The Quantum/Classical Connection

B. F. Riley

By way of the Quantum/Classical Connection, the radii of nearby G and K-type Main Sequence stars map onto the atomic masses of Period 4 transition metals while the masses of the stars map onto the atomic radii of the same elements.

'Classical' length scales (specifically the radii of astrophysical bodies) r_C map onto related 'quantum' masses $m_Q < m_{Planck}$ by way of the Quantum/Classical Connection [1, 2]:

$$r_C{}^2 = 2m_Q{}^{-5} \tag{1}$$

found in the Planck Model [3]. Planck units (ħ=c=G=1) are used in (1) and in equations throughout the paper, which explains the apparently unbalanced dimensions.

First, mass values m_Q are calculated from stellar radii $r_C = r_S$ using (1) for six G and K-type Main Sequence stars within 4 pc of earth: the Sun, Alpha Centauri A & B, Tau Ceti, Epsilon Eridani and Epsilon Indi. The uncertainty in radius measurement for each of these stars is <1%. The stellar radii and masses are presented in Table 1.

Star	Type	Radius (R⊙)	Mass (M⊙)	References
Sun	G	1	1	
Alpha Centauri A	G	1.2234(53)	1.100(6)	4, 5
Alpha Centauri B	K	0.8632(37)	0.907(6)	4, 5
Tau Ceti	G	0.793(4)	0.783(12)	6
Epsilon Eridani	K	0.735(5)	0.82(2)	7, 8
Epsilon Indi	K	0.732(6)	0.762(38)	7

Table 1: Stellar parameters of six nearby G and K-type Main Sequence stars

The mass values m_Q calculated from (1) are plotted in Figure 1 on the levels and sublevels of Planck Sequences 1 and 3, which descend in geometric progression from the Planck Mass with common ratio 1/π and 1/e, respectively,[1] and which may derive from an extra-dimensional geometry [9]. The level-numbers n_1 and n_3 corresponding to m_Q are calculated from:

$$n_1 = \ln(m_{Planck}/m_Q)/\ln(\pi) \tag{2}$$

[1] Planck Sequence 2 is of common ratio 2/π.

$$n_3 = \ln(m_{Planck}/m_Q) \quad (3)$$

Figure 1 should be compared with Figure 2, in which the atomic masses of nuclides with A in the range 48-61 are presented[2]. Atomic masses corresponding, through (1), to the radii of stars are marked.

Figure 1: Masses m_Q corresponding through the Quantum/Classical Connection of (1) to the radii r_S of nearby G and K-type Main Sequence stars - shown on the mass levels of Planck Sequences 1 and 3. The levels have been drawn at right angles. In such a representation the masses lie on a straight line since for all values of mass the corresponding level-numbers n_1 and n_3 are in constant ratio.

