \{u, v, u_i, v_i : 1 \le i \le n\}$ and $E(B_{n,n}) = \{uv, uu_i, vv_i : 1 \le i \le n\}$. Let $V(DS(B_{n,n})) = V(B_{n,n}) \cup \{w_1, w_2\}$ and $E(DS(B_{n,n})) = E(B_{n,n}) \cup \{w_1u_i, w_1v_i, w_2u, w_2v : 1 \le i \le n\}$. Clearly $DS(B_{n,n})$ has $2n + 4$ vertices and $4n + 3$ edges.

Case 1. $n \equiv 1 \pmod 4$.

Let $n = 4t + 1$. Assign the label 3 to the vertices $v_1, v_2, \cdots, v_{2t+1}$ and 1 to the vertices $v_{2t+2}, v_{2t+3}, \cdots, v_{4t+1}$. Next assign the label 4 to the vertices $u_1, u_2, \cdots, u_{2t+2}$ and 2 to the vertices $u_{2t+3}, u_{2t+4}, \cdots, u_{4t+1}$. Finally, assign the labels $1, 2, 2$ and 2 to the vertices w_2, u, v and w_1 respectively.

Case 2. $n \equiv 3 \pmod 4$.

As in case 1 assign the labels to the vertices u_i, v_i, u, v, w_1 and w_2 $(1 \le i \le n - 2)$. Next assign the labels $1, 3, 2$ and 4 respectively to the vertices v_{n-1}, v_n, u_{n-1} and u_n. The Table 4 establishes that this vertex labeling f is a 4-prime cordial labeling. □

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$4t+1$	$2t+1$	$2t+2$	$2t+1$	$2t+2$	$8t+3$	$8t+4$
$4t+3$	$2t+2$	$2t+3$	$2t+2$	$2t+3$	$8t+7$	$8t+8$

Table 4

The final investigation is about 4-prime cordiality of jelly fish graph.

Theorem 2.7 *The Jelly fish $J(n, n)$ is 4-prime cordial.*

Proof Let $V(J(n, n)) = \{u, v, u_i, v_i, w_1, w_2 : 1 \leq i \leq n\}$ and $E(J(n, n))$ =$\{uu_i$, vu_i, uw_1, w_1v, vw_2, uw_2, $w_1w_2 : 1 \leq i \leq n\}$. Note that $J(n, n)$ has $2n + 4$ vertices and $2n + 5$ edges.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$. Assign the label 1 to the vertices $u_1, u_2, \cdots, u_{2t+1}$. Next assign the label 3 to the vertices $u_{2t+2}, u_{2t+3}, \cdots, u_{4t}$. We now move to the other side pendent vertices. Assign the label 3 to the vertices u_1, u_2. Next assign the label 2 to the vertices $u_3, u_4, \ldots, u_{2t+3}$. Then assign the label 4 to the remaining pendent vertices. Finally assign the label 4 to the vertices u, v, w_1, w_2.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4t + 1$. In this case, assign the label 1 to the vertices $v_1, v_2, \cdots, v_{2t+1}$ and 3 to the vertices $v_{2t+1}, v_{2t+3}, \cdots, v_{4t+1}$. Next assign the label 2 to the vertices $u_1, u_2, \cdots, u_{2t+2}$, and 3 to the vertices u_{2t+3} and u_{2t+4}. Next assign the label 4 to the remaining pendent vertices $u_{2t+5}, u_{2t+6}, \cdots, u_{4t+1}$. Finally assign the label 4 to the vertices u, v, w_1, w_2.

Case 3. $n \equiv 2 \pmod 4$.

As in Case 2, assign the label to the vertices $u_i, v_i (1 \leq i \leq n - 1), u, v, w_1, w_2$. Next assign the labels $1, 4$ respectively to the vertices u_n and v_n.

Case 4. $n \equiv 3 \pmod 4$.

Assign the labels to the vertices $u, v, w_1, w_2, u_i, v_i (1 \leq i \leq n - 1)$ as in case 3. Finally assign the labels $2, 1$ respectively to the vertices u_n, v_n. The Table 5establishes that this vertex labeling f is obviously a 4-prime cordial labeling. □

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$4t$	$2t+1$	$2t+1$	$2t+1$	$2t+1$	$4t+3$	$4t+2$
$4t+1$	$2t+1$	$2t+2$	$2t+2$	$2t+1$	$4t+4$	$4t+3$
$4t+2$	$2t+2$	$2t+2$	$2t+2$	$2t+2$	$4t+5$	$4t+4$
$4t+3$	$2t+3$	$2t+3$	$2t+2$	$2t+2$	$4t+6$	$4t+5$

Table 5

Corollary 2.1 *The Jelly fish $J(m, n)$ where $m \geq n$ is 4-prime cordial.*

Proof Let $m = n + r$, $r \geq 0$. Use of the labeling f given in theorem **??** assign the label to the vertices u, v, w_1, w_2, u_i, v_i $(1 \leq i \leq n)$.

Case 1. $r \equiv 0 \pmod 4$.

Let $r = 4k$, $k \in N$. Assign the label 2 to the vertices $u_{n+1}, u_{n+2}, \cdots, u_{n+k}$ and to the vertices $u_{n+k+1}, u_{n+k+2}, \cdots, u_{n+2k}$. Then assign the label 1 to the vertices $u_{n+2k+1}, u_{n+2k+2}, \cdots, u_{n+3k}$ and 3 to the vertices u_{n+3k+1}, u_{n+3k+2}, $\ldots$, u_{n+4k}. Clearly this vertex labeling is a 4-prime cordial labeling.

Case 2. $r \equiv 1 \pmod 4$.

Let $r = 4k + 1$, $k \in N$. Assign the labels to the vertices u_{n+i} $(1 \leq i \leq r - 1)$ as in case 1. If $n \equiv 0, 1, 2 \pmod 4$, then assign the label 1 to the vertex u_r; otherwise assign the label 4 to the vertex u_r.

Case 3. $r \equiv 2 \pmod 4$.

Let $r = 4k + 2$, $k \in N$. As in Case 2 assign the labels to the vertices u_{n+i} $(1 \leq i \leq r - 1)$. Then assign the label 4 to the vertex u_r.

Case 4. $r \equiv 3 \pmod 4$.

Let $r = 4k + 3$, $k \in N$. In this case assign the label 3 to the last vertex and assign the label to the vertices u_{n+i} $(1 \leq i \leq r - 1)$ as in Case 3. □

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