In Search of Schrödinger's Electron

Jean Louis Van Belle, Drs, MA*Ec*, BA*Ec*, B*Phil*[1]

Abstract

This article continues to explore a possible physical interpretation of the wavefunction that has been elaborated in previous papers (see http://vixra.org/author/jean_louis_van_belle). It basically zooms in on the physical model it implies for an electron in free space.

It concludes that the mainstream interpretation of quantum physics (the Copenhagen interpretation) is and remains the most *parsimonious* explanation, but that one or two extra assumptions – the wavefunction as a two-dimensional self-sustaining oscillation of a pointlike charge in space – make more frivolous explanations (many-worlds, pilot-wave, etc.) redundant.

Contents

[1] Website: www.readingfeynman.org

In Search of Schrödinger's Electron[2]

Epistemology versus physics

It is a *cliché* to say that quantum-mechanical concepts and principles are very non-intuitive. However, it explains why several *interpretations* of quantum mechanics have emerged and continue to vie for attention – even if the basic math has been formulated (almost) a century ago.

The mainstream interpretation of quantum mechanics is referred to as the Copenhagen interpretation. It mainly distinguishes itself from more frivolous interpretations – such as the many-worlds and the pilot-wave interpretations – because it respects Ockham's *lex parsimoniae*: the Copenhagen interpretation makes fewer assumptions.

Unfortunately, the Copenhagen interpretation itself is subject to many interpretations.[3] One such interpretation may be referred to as radical skepticism or empiricism[4]: we can only say something meaningful about Schrödinger's cat if we open the box and observe its state. According to this rather particular viewpoint, we cannot be sure of its reality as long if we don't observe it. All we can do is *describe* its reality by a superposition of the two *possible* states: dead or alive. That's Hilbert's logic[5]: the two states (dead or alive) are mutually exclusive but we add them anyway. If a tree falls in the wood and no one hears it, then it is both standing and not standing. Richard Feynman – who may well be the most eminent representative of mainstream physics – thinks this epistemological position is nonsensical:

> "A real tree falling in a real forest makes a sound, of course, even if nobody is there. Even if no one is present to hear it, there are other traces left. The sound will shake some leaves, and if we were careful enough we might find somewhere that some thorn had rubbed against a leaf and made a tiny scratch that could not be explained unless we assumed the leaf were vibrating." (*Feynman's Lectures*, III-2-6)

It is hard to not agree with him. So what is the mainstream physicist's interpretation of the Copenhagen interpretation of quantum mechanics then? Any academic course in quantum mechanics will answer that question, but then you are reading this article because you want an exceptionally smart summary of such course. Hence, let me quote from Feynman's introductory Lecture on the Uncertainty Principle: "Making an observation affects the phenomenon. *The point is that the effect cannot be disregarded or*

---

[2] The title obviously refers to John Gribbins' *In Search of Schrödinger's Cat: Quantum Physics and Reality* (1984). I hope the reader will appreciate the reference.

[3] See, for example, Don Howard, *Who Invented the "Copenhagen Interpretation"? A Study in Mythology*, December 2004, https://www3.nd.edu/~dhoward1/Copenhagen%20Myth%20A.pdf, accessed on 17 September 2018

[4] Radical empiricism and radical skepticism are actually very different epistemological positions, but then we are discussing some basic principles in physics here rather than epistemological theories and so I'll let the reader criticize.

[5] The reference to Hilbert's logic refers to Hilbert spaces: a Hilbert space is an abstract vector space. Its properties allow us to work with quantum-mechanical states, which become *state vectors*. You should not confuse them with the real or complex vectors you're used to. The only thing state vectors have in common with real or complex vectors is that (1) we also need a *base* (aka as a *representation* in quantum mechanics) to define them and (2) that we can make linear combinations.

minimized or decreased arbitrarily by rearranging the apparatus. When we look for a certain phenomenon we cannot help but disturb it in a certain minimum way." (ibidem, *my italics*)

We should separate measurement and consciousness. The Copenhagen interpretation has nothing to do with consciousness. Reality, a measurement, and consciousness are very different things. After having concluded the tree did make a noise, even if no one was there to hear it, Feynman wraps up the philosophical discussion as follows: "We might ask: was there a *sensation* of sound? No, sensations have to do, presumably, with consciousness. And whether ants are conscious and whether there were ants in the forest, or whether the tree was conscious, we do not know. Let us leave the problem in that form." (ibidem)

In short, I think we can all agree that the cat is dead *or* alive, or that the tree is standing or not standing—regardless of the observer. It's a binary situation. Not something in-between. The box just obscures our view. That's all. There is nothing more to it. However, having said that, quantum physicists don't study cats in boxes or trees in forests. Those are big things. They study the behavior of photons and electrons, and smaller things[6], and then the Uncertainty Principle *does* come into play.