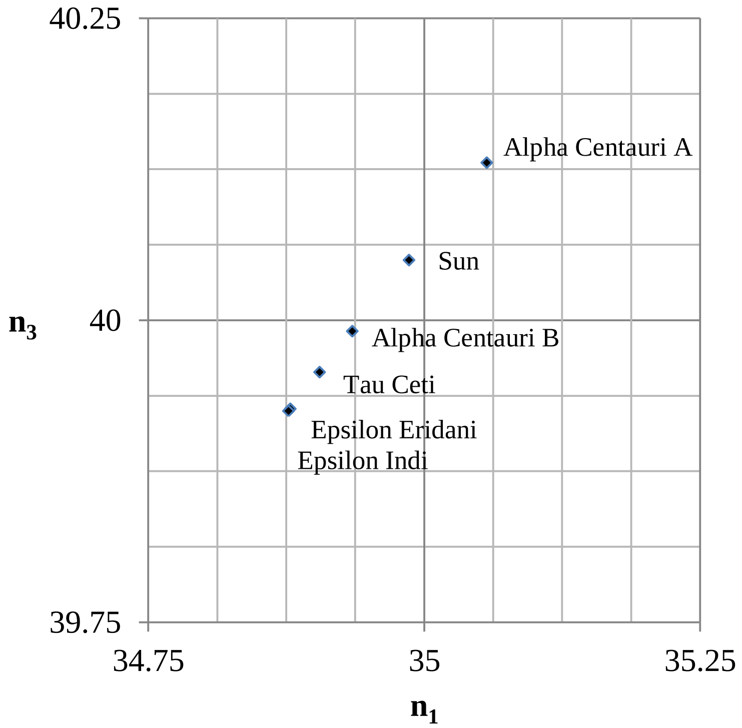
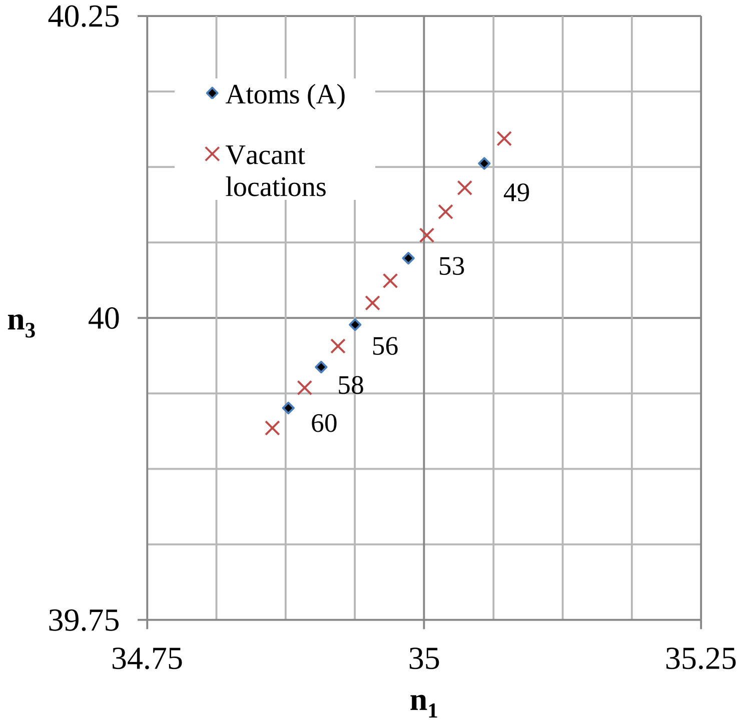


Figure 2: Masses of atoms with mass numbers A - shown on the mass levels of Planck Sequences 1 and 3.



[2] The mass excess is set to zero.

The radii of G and K-type stars are mapped onto the atomic masses of Period 4 transition metals using (1).[3] Stable nuclides tend to occupy prominent levels and sublevels in the Planck sequences [10]. The tightly bound ^{52}Cr and ^{56}Fe, of which massive stellar cores are finally composed, lie close to superlevels[4] at the near-coincidence (35, 40) in Planck Sequences 1 and 3. Nearly-coincident levels and superlevels are important locations for physics [3, 11].

We will now show that the *masses* of the G and K-type stars of Figure 1 map onto the atomic *radii* of the Period 4 transition metals of Figure 2 by way of the symmetrical counterpart of (1). The analysis will include other types of Main Sequence stars. The 'classical' stellar masses are measured relative to the inverse Bohr radius, $a_0{}^{-1} = 3.73$ keV, and not relative to the Planck Mass. The symmetrical counterpart of (1) is then:

$$(a_0 m_S)^2 = 2 r_Q{}^5 \tag{4}$$

The Bohr radius, $a_0 = (\pi/2)^{125}\, l_{Planck}$, is a key scale in the Planck Model. It lies on the superlevel near-coincidence (50, 125) in Sequences 1 and 2, as shown in Figure 4.[5]

