Hubble Law Extended for the Accelerating Universe

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Abstract

Hubble law describes both the uniformly expanding universe originally and then the accelerating universe. We argue that for describing an accelerating system, physics laws should include terms of acceleration for accuracy and completeness. We extend Velocity-Distance and Distance-Redshift relations of Hubble law to Acceleration-Velocity-Distance and Acceleration-Velocity-Distance-Redshift relation of Extended-Hubble-Law (denoted as **EHL**). The latter contains Doppler effect, Acceleration-Redshift, Hubble's Distance-Redshift, and the cosmological redshift; and, thus, is appropriate for small and large redshift, slow and fast velocity, near and faraway source. **EHL** shows: (1) acceleration causes both redshift and the time change of redshift, which may be tested by sound wave; (2) the age of the accelerating universe is older. **EHL** provides an approach to test jerking motion of the universe. We suggest that **EHL** is worth to pursue for restudying Hubble law related phenomena including measurement of Hubble constant.

1. Introduction

In the 1998, scientists report a revolutionary discovery that the expansion of the universe is accelerating [1]. In the 2016, scientists report even that acceleration is faster than expected [2], call it jerk. Hubble law is one of the most important laws in characterizing the accelerating universe. On the other hand, several issues relate to Hubble law, such as the discrepancy in measurements of Hubble constant, and the age of several star/galaxy being close to or older than the universe. It is suggested that those issues could imply that Hubble law misses some ingredients [3]. We notice a hitherto un-emphasized fact that Hubble law does not contain acceleration, because Hubble's original data indicated that the universe was uniformly expanding.

We believe that, in order to describe an accelerating system, physics laws should contain acceleration for accuracy and completeness; and, therefore, suggest that Hubble law could miss acceleration when describing the accelerating universe.

In this article, we extend Hubble law to include acceleration.

2. Extended Hubble Law (**EHL**)

2.1. Acceleration-Velocity-Distance Relation of **EHL**

Hubble established his Velocity-Distance relation, now denoted as Hubble's law,

$$\dot{r} = H_0 r,$$

where r is the physical distance between a star and an observer; H_0 is the Hubble constant. Note since there is no term of acceleration in Hubble's law, it is correct for describing a uniformly moving universe, but not accurate for describing the accelerating universe.

In this form, it is not obvious how to extend Hubble's law to contain acceleration. For this aim, let's rewrite Hubble's law as,

$$r = \dot{r}\left(\frac{1}{H_0}\right),$$

which implies that present distance r equals to present velocity $\dot{r}$ times present Hubble time $1/H_0$, and nothing to do with acceleration. Comparing Hubble's law in this form with the equation of motion in classic mechanism,

$$r = \dot{r}t + \frac{1}{2}\ddot{r}t^2,$$

To extend Hubble law, we propose Acceleration-Velocity-Distance relation of **EHL** as,

$$r(t) = \dot{r}(t_1)\left(\frac{1}{H_e}\right) + \frac{1}{2}\ddot{r}\left(\frac{1}{H_e}\right)^2, \quad (1)$$

where H_e is Extended-Hubble-Parameter; time t_1 is prior to an arbitrary time t; acceleration $\ddot{r}$ is constant, $\ddot{r}(t) = \ddot{r}(t_1)$. The faster the receding velocity and acceleration, the farther the object is away.

Hubble law is the first approximation of **EHL**. Moreover, **EHL** agrees with kinematics of an accelerating system.

Converting velocity at time t_1 to that at time t, **EHL** in terms of scale factor becomes,

$$a(t) = \dot{a}(t)\left(\frac{1}{H_e}\right) - \frac{1}{2}\ddot{a}\left(\frac{1}{H_e}\right)^2, \tag{1a}$$

$$\frac{\dot{a}(t)}{a(t)} = H_e\left[1 + \frac{1}{2H_e^2}\frac{\ddot{a}(t)}{a(t)}\right], \tag{1b}$$

$$H_e = \frac{1}{2}\frac{\dot{a}(t)}{a(t)}\left\{1 + \sqrt{1 - 2\frac{\ddot{a}(t)a(t)}{[\dot{a}(t)]^2}}\right\}. \tag{1c}$$

Eq. (1a) implies that present scale factor equals to its present velocity times Extended-Hubble-Time, subtracts the distance due to the acceleration. Eq. (1c) shows that Extended-Hubble-Parameter is reduced by acceleration, i.e., smaller than Hubble-Parameter. Thus Extended-Hubble-Time is longer than Hubble time.