The question then becomes: what can we say about the electron – or the photon – before we observe it, or before we make any measurement? Think of the Stein-Gerlach experiment, which tells us that we'll always measure the angular momentum of an electron – along any axis we choose – as either $+\hbar/2$ or, else, as $-\hbar/2$. What is its *reality* before it enters the apparatus? Do we have to assume it has some *definite* angular momentum, and that its value is as binary as the state of our cat: *up* or *down*—no in-between?

Physics, fuzziness and mathematical tricks

To answer that question, we should first explain what we mean by a *definite* angular momentum. It's a concept from classical physics, and it assumes a precise *value* (or magnitude) along some precise *direction*.

We can easily challenge these assumptions. The direction of the angular momentum may be changing all the time, for example. If we think of the electron as a pointlike charge in some regular or irregular orbit, whizzing around in its own three-dimensional space, then the concept of a precise direction of its angular momentum becomes quite fuzzy, because its direction changes all the time. And if its direction is fuzzy, then its value might be fuzzy as well. In classical physics, such fuzziness is not allowed, because angular momentum is conserved: it takes an outside force – or *torque* – to change it.

Presumably, it would also require a force or a torque to change angular momentum in quantum mechanical, right? Yes, and no. We have the Uncertainty Principle in quantum physics: some energy (force over a distance, remember) can be borrowed, so to speak, as long as it's swiftly being returned—within the limits set by the Uncertainty Principle, that is: $\Delta E \cdot \Delta t = \hbar/2$.

Mainstream physicists – including Feynman – do not try to think about this. For them, the electron that enters the Stern-Gerlach apparatus is just like Schrödinger's cat in a box: the only property we're interested in is its spin (up or down), and there's some box obscuring the view. The cat is dead *or* alive.

[6] We limit our analysis to quantum electrodynamics. Hence, this article doesn't try to discuss quarks or other *sectors* of the so-called Standard Model of particle physics.

Likewise, the electron spin is up or down, and each of the two states has some probability – both of which must obviously add up to one. In short, they will write the *state* of the electron before it enters the apparatus as the superposition of the *up* and *down* states:

$$|\psi\rangle = C_{\text{up}}\,|\text{up}\rangle + C_{\text{down}}\,|\text{down}\rangle$$

We are all familiar with this[7]: C_{up} is the amplitude for the electron spin to be equal to $+\hbar/2$ along the chosen direction (which we refer to as the *z*-direction because we will choose our reference frame such that the *z*-axis coincides with this chosen direction) and, likewise, C_{up} is the amplitude for the electron spin to be equal to $-\hbar/2$ (along the same direction, obviously). C_{up} and C_{up} will be functions, and the associated probabilities will vary sinusoidally – with a phase difference to make sure both add up to one.

The math is consistent, but most would agree this feels more like a mathematical trick than a true model of the electron. This description of reality – if that's what it is – does not *feel* like a model of a *real* electron. It's just like reducing the cat in our box to the mentioned fuzzy state of being dead and alive at the same time: that doesn't feel very real either!

What we are actually doing here, is to reduce a three-dimensional object – our electron – to some *flat* mathematical object.[8] Can't we come up with something more exciting? Perhaps we can.

Schrödinger's electron

Physicists describe the reality of electrons by a *wavefunction*. If you are reading this article, you obviously know how a wavefunction looks like: it is a superposition of *elementary* wavefunctions. These elementary wavefunctions are written as $A_i \cdot \exp(-i\theta_i)$, so they have an amplitude A_i and an argument $\theta_i = (E_i/\hbar)\cdot t - (p_i/\hbar)\cdot x$. Let's forget about uncertainty, so we can drop the index (*i*) and think of a geometric interpretation of the simpler $A\cdot\exp(-i\theta) = A\cdot e^{-i\theta}$ expression.