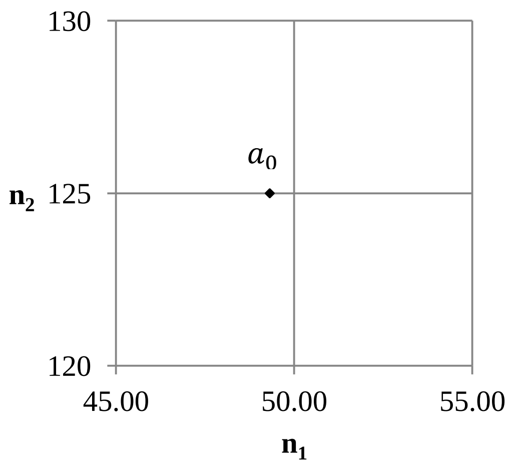


Figure 4: The Bohr radius on levels of length scale in Planck Sequences 1 and 2

Mass values m_Q have been calculated from stellar radii r_S using (1) and radius values r_Q have been calculated from stellar masses m_S using (4) for nearby, bright or spectral standard Main Sequence stars of types O, B, A, F, G, K and M, including the stars of Table 1. The stellar radii and masses of those stars not included in Table 1 are presented in Table 2. The values of r_Q calculated from (4) for stars of type B, A, F, G, K and M are plotted in Figure 5 against the atomic mass numbers A corresponding to the values of m_Q calculated from (1). Values of atomic (covalent) radius [12] are

[3] Values of A for the Period 4 transition metals lie in the range 45-65 (stable nuclides).
[4] Superlevels have level-numbers that are multiples of 5.
[5] In the Planck Sequences, levels of length scale ascend from Planck scale while mass levels descend from Planck scale.

plotted against relative atomic mass for each element of Period 4 for comparison with the radius values r_Q. The values of m_Q and r_Q found for O-type stars will be considered later.

Star	Type	Radius (R⊙)	Mass (M⊙)	References
BI253	O	10.7	84	13, 14
HD93129 Aa	O	22.5	110	15, 16
HD93129 Ab	O	13.1	70	17, 16
HD93129 B	O	13	52	18
Regulus A	B	3.092	3.8	19, 20
Sirius A	A	1.711	2.063	21, 22
γ Ursae Majoris	A	3.04	2.94	23, 24
Formalhaut	A	1.842	1.92	25
Tabby's Star	F	1.58	1.43	26
π^3 Orionis	F	1.323	1.236	27, 28
61 Cygni A	K	0.665	0.70	29, 30
61 Cygni B	K	0.595	0.63	29, 30
Struve 2398 A	M	0.351	0.334	31
Groombridge 34 A	M	0.3863	0.404	32, 33
Lalande 21185	M	0.393	0.46	7, 34
Lacaille 9352	M	0.459	0.503	7
Luyten's Star	M	0.35	0.26	35, 34

Table 2: Stellar parameters of the Main Sequence stars studied that are not in Table 1

It can be seen from Figure 5 that (4) maps the masses of types A, F, G and K-type Main Sequence stars onto values of r_Q that are closely aligned with the atomic (covalent) radii of Period 4 elements. Values of r_Q calculated from the radii of M-type stars diverge from the Period 4 atomic radii. A stellar mass m_S of 0.08 M⊙ (∼smallest mass of a true star) maps onto an r_Q value of 0.05 nm (~Bohr radius).