For a jerking universe, **EHL** becomes,

$$H_e a(t) = \dot{a}(t) - \frac{1}{2H_e}\ddot{a}(t) + \frac{1}{3!H_e^2}\dddot{a}(t). \tag{1d}$$

2.2. Acceleration-Velocity-Distance-Redshift Relation of **EHL**

Now we demonstrate that acceleration does shift wavelength.

Distance-Redshift relation of Hubble law,

$$cz = HD,$$

has no acceleration term. Thus, for describing the accelerating universe, Hubble law should be extended to include acceleration. For this aim, separating redshift into two parts: first part is created during emitting process; second part is created during propagation and, thus, is a pure cosmological redshift.

(A) Redshift during emitting process. Special Relativity doesn't hold for accelerating systems, thus, we don't take into account effects of SR. When a receding source is emitting, its accelerating motion stretches the wavelength. Let's consider a wave with two wave crest #1 and #2 emitted at time t_1 and t_2 respectively; and $(t_2 - t_1) = {}^{\lambda_s}/_c$, c is the speed of the wave. During the time interval $(t_2 - t_1)$, the source moves a distance d,

$$d = v(t_1)(t_2 - t_1) + \frac{1}{2}\dot{v}(t_2 - t_1)^2.$$

$v(t_1)$ and $\dot{v}$ are respectively the velocity and acceleration of the source at time t_1; and $\dot{v} = \dot{v}(t_2) = \dot{v}(t_1)$. The wavelength is stretched from λ_s to λ_{12}. We obtain Extended-Doppler-Redshift relation for an accelerating source,

$$cz_{12} = v(t_1) + \frac{\dot{v}}{2}\frac{\lambda_s}{c}. \tag{2}$$

Taking time derivative, the uniform acceleration of the source leads to,

$$c\dot{z}_{12} = \dot{v}, \tag{3}$$

which provides a new way to confirm whether a source is accelerating.

Extended-Doppler-Redshift and its time change may be tested with sound wave.

Extended-Doppler-Redshift can be extended to contain jerk, as,

$$cz_{12} = v(t_1) + \frac{\dot{v}}{2}\frac{\lambda_s}{c} + \frac{\ddot{v}}{6}\left(\frac{\lambda_s}{c}\right)^2. \tag{2a}$$

During the emitting process, Extended-Doppler-Redshift relation can also be derived from the accelerating expansion of the scale factor $a(t)$. Thus redshift created during the emitting process may be interpreted as either Extended-Doppler-Redshift or the cosmological redshift. We adopt the former in this essay.

(B) Redshift during propagation through the accelerating universe. Once wave crest #2 left the source at time t_2, the wave is no longer related to the source. During propagation, its wavelength is stretched by the accelerating expansion of the universe. Assuming that an observer receives wave crest #2 at time t_0, the wave's propagation time is $(t_0 - t_2)$. Then $c(t_0 - t_2) = D$, where D is the physical distance between the source and observer. Wavelength is stretched from λ_{12} to λ_{20} as $\frac{\lambda_{20}}{\lambda_{12}} = \frac{a(t_0)}{a(t_2)}$. Applying Taylor series of the scale factor $a(t_0)$,

$$a(t_0) = a(t_2) + \dot{a}(t_2)(t_0 - t_2) + \frac{1}{2}\ddot{a}(t_2)(t_0 - t_2)^2 + \frac{1}{6}\dddot{a}(t_2)(t_0 - t_2)^3 + \cdots,$$

Acceleration-Velocity-Distance-Redshift created during propagation is,

$$cz_{20} = \frac{\dot{a}(t_2)}{a(t_2)}D + \frac{\ddot{a}(t_2)}{2a(t_2)}\frac{D^2}{c}. \tag{4}$$

Where jerk and higher order terms have been ignored for the accelerating universe.