Now here we have a weird thing: for some reason, mainstream physicists think the minus sign in the exponent ($-i\theta$) should always be there. However, if we are seeking a geometric interpretation, then we should explore the two mathematical possibilities. Indeed, the convention is that we get the *imaginary unit* (*i*) by a *counterclockwise* 90° rotation of the real unit (1). However, I like to think a rotation in the *clockwise* direction must also describe something real. To be clear, I think to think $A\cdot e^{-i\theta}$ and $A\cdot e^{+i\theta}$ describe the same electron but with opposite spin. How should we visualize this? I like to think of $A\cdot e^{-i\theta}$ and $A\cdot e^{+i\theta}$ as two-dimensional harmonic oscillators:

$$A\cdot e^{-i\theta} = \cos(-\theta) + i\cdot\sin(-\theta) = \cos\theta - i\cdot\sin\theta$$

$$A\cdot e^{+i\theta} = \cos\theta + i\cdot\sin\theta$$

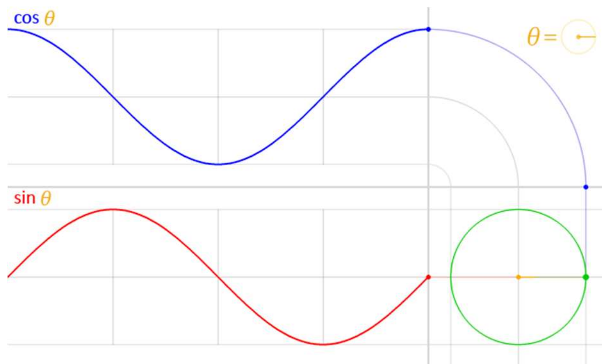
---

[7] It is the *Dirac* or *bra-ket* notation. This simple formula also summarizes the essence of what one should know about *Hilbert spaces*: the two states are *Hilbertian* state vectors which we can combine linearly. These state vectors are always defined in terms of a *base*, or a representation. A representation in quantum mechanics basically establishes a line of sight between the observer and the object or – if you don't like the idea of an observer (consciousness has nothing to do with it) – it establishes the geometric relation between the electron (or whatever other thing we're measuring) and the measurement apparatus (this is a Stern-Gerlach apparatus in this case).

[8] We call it flat because it has two (mathematical) dimensions only.

So we may want to imagine our electron as a pointlike electric charge (see the green dot in the illustration below) to spin around some center in either of the two possible directions. The cosine keeps track of the oscillation in one dimension, while the sine (plus or minus) keeps track of the oscillation in a direction that is perpendicular to the first one.

Figure 1: A pointlike charge in orbit



So we have a weird two-dimensional oscillator here, and we may calculate the energy in this oscillation. To calculate such energy, we need a mass concept. We only have a charge here – so no mass. However, a (moving) charge has an *electromagnetic* mass. Now, the electromagnetic mass of the electron's charge may or may not explain all the mass of the electron (mainstream physicists think it doesn't) but let's assume it does for the sake of the model that we're trying to build up here.[9] So we have some mass oscillating in two directions simultaneously: we basically assume space is, somehow, elastic. How do we *visualize* that?

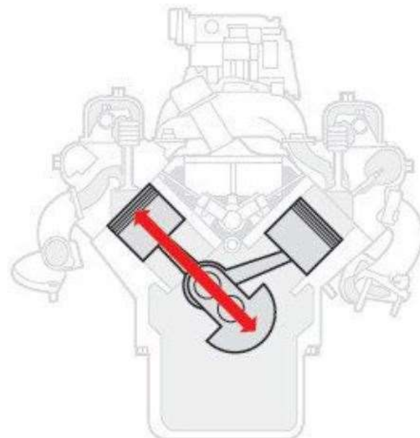
We have worked out a V-2 engine *metaphor* before, so we won't repeat ourselves here.[10] We will limit ourselves here to noting the metaphor does require us to make sense of the real and imaginary part of ψ: we need to associate a physical dimension with them. What could it be? We argued it should be the same as the dimensions of the electric and magnetic field vectors **E** and **B** because, when everything is said and done, a force needs something to grab on, and because our charge has no (rest) mass (no mechanical mass), the charge is the only thing the force can grab onto.[11]

[9] The theory of electromagnetic mass gives us a very simple explanation for the concept of mass here, and so that's why we'll use it for the time being. Otherwise we need an alternative theory: a charge is supposedly pointlike and, therefore, should not have any mechanical mass.

[10] Jean Louis Van Belle, The Wavefunction as an Energy Propagation Mechanism, http://vixra.org/pdf/1806.0106v1.pdf, accessed on 17 September 2018

[11] For more details, see the above-mentioned paper.

Figure 2: A perpetuum mobile?