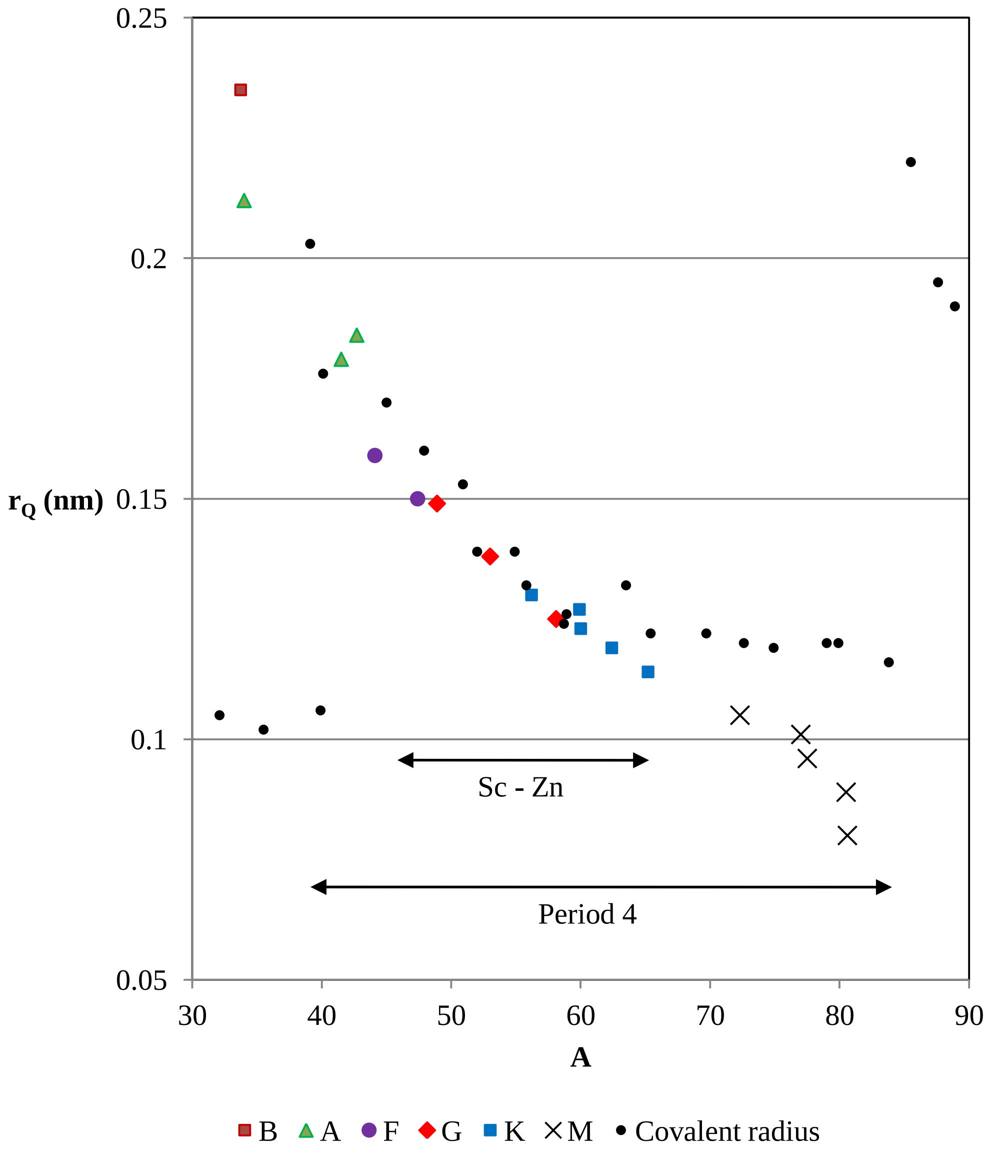


Figure 5: Values of r_Q calculated from stellar masses m_S using (4), plotted against atomic mass numbers A corresponding to the values of m_Q calculated from stellar radii r_S using (1). Also shown are values of atomic (covalent) radius [12] for each element of Period 4, plotted against relative atomic mass. The range of relative atomic mass for the Period 4 transition metals (Sc – Zn) is marked.

From Figure 5, it can also be seen that the masses of G and K-type Main Sequence stars map onto the atomic radii of Period 4 transition metals. Do the stellar mass and radius map onto the radius and mass, respectively, of the same nuclide? Consider the values of corresponding mass number A found for the six G and K-type stars of Figure 1. For each of these stars there is only one corresponding stable nuclide: for Alpha Centauri A, ^{49}Ti; for the Sun, ^{53}Cr; for Alpha Centauri B, ^{56}Fe; for Tau Ceti,

^{58}Fe; and for Epsilon Eridani and Epsilon Indi, ^{60}Ni. The value of r_Q found for each of the six stars is plotted against atomic number Z in Figure 6. Also plotted are values of atomic (covalent) radius. The agreement between r_Q and atomic radius is good. By way of the Quantum/Classical Connection - at least for these G and K-type Main Sequence stars - stellar radii map onto atomic masses and stellar masses map onto the radii *of the same atoms*.