The accelerating motion of the source stretches wavelength from λ_s to λ_{12}, the accelerating expansion of the universe stretches wavelength from λ_{12} to λ_{20}. Net-Redshift relation of **EHL** is the sum of two parts,

$$cz_N = cz_{12} + cz_{20} = v(t_1) + \frac{\dot{v}}{2}\frac{\lambda_s}{c} + H(t_2)D + \frac{\ddot{a}}{2a(t_2)}\frac{D^2}{c}, \tag{5}$$

which changes with time as,

$$c\dot{z}_N = \dot{v}(t_1) + \dot{H}(t_2)D - \frac{\ddot{a}\dot{a}(t_2)}{2a^2(t_2)}\frac{D^2}{c}. \tag{6}$$

Indeed, acceleration causes both the redshift and its time change. On the right hand side of Eq. (5), first term is Doppler effect, second term is Acceleration-Redshift, third tem Hubble's Distance-redshift, and fourth term Acceleration-Distance-Redshift. Thus Net-Redshift relation of **EHL** is appropriate for describing small and large redshift, slow and fast velocity, and near and faraway source. In this form, the physical origin of each portion of redshift is clear.

Rewriting Eq. (5), we obtain Extended-Cosmological-Redshift relation,

$$1 + z_N = \frac{a(t_0)}{a(t_2)} + \frac{v(t_1)}{c} + \frac{\dot{v}}{2}\frac{\lambda_s}{c^2}, \tag{5a}$$

For a jerking universe, Net-Redshift relation is

$$cz_{20} = v(t_1) + \frac{\dot{v}}{2}\frac{\lambda_s}{c} + \frac{\ddot{v}}{6}\left(\frac{\lambda_s}{c}\right)^2 + \frac{\dot{a}(t_2)}{a(t_2)}D + \frac{\ddot{a}(t_2)}{2a(t_2)}\frac{D^2}{c} + \frac{\dddot{a}}{6a(t_2)}\frac{D^3}{c^2}. \tag{5b}$$

(C) Turning Point. According to Eq. (5), we propose Primary-Turning-Point,

$$v(t_1) + \frac{\dot{v}}{2}\frac{\lambda_s}{c} = \frac{\dot{a}(t_2)}{a(t_2)}D + \frac{\ddot{a}(t_2)}{2a(t_2)}\frac{D^2}{c}, \tag{7}$$

and Secondary-Turning-Point,

$$\frac{\dot{a}(t_2)}{a(t_2)}D = \frac{\ddot{a}(t_2)}{2a(t_2)}\frac{D^2}{c}. \tag{7a}$$

When $\frac{\dot{a}(t_2)}{a(t_2)}D + \frac{\ddot{a}(t_2)}{2a(t_2)}\frac{D^2}{c} > v(t_1) + \frac{\dot{v}}{2}\frac{\lambda_s}{c}$, the cosmological redshift is dominant. When $\frac{\dot{a}(t_2)}{a(t_2)}D + \frac{\ddot{a}(t_2)}{2a(t_2)}\frac{D^2}{c} > v(t_1) + \frac{\dot{v}}{2}\frac{\lambda_s}{c}$ and $\ddot{a}(t_2)\frac{D}{2c} > \dot{a}(t_2)$, the product of the acceleration and distance is the dominant contribution to redshift.

2. 3. Extend Deceleration Parameter

2.3.1. Motion Parameter

With the extended Hubble parameter, let's extend Deceleration Parameter q to "Motion Parameter" q_n that judges the motion status, and is defined as,

$$q_n \equiv -\frac{a^{(n)}(t_0)}{n!a(t_0)H_e^n}, \tag{8}$$

where n is positive integer. "Velocity Parameter" $q_1 < 0$ represents expansion; $q_1 > 0$ represents collapse. $2q_2$ is the ordinary Deceleration Parameter, $q_2 < 0$ represents acceleration; $q_2 > 0$ represents deceleration. "Jerk Parameter" $q_3 < 0$ represents accelerating acceleration; $q_3 > 0$ represents decelerating acceleration.

2.3.2. **EHL** in Terms of Motion Parameter

EHL can be rewritten in terms of Motion Parameter. Substituting Eq. (8) into **EHL**, Eq. (1a-1d), we obtain,

$$1 = -q_1 + q_2, \tag{1a-1}$$

$$\frac{\dot{a}(t_0)}{a(t_0)} = H_e\{1 - q_2\}, \tag{1b-1}$$

$$H_e = \frac{1}{2}H\left\{1 + \sqrt{1 + 4\frac{q_2}{q_1^2}}\right\}, \tag{1c-1}$$

$$1 = -q_1 + q_2 - q_3. \tag{1d-1}$$

Eq. (1a-1) can be used to confirm the acceleration of the universe.

3. Application of **EHL**

3.1. Extended-Hubble-Time and age of the accelerating universe

The fact that the ages of several stars/Galaxies are close to or older than t_H could imply that the universe may be older. Now we demonstrate that the age of the accelerating universe obtained from **EHL** is older than that given by Hubble law.

