On the impossibility of a Yang-Mills finite mass gap in the flat spacetime

Open letter to the Clay Mathematics Institute from V. Paromov, PhD

There is an unproven conjecture that a non-trivial quantum Yang-Mills theory exists on $\mathbf{R}^4$ **a**nd its vacuum state excitations in the absence of matter fields have a finite mass gap. Notably, both Abelian and non-Abelian Lie group-based gauge theories naturally produce massless quantum fields, and hence, the introduction of mass-inducing quantum fields is always *ad hoc*. Moreover, taken to the account the fundamental physical quality of any massive field, the ability to induce geometrical curvature on flat spacetime, such quantum Yang-Mills theory, if exists, cannot be complete when formulated on $\mathbf{R}^4$. In case the mass gap exists, the quantum field corresponding to this gap induces a non-zero geometrical curvature on $\mathbf{R}^4$ transforming the latter into a curved hypersurface in $\mathbf{R}^5$. Although the effect may be infinitesimal, it cannot be neglected completely. Hence, the theory formulated on $\mathbf{R}^4$ will be incomplete requiring additional manipulations, e.g. renormalization. Thus, it is logical to expect the theory to be initially formulated on a manifold with a higher dimensionality ($n > 4$), and then translated to $\mathbf{R}^4$ or, more conveniently, to $\mathbf{R}^3$ and absolute time.

Alternatively, the theory may be formulated assuming a higher dimensionality of the spacetime. Such an approach avoids *ad hoc* introductions of mass-inducing fields. As an example, the quantum electrodynamics (QED) can be initially formulated on a simple symmetrical hypersurface in $\mathbf{R}^5$, S^4 (in case time curvature is disregarded) and translated to $\mathbf{R}^3$ (and absolute time) using the Hopf fibration: $U(1) \rightarrow S^3 \rightarrow \mathbf{CP}^1$. [1]. Then, the Higg's field comes naturally as the geometrical curvature of S^2, the geometrical intersection of S^3 with $\mathbf{R}^3$ [1]. Furthermore, the Yang-Mills theory can be initially formulated on S^7 (in case time curvature is disregarded) and then translated to $\mathbf{R}^3$ and absolute time using two consecutive Hopf fibrations: $SU(2) \rightarrow S^7 \rightarrow S^4$ and $U(1) \rightarrow S^3 \rightarrow \mathbf{CP}^1$ [1]. Then, the postulated geometry of "total" spacetime with the global space topology $S^3 \times S^1 \times S^3$ appears as the geometry behind the Lie algebras used in the Standard Model. This geometry can explain naturally the gauge symmetry, color confinement, renormalization requirement, and weak interactions' chirality.

Reference:

1. V. Paromov. Fractal Structure of the Spacetime, the Fundamentally Broken Symmetry. http://vixra.org/abs/1806.0181