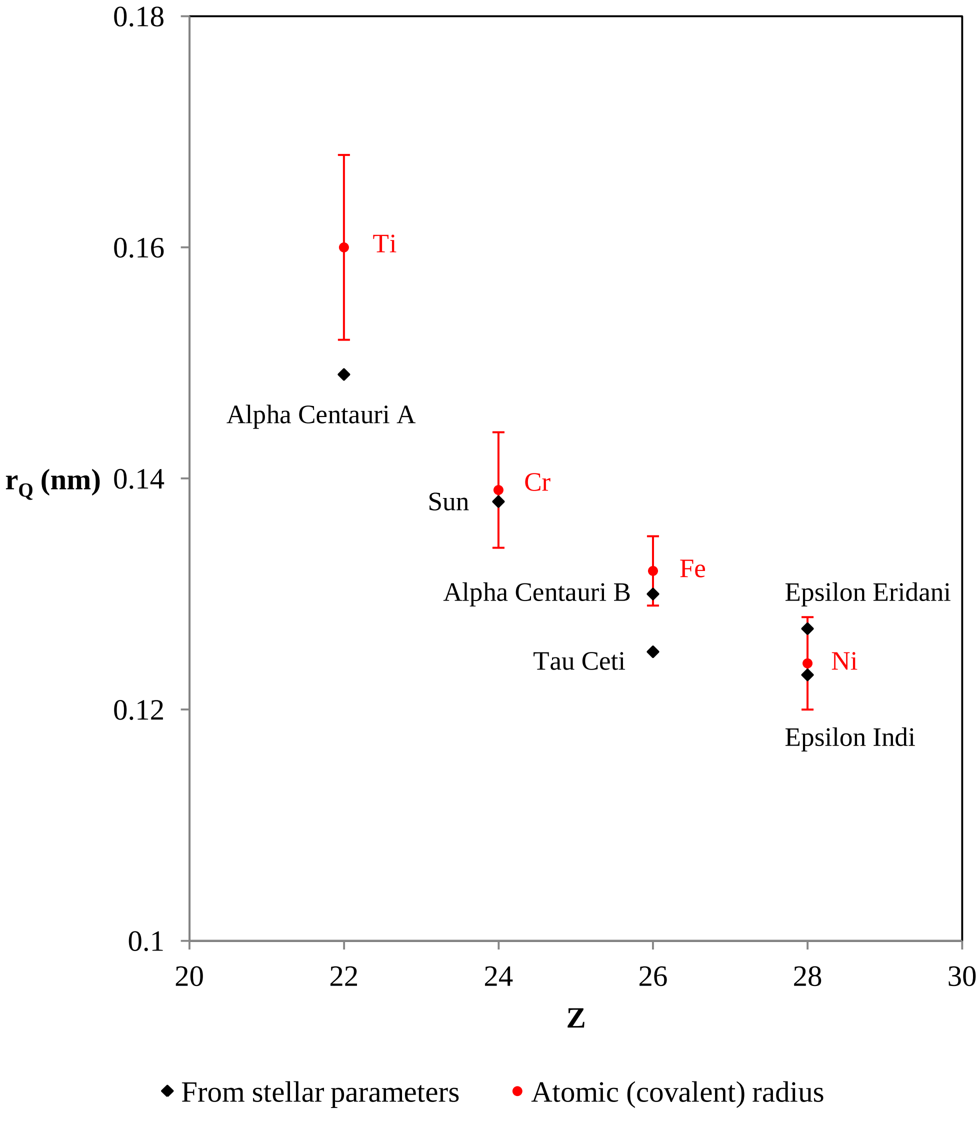


Figure 6: Values of r_Q calculated from stellar masses m_S using (4), plotted against atomic number Z of the corresponding nuclides identified from the values of m_Q calculated from stellar radii r_S using (1). Also plotted are values of atomic (covalent) radius [12]. The error bars signify one standard deviation.

We will now consider giant stars that have left the Main Sequence after having burned up their core hydrogen. Values of m_Q calculated, using (1), from the radii r_S of the stars of Table 3 are plotted on the mass levels and sublevels of Planck Sequences 1 and 3 in Figure 6. The values of m_Q coincide with sublevels rather than with atomic masses.