(A) Kinematics: Eq. (1c-1) of **EHL** gives Extended-Hubble-Time,

$$t_e = t_H \left[\frac{2}{1+\sqrt{1+4\frac{q_2}{q_1^2}}} \right], \tag{9}$$

where t_e and t_H are respectively Extended-Hubble-Time and Hubble-Time. Acceleration increases universe's age: $t_e > t_H$.

(B) Dynamics: Freidmann equation gives the age of the accelerating universe as,

$$t_H = \frac{1}{H_0} F(z, \Omega_m, \Omega_\Lambda, ..),$$

where $F(z, \Omega_m, \Omega_\Lambda, ..)$ is the function of $z, \Omega_m, \Omega_\Lambda,$ etc.

Let's introduce acceleration, Eq. (1b-1), into the Freidmann equation, we obtain,

$$\left(\frac{\dot{a}}{a}\right)^2 = [H_e(1 - q_2)]^2 = \frac{8\pi G}{3}\rho - \frac{k}{r^2} + \frac{\Lambda}{3}.$$

The Extended Age, t_e, of the accelerating universe is

$$t_e = \{1 - q_2\} t_H, \tag{9a}$$

i.e., the age of the accelerating universe is older.

The accelerating universe was expanding slower in the past than it is today, so it took a longer time to expand to its present size than that calculated without taking into account effects of acceleration.

3.2. Alternative Validation Test of the 2016 Observation

If confirmed, the 2016 observation indicates that the acceleration is accelerating, which puzzles dynamically. First, theories of gravity are second order derivative and a jerk is third order derivative. Second, need a mechanism to drive a jerk universe. The confirmation of the 2016 observation is profoundly important.

Besides the observational confirmation, **EHL** provides an alternative approach to test the validation of the 2016 observation. Once q_1 and q_2 are determined, **EHL,** Eq. (1d-1), indicates that if

$$1 + q_1 - q_2 \neq 0, \tag{10}$$

then $q_3 \neq 0$, i.e., $\dddot{r}(t) \neq 0$. Thus there is jerk; otherwise there will be no jerk.

The significance of this indirect confirmation is that the confidence level of the conclusion is the same as that of measurements of velocity and acceleration.

3.3. On Measurements of Hubble Constant

Hubble constant can be measured by the distance modulus formula. For the accelerating universe, instead of substituting Hubble's Distance-Redshift relation into it, we suggest substituting Net-Redshift relation of **EHL**. Solving Eq. (5) gives the distance D. Applying D, the distance modulus formula becomes

$$\mathrm{m} - \mathrm{M} = 5\log\left\{\frac{c\dot{a}(t_2)}{\ddot{a}}\left\{\sqrt{1 + \frac{2\,a(t_2)\ddot{a}}{\dot{a}^2(t_2)}\left[z_N - \frac{v(t_1)}{c} - \frac{\dot{v}}{2}\frac{\lambda_s}{c^2}\right]} - 1\right\}\right\} + 5. \qquad (11)$$

For $1 > \frac{2\,a(t_2)\ddot{a}}{\dot{a}^2(t_2)}\left[z_T - \frac{v(t_1)}{c} - \frac{\dot{v}}{2}\frac{\lambda_s}{c^2}\right]$, we obtain,

$$\mathrm{m} - \mathrm{M} = 5\log\{cz_N - v(t_1)\} - 5\log\{A\} + 5, \qquad (11a)$$

where $A \equiv H(t_0)\frac{1-2\frac{q_2}{q_1}H(t_0)\frac{D}{c}}{1-H(t_0)\frac{D}{c}+q_2H^2(t_0)\left(\frac{D}{c}\right)^2}$; and $\frac{\dot{v}}{2}\frac{\lambda_s}{c^2} \approx 0$. Once we obtain A, $H(t_0)$ can be obtained. $H(t_0)$ is the function of velocity, acceleration, and distance.

4. Summary and Discussion

For describing the accelerating universe, we extend Hubble law to **EHL** and show several effects: (1) the age of the accelerating universe is older; (2) provides approaches for confirming the accelerating universe and for exploring a jerk universe respectively; (3) affects the result of measurement of Hubble constant. Indeed, acceleration causes redshift and its time change.

We suggest testing both Extended-Doppler-Redshift relation and time change of redshift of sound wave, and replacing Hubble law by **EHL** in studying cosmology.

Reference

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