Can I Embed the Mass/Energy into the Spacetime Structure?

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Some physicists believe that the spacetime, in absence of the matter, is empty. In this paper I presented an opposing point of view.

1. Introduction

Until the early twentieth century the space and time were considered separate beings. In 1907, German mathematician H. Minkowski connected together the three-dimensional space and one-dimensional time into single idea, creating a new Minkowski's four-dimensional spacetime [1] (or shorter *the Minkowski spacetime M*).

This idea enjoyed success in the *Special Relativity* (SR) and the *General Relativity* (GR), correctly describing a wide range of a physical phenomena. Some physicists believe that, in absence of the matter, the Minkowski spacetime *M* is empty.

2. The spacetime M^*

I assumed that Minkowski's a four-dimensional spacetime is not empty, but is filled by the bare mass/energy, **forming together a new entity:** *the Minkowski spacetime with the mass/energy* (or shorter *the Minkowski spacetime* M^*). The process of a embedding the mass/energy into the spacetime structure I will call *the massification of this spacetime*.

Postulate I. In the absence of the gravitational field the massification of the spacetime takes the form:

$$\left(M\ \eta\right) \xrightarrow{\text{the massification}} \left(M^*\ \rho^{bare}\right) \tag{1}$$

where: $\left(M\ \eta\right)$ is the Minkowski a differentiable manifold *M* equipped with the Minkowski tensor η, $\left(M^*\ \rho^{bare}\right)$ is the Minkowski a differentiable manifold M^* equipped with the symmetric bare mass/energy density tensor ρ^{bare}.

I created the spacetime with built-in *"transparent substance"*, endowed with the mass/energy, tightly filling whole Universe, through which the electromagnetic (EM) interactions are propagate[1] with $c = f\left(\varepsilon_0, \mu_0\right)$, where c is speed of light, ε_0 is the vacuum permittivity, μ_0 is the vacuum permeability.

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[1] In physics there was a proposition of the existence of a medium (*an ether*) a space-filling substance, which seemed to be necessary as *a transmission medium* for the propagation of EM waves. About Einstein ether you can read in [2, 3].

Uniform motion of bodies has no impact on the structure of the *"transparent substance"*. The symmetric bare mass/energy density tensor $\rho_{\mu\nu}^{bare}$ describes the inertial field in the Minkowski spacetime M^*, Greek indices: μ, ν = 0, 1, 2, 3.

Postulate II. The gravitational field changes the structure of the *"transparent substance"* causes that

$$\left(M^* \ \rho^{bare}\right) \xrightarrow{\text{the gravitational field}} \left(M^* \ \rho\right) \tag{2}$$

where: $\left(M^* \ \rho^{bare}\right)$ is the Minkowski a differentiable manifold M^* equipped with the symmetric bare mass/energy density tensor ρ^{bare}, $\left(M^* \ \rho\right)$ is the pseudo-Riemannian a differentiable manifold M^* equipped with the symmetric mass/energy density tensor ρ.

Influence of the gravitational field causes, that the Minkowski M^* spacetime become *the pseudo-Riemannian spacetime* M^*. The symmetric mass/energy density tensor ρ describes the gravitational field in the pseudo-Riemannian spacetime M^*.

Postulate III. The metric ds^2 of the pseudo-Riemannian spacetime M^* takes the form:

$$ds^2 \overset{def}{=} \frac{\rho_{\mu\nu}}{\rho^{bare}} \cdot dx^\mu dx^\nu \tag{3}$$

where: $\frac{\rho_{\mu\nu}}{\rho^{bare}}$ can plays role of *the symmetric metric tensor* $g_{\mu\nu}$.

In the absence of the gravitational field the metric (3) takes the form of the Minkowski metric:

$$ds^2 = \eta_{\mu\nu} \cdot dx^\mu dx^\nu \tag{4}$$

which well suited to describes all the physical phenomena occurring in SR.