Star	Type	Radius (R⊙)	Mass (M⊙)	References
Pollux	K	8.8(1)	1.91(9)	36, 37
Arcturus	K	25.4(2)	1.08(6)	38
Aldebaran	K	44.13(84)	1.16(7)	39, 40
Canopus	A	71(4)	8.0(3)	41

Table 3: Stellar parameters of giant stars

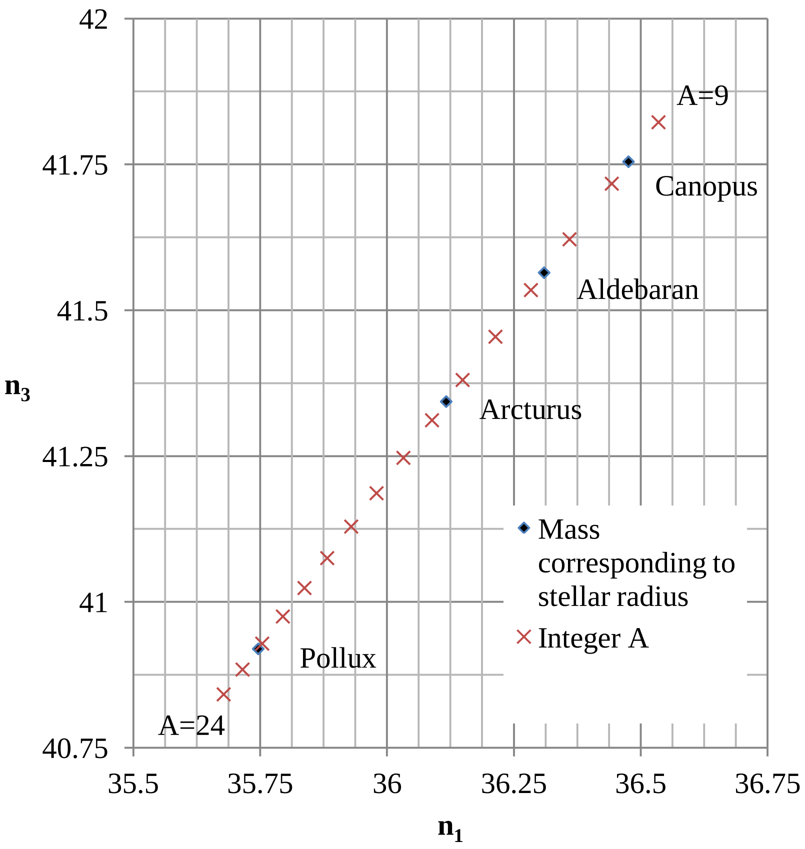


Figure 7: Masses m_Q corresponding, through the Quantum/Classical Connection in (1), to the radii r_S of giant stars - shown on the mass levels and sublevels of Planck Sequences 1 and 3.

Values of r_Q calculated for the giant stars of Figure 6 and all the stars of Table 2, including those of O-type, are plotted against values of A (corresponding to m_Q) in Figure 7. A 'quantum' Main Sequence can be seen.