We showed in our previous papers how this metaphor relates apparently unrelated but *structurally similar* formulas:

1. The energy of an oscillator: $E = (1/2)\cdot m\cdot a^2\cdot\omega^2$
2. Kinetic energy: $E = (1/2)\cdot m\cdot v^2$
3. The rotational (kinetic) energy that's stored in a flywheel: $E = (1/2)\cdot I\cdot\omega^2 = (1/2)\cdot m\cdot r^2\cdot\omega^2$
4. Einstein's energy-mass equivalence relation: $E = m\cdot c^2$

Of course, we are mixing relativistic and non-relativistic formulas here, and there's the 1/2 factor – but these are minor issues that can be solved. For example, we were talking not one but *two* oscillators, so – for the first formula – we should add their energies: $(1/2)\cdot m\cdot a^2\cdot\omega^2 + (1/2)\cdot m\cdot a^2\cdot\omega^2 = m\cdot a^2\cdot\omega^2$. Hence, the 1/2 factor disappears.

Also, one can show that the classical formula for kinetic energy (i.e. $E = (1/2)\cdot m\cdot v^2$) morphs into $E = m\cdot c^2$ when we use the relativistically correct force equation for an oscillator. So, yes, our metaphor – or our suggested physical interpretation of the wavefunction, I should say – makes sense.

The mathematical derivation of the electromagnetic mass gives us the classical electron radius, aka the *Thomson* radius (*Feynman's Lectures*, II-28-3). It's the smallest of a trio of radii that are relevant when discussing electrons: the other two radii are the Bohr radius and the Compton scattering radius respectively. The Thomson radius is used in the context of elastic scattering: the frequency of the incident particle (usually a photon), and the energy of the electron itself, do not change. In contrast, Compton scattering does change the frequency of the photon that is being scattered, and also impacts the energy of our electron. [As for the Bohr radius, you know that's the radius of an electron orbital, roughly speaking – or the size of a hydrogen atom, I should say.]

Now, if we combine the $E = m\cdot a^2\cdot\omega^2$ and $E = m\cdot c^2$ equations, then $a\cdot\omega$ must be equal to c, right? Can we show this? Maybe. It is easy to see that we get the desired equality by substituting the amplitude of the oscillation (a) for the Compton scattering radius $r = \hbar/(m\cdot c)$, and ω (the (angular) frequency of the oscillation) by using the Planck relation ($\omega = E/\hbar$):

$$a\cdot\omega = [\hbar/(m\cdot c)]\cdot[E/\hbar] = E/(m\cdot c) = m\cdot c^2/(m\cdot c) = c$$

We can, of course, also calculate a *tangential* velocity of our charge in orbit. The tangential velocity – which we'll just write as v here – is the product of the radius and the angular velocity: $v = r \cdot \omega = a \cdot \omega = c$.

We get a wonderfully simple geometric model of an electron here: an electric charge that spins around in a plane. Its radius is the *Compton* electron radius – which makes sense – and the radial velocity of our spinning charge is the speed of light – which may or may not make sense.

Of course, we need an explanation of why this spinning charge doesn't radiate its energy away – but then we don't have such explanation anyway. All we can say is that the electron charge seems to be spinning around in its own space: it is moving along a geodesic, so to speak. Hence, it is just like some (rest) mass creating its own space: according to Einstein's general relativity theory, gravity becomes a *pseudo*-force—literally: no *real* force. How? I am not sure: the model here assumes the medium – empty space – is, somehow, perfectly elastic: the electron constantly borrows energy from one direction and then returns it to the other – so to speak. It is a crazy model, yes – but is there anything better? We only want to present a metaphor here: a possible *visualization* of quantum-mechanical models.

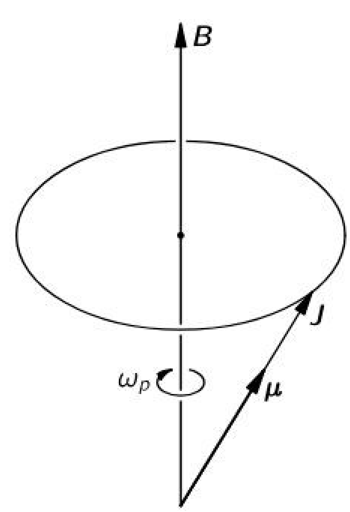
Of course, if this model is to represent anything real, then we also need to answer other questions. Let us think about a possible *interpretation* of the results of the Stern-Gerlach experiment.

Stern-Gerlach's experiment revisited

A spinning charge is a tiny magnet – and so it's got a *magnetic moment*, which we need to explain the Stern-Gerlach experiment. But it doesn't explain the *discrete* nature of the electron's angular momentum: it's either $+\hbar/2$ or $-\hbar/2$, nothing in-between, and that's the case *along any direction* we choose. How can we explain this? Also, space is three-dimensional. Why would electrons spin in a perfect plane? The answer is: they don't.

Indeed, the corollary of the above-mentioned binary value of the angular momentum is that the angular momentum – or the electron's spin – is never completely along any direction. This may or may not be explained by the *precession* of a spinning charge in a field, which is illustrated below (illustration taken from *Feynman's Lectures*, II-35-3).