3. Free particle in the pseudo-Riemannian spacetime M^*

The Lagrangian function for the body with mass density ρ_0 in the pseudo-Riemannian spacetime M^* takes the form:

$$L = \frac{1}{2}\rho_0 \cdot \frac{\rho_{\mu\nu}}{\rho^{bare}} \cdot \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} \tag{5}$$

The geodesic equation takes the form:

$$\frac{d}{d\tau}\left(\rho_{\gamma\mu} \cdot \frac{dx^\mu}{d\tau}\right) - \frac{1}{2}\frac{\partial \rho_{\mu\nu}}{\partial x^\gamma}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0 \tag{6}$$

where: τ is *the proper time*.

The geodesic equation in GR takes the form:

$$\frac{d}{d\tau}\left(g_{\gamma\mu}\cdot\frac{dx^{\mu}}{d\tau}\right)-\frac{1}{2}\frac{\partial g_{\mu\nu}}{\partial x^{\gamma}}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}=0 \tag{7}$$

Equations (6) and (7) **are the same**, if and only if, there is a relationship between the symmetric metric tensor $g_{\mu\nu}$ and symmetric the mass/energy density tensor $\rho_{\mu\nu}$, which satisfies the condition:

$$g_{\mu\nu}=\frac{\rho_{\mu\nu}}{\rho^{bare}} \tag{8}$$

This leads to the hypothesis: in the absence of the matter the pseudo-Riemannian spacetime M **cannot be empty**.

4. Summary and conclusion

GR

$$\left(M\ \ \eta\right)\xrightarrow{\textit{the gravitational field}}\left(M\ \ g\right) \tag{9}$$

where: $\left(M\ \ g\right)$ is the pseudo-Riemannian a differentiable manifold M equipped with the symmetric metric tensor g.

The metric ds^2 of the pseudo-Riemannian spacetime M takes the form:

$$ds^2=g_{\mu\nu}\cdot dx^{\mu}dx^{\nu} \tag{10}$$

In absence of the gravitational field the metric (10) becomes the Minkowski metric:

$$ds^2=\eta_{\mu\nu}\cdot dx^{\mu}dx^{\nu} \tag{11}$$

Our model

$$\left(M^{*}\ \ \rho^{bare}\right)\xrightarrow{\textit{the gravitational field}}\left(M^{*}\ \ \rho\right) \tag{12}$$

The metric ds^2 of the pseudo-Riemannian spacetime M^{*} takes the form:

$$ds^2\overset{def}{=}\frac{\rho_{\mu\nu}}{\rho^{bare}}\cdot dx^{\mu}dx^{\nu} \tag{13}$$

In absence of the gravitational field the metric (13) becomes the Minkowski metric (11).

The massification diagram

$$\begin{array}{ccc} \left(M\ \ \eta\right) & \xrightarrow{\textit{the massification}} & \left(M^{*}\ \ \rho^{bare}\right) \\ \Big\downarrow & \textit{the gravitational field} & \Big\downarrow \\ \left(M\ \ g\right) & \xrightarrow{\textit{the massification}} & \left(M^{*}\ \ \rho\right) \end{array}$$

The relationship between the symmetric metric tensor $g_{\mu\nu}$ and the symmetric mass/energy density tensor $\rho_{\mu\nu}$

$$\left(M\ \ g\right) \overset{g_{\mu\nu}=\frac{\rho_{\mu\nu}}{\rho^{bare}}}{\longleftrightarrow} \left(M^{*}\ \ \rho\right)$$

leads to a surprising hypothesis that, in the absence of matter, $\left(M\ \ g\right)$ is not empty at all.

Reference

1. Minkowski H., *Raum und Zeit*, Physikalische Zeitschrift, **10**, 1909, pp. 104–111. English translation *Space and Time: Minkowski's Papers on Relativity*, ed. V. Petkov, Minkowski Institute Press, Montréal – Québec, 2012.
2. Kostro L., *Alberta Einsteina koncepcja eteru relatywistycznego*, Uniwersytet Gdański 1992. In the English version: *Einstein and the Ether*, Apeiron (December 1, 2000), https://www.amazon.com/Einstein-Ether-Ludwik-Kostro/dp/0968368948 .
3. Kostro L., *Albert Einstein's new Ether and his General Relativity*, http://www.mathem.pub.ro/proc/bsgp-10/K10-KOSTRO.PDF .