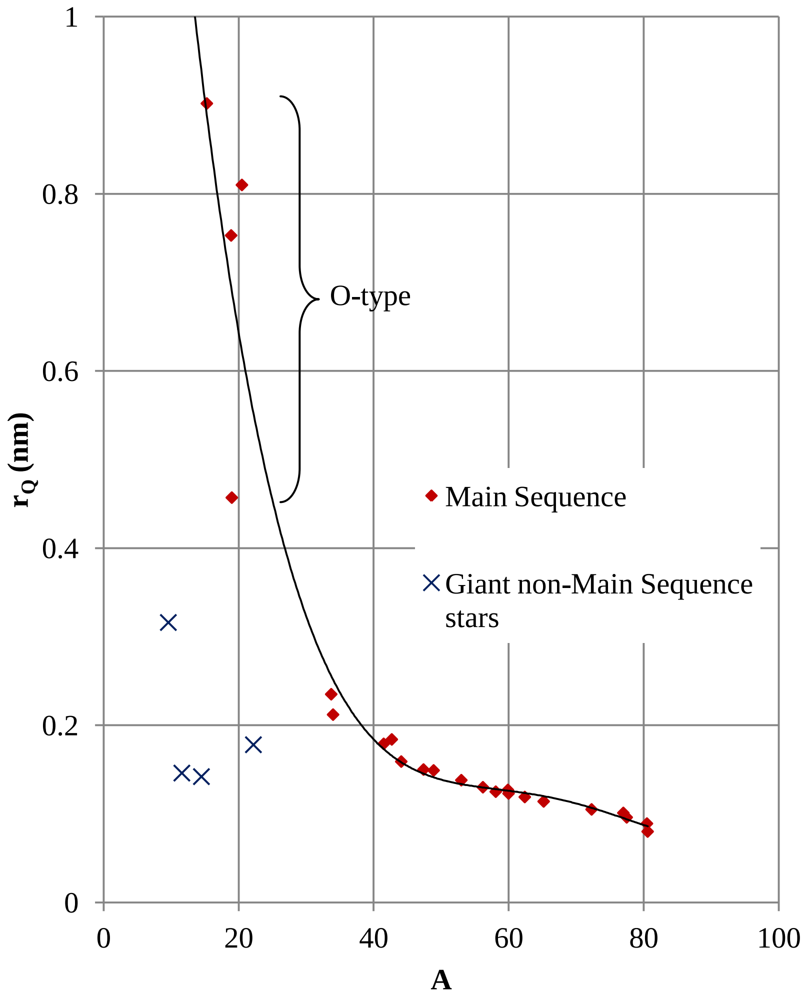


Figure 8: Values of r_Q calculated from stellar masses m_S using (4), plotted against atomic mass numbers A corresponding to the values of m_Q calculated from stellar radii r_S using (1). Included are type-O Main Sequence stars and giant stars that have left the Main Sequence.

Conclusions

1. The Quantum/Classical Connection maps the radii of nearby G and K-type Main Sequence stars onto the masses of Period 4 transition metals.
2. The Quantum/Classical Connection maps the masses of the G and K-type Main Sequence stars onto the atomic radii of the same Period 4 transition metals as above.
3. The smallest mass characterising a true star maps onto a length scale of ~Bohr radius.
4. The radii of giant non-Main Sequence stars map onto mass sublevels in the Planck sequences rather than onto atomic masses.

References

1. B. F. Riley, '10D/4D correspondence and the Big Bang', viXra:1503.0104
2. B. F. Riley, 'The correlation between stellar radii and the masses of stable atomic nuclei', viXra:1704.0049
3. B. F. Riley, 'The Planck Model', viXra:1311.0053
4. P. Kervella et al, 'The radii and limb darkening of Alpha Centauri A and B – Interferometric measurements with VLTI/PIONIER' Astronomy and Astrophysics **597:** A137 (2017)
5. F. Thevenin et al, 'Asteroseismology and calibration of alpha Cen binary system', Astronomy & Astrophysics **392**: L9 (2002)
6. T. C. Teixeira et al, 'Solar-like oscillations in the G8 V star tau Ceti', Astronomy & Astrophysics **494** (1): 237-42 (2009)
7. B-O. Demory et al, 'Mass-radius relation of low and very low-mass stars revisited with the VLTI', Astronomy and Astrophysics **505** (1): 205-15 (2009)
8. E. K. Baines and J. T. Armstrong, 'Confirming fundamental parameters of the exoplanet host star epsilon Eridani using the Navy Optical Interferometer', The Astrophysical Journal **748**: 72 (2011)
9. B. F. Riley, 'Standard Model scales from warped extra dimensions', arXiv:0809.0111
10. B. F. Riley, 'Particle mass levels', viXra:0909.0006
11. B. F. Riley, 'The intersecting brane world', arXiv:physics/0607152
12. Cambridge Structural Database
13. J. G. Rivero Gonzalez et al, 'Nitrogen line spectroscopy of O-stars II. Surface nitrogen abundances for O-stars in the Large Magellanic Cloud', Astronomy & Astrophysics **537**: A79 (2012)
14. J. M. Bestenlehner et al, 'The VLT-FLAMES Tarantula Survey XVII. Physical and wind properties of massive stars at the top of the main sequence', Astronomy & Astrophysics **570**: A38 (2014)