Figure 3: Precession of an electron in a magnetic field



So we do have an oscillation in three dimensions here, really – even if our wavefunction is a two-dimensional mathematical object. Note that the measurement (or the Stein-Gerlach apparatus in this case) establishes a line of sight and, therefore, a reference frame, so 'up' and 'down', 'left' and 'right', and 'in front' and 'behind' get meaning. In other words, we establish a *real* space. The question then becomes: how and why does an electron sort of *snap into place* as soon as it enters the apparatus?

The geometry of the situation suggests the angle of the angular momentum vector should be 45° (from the horizontal or the vertical). Now, if the value of its z-component (i.e. its *projection* on the z-axis) is to be equal to $\hbar/2$, then the magnitude of $\boldsymbol{J}$ itself should be *larger*. To be precise, it should be equal to $\hbar/\sqrt{2}) \cong 0.7\cdot\hbar$ (just apply Pythagoras' Theorem). Is that value compatible with our flywheel model?

Maybe. Let's see. The *classical* formula for the magnetic moment is $\mu = I\cdot A$, with I the (effective) current and A the (surface) area. The notation is confusing because I is also used for the moment of inertia, or rotational mass, but… Well… Let's do the calculation. The effective current is the electron charge (q_e) divided by the *period* (T) of the orbital revolution: : $I = q_e/T$. The period of the orbit is the *time* that is needed for the electron to complete one loop. That time (T) is equal to the circumference of the loop ($2\pi\cdot a$) divided by the tangential velocity (v_t). Now, we suggest $v_t = r\cdot\omega = a\cdot\omega = c$, and the circumference of the loop is $2\pi\cdot a$. For a, we still use the Compton radius $a = \hbar/(m\cdot c)$. Now, the formula for the area is $A = \pi\cdot a^2$, so we get:

$$\mu = I\cdot A = [q_e/T]\cdot\pi\cdot a^2 = [q_e\cdot c/(2\pi\cdot a)]\cdot[\pi\cdot a^2] = [(q_e\cdot c)/2]\cdot a = [(q_e\cdot c)/2]\cdot[\hbar/(m\cdot c)] = [q_e/(2m)]\cdot\hbar$$

In a classical analysis, we have the following relation between angular momentum and magnetic moment:

$$\mu = (q_e/2m)\cdot J$$

Hence, we find that the angular momentum J is equal to $\hbar$, so that's *twice* the measured value. We've got a problem. We would have hoped to find $\hbar/2$ or $\hbar/\sqrt{2}$. Perhaps it's because $a = \hbar/(m\cdot c)$ is the so-called *reduced* Compton scattering radius but… Well… No.

Maybe we'll find the solution one day. I think it's already quite nice we have a model that's accurate up to a factor of 2 or $\sqrt{2}$. The point is: the precise $+\hbar/2$ or $-\hbar/2$ value for the angular momentum of an electron – along any direction – suggests a definite value and direction, which isn't there. The classical analysis suggests precession, which implies the direction of the angular momentum is changing all of the time. The precession phenomenon just ensures it changes in an equally precise way. Hence, while the idea of a specific *direction* for the angular momentum cannot be maintained, the idea of a specific value can be maintained.

From there, it is easy to take the following logical step: what if there is no magnetic field to line our electron up? The direction of the angular momentum might really wander around then, and we have a three-dimensional model.

The model that is offered here is structurally similar to the spherically symmetric solutions to Schrödinger's equation for electron orbitals (the *s*-states). The wavefunction ψ does *not* depend on the angles here. In other words, we do not worry about the line of sight between the observer and the object (the hydrogen atom, in this case). Hence, it could, perhaps, represent something real. Of course, there are several differences with the mentioned *s*-states: the radius is different, and the electron is actually spinning *around* some center – rather than filling an entire sphere. But it's basically the same.

Can we prove this model? Absolutely not. This is just an *interpretation*, and it does *not* respects Ockham's *lex parsimoniae*: the Copenhagen interpretation makes fewer assumptions and is, therefore, the better explanation. However, playful interpretations like this may help us to appreciate that quantum physics does describe reality.

18 September 2018

References

This paper discusses general principles in physics only. Hence, references were limited to references to basic physics textbooks and online material only. All of the illustrations in this paper are open source or have been created by the author. The author hopes the reviewer(s) will give him, once again, the benefit of the doubt. As the author wrote this paper for his son, who is about to start a course in quantum mechanics as part of his second year of engineering studies, he also hopes the reviewer(s) will forgive his casual tone.