15. T. Repolust et al, 'Stellar and wind parameters of galactic O-stars. The influence of line-blocking/blanketing', Astronomy & Astrophysics **415**: 349-76 (2004)
16. E. P. Nelan et al, 'Resolving OB systems in the Carina Nebula with the Hubble Space Telescope fine guidance sensor', The Astronomical Journal **128**: 323-9 (2004)
17. S. Del Palacio et al, 'A model for the non-thermal emission of the very massive colliding wind binary HD 93129A', Astronomy & Astrophysics **591**: A139 (2016)
18. J. S. Vink et al, 'On the presence and absence of disks around O-type stars', Astronomy & Astrophysics **505** (2): 743 (2009)
19. G. T. van Belle and K. von Braun, 'Directly determined linear radii and effective temperatures of exoplanet host stars', The Astrophysical Journal **694** (2): 1085-98 (2009)
20. M. L. Malagnini and C. Morossi, 'Accurate absolute luminosities, effective temperatures, radii, masses and surface gravities for a selected sample of field stars', Astronomy and Astrophysics Supplement Series **85** (3): 1015-19 (1990)
21. J. Liebert et al, 'The age and progenitor mass of Sirius B', The Astrophysical Journal **630** (1): L69-72 (2005)
22. H. E. Bond et al, 'The Sirius system and its astrophysical puzzles: Hubble Space Telescope and ground-based astrometry', The Astrophysical Journal **840** (2): 70 (2017)
23. E. L. Fitzpatrick and D. Massa, 'Determining the physical properties of the B stars II. Calibration of synthetic photometry', The Astrophysical Journal **129** (3): 1642-62 (2005)
24. S. Eggl et al, 'Circumstellar habitable zones of binary-star systems in the solar neighbourhood', Monthly notices of the Royal Astronomical Society **428** (4): 3104 (2012)
25. E. E. Mamajek, 'On the age and binarity of Formalhaut', Astrophysical Journal Letters **754** (2): L20 (2012)
26. T. S. Boyajian et al, 'Planet hunters IX. KIC8462852 – where's the flux?', Monthly notices of the Royal Astronomical Society **457** (4): 3988-4004 (2012)
27. T. S. Boyajian et al, 'Stellar diameters and temperatures I. Main Sequence A, F and G stars', The Astrophysical Journal 746 (1): 101 (2012)
28. G. Takeda et al, 'Stellar parameters of nearby cool stars II. Physical properties of ~1000 cool stars from the SPOCS catalog', Astrophysical Journal Supplement Series 168: 297-318 (2007)
29. P. Kervella et al, 'The radii of the nearby K5V and K7V stars 61 Cygni A & B, CHARA/FLUOR interferometry and CESAM2k modeling', Astronomy and Astrophysics **488** (2): 667-74 (2008)
30. Research consortium on nearby stars, Georgia State University, 'List of the nearest 100 stellar systems' (2007)
31. A. W. Mann et al, 'How to constrain your M dwarf: measuring effective temperature, bolometric luminosity, mass and radius', The Astrophysical Journal **804** (1): 38 (2015)

32. A. W. Howard et al, ‘The NASA-UC-UH ETA-Earth Program IV. A low-mass planet orbiting an M dwarf 3.6 pc from Earth’, The Astrophysical Journal **794** (1): 9 (2014)
33. D. H. Berger et al, ‘First results from the CHARA array IV. The interferometric radii of low-mass stars’, The Astrophysical Journal **644** (1): 475-83 (2006)
34. Research consortium on nearby stars, Georgia State University, ‘List of the nearest 100 stellar systems’ (2009)
35. C. H. Lacy, ‘Radii of nearby stars: an application of the Barnes-Evans relation’, Astrophysical Journal Supplement Series **34**: 479-92 (1977)
36. A. P. Hatzes et al, ‘Confirmation of the planet hypothesis for the long-period radial velocity variations of β Geminorum’, Astronomy and Astrophysics **457**: 335-41 (2006)
37. A. P. Hatzes et al, ‘The mass of the planet-hosting giant star β Geminorum determined from its p-mode oscillation spectrum’, Astronomy and Astrophysics **543**: 9 (2012)
38. I. Ramirez and C. Allende Prieto, ‘Fundamental parameters and chemical composition of Arcturus’, The Astrophysical Journal **743** (2): 135 (2011)
39. L. Piau et al, ‘Surface convection and red-giant radius measurements’, Astronomy and Astrophysics **526**: A100 (2011)
40. W. M. Farr et al, ‘Aldebaran b’s temperate past uncovered in planet search data, arXiv:1802.09812
41. P. Cruzalebes et al, ‘Fundamental parameters of 16 late-type stars derived from their angular diameter measured with VLTI/AMBER’, Monthly notices of the Royal Astronomical Society **434**: 437 (2013)