Logical Inconsistency in the Einsteinian Relativistic Time-dilation Interpretation of the Increased Lifetime of High Speed Particles

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Abstract:

This study has examined the logic in the interpretation of the increased lifetime of high speed elementary particles, an important experimental evidence of the Lorentzian or Einsteinian relativistic time dilation. When analyzed with the particle frame being the rest frame, the interpretation based on the Einsteinian special relativity leads to a velocity paradox and inconsistency between the invariance of space-time intervals in the Minkowski space and the constancy of the speed of light. The Lorentz ether theory and the Standard Model Extension might avoid this inconsistency because they allow privileged reference frames. Therefore, although the mainstream physicists have chosen Einstein's special relativity as the correct theory, the Lorentz ether theory might be superior in explaining experimental evidence of time dilation.

Keywords: Lorentz ether theory; special relativity; space-time interval; Minkowski space; Standard Model Extension.

1. Introduction

Time dilation is an important result of relativistic theories. In order to explain the null result of the Michelson-Morley experiments and the invariance of Maxwell's electromagnetic equations across different reference frames, Lorentz proposed time dilation in addition to length contraction in the form of the Lorentz transformation (Lorentz 1904). The meaning of time dilation is that clocks moving relative to the ether frame are slower than clocks at rest in the ether frame. Later, Einstein derived the Lorentz transformation from two postulates, the principle of relativity and the constancy of the speed of light. He obtained time dilation and length contraction from the Lorentz transformation (Einstein 1905). Since then, there are two competing theories that explain physical phenomena at high velocity; the Lorentz ether theory (LET) is based on velocity relative to the ether frame, while Einstein's special relativity is based on velocity between any two inertial reference frames. All the experiments conducted so far to verify their predictions cannot distinguish between them (Zhang 1995). However, the mainstream physicists have chosen Einstein's special relativity as the correct theory mainly because of philosophical or aesthetical considerations. The theory of relativity and quantum mechanics are considered to be the two most important advances in physics of the twentieth century.

Both the LET and the Einsteinian special relativity can insure that laws of physics are invariant under a transformation between two coordinate frames moving at constant velocity with respect to each other, but the two theories are not identical in all their predictions. The LET does not lead to paradoxes, because its Lorentz time dilation and length contraction are effects due to an object's velocity in the ether frame. The Einsteinian special relativity leads to many apparent paradoxes being raised since the twin paradox was first noticed by Langevin (1911), because time dilation and other relativistic effects are observational effects due to an object's velocity relative to the observer. More recently, a paradox published in *Physical Review Letters* by Masud Mansuripur in 2012 describes a scenario in which a pointlike electric charge sits a fixed distance from a tiny magnet without charge. The magnet can be viewed as a tiny loop of wire in which negatively charged electrons run through stationary positive ions. In the rest frame of reference the unmagnetized charge does not interact with the magnet's magnetic field. In a “moving frame of reference”, the magnet appears to have more positive charge on one side and more negative charge on the other thanks to the effects of relativity, so the point charge will pull on one side of the magnet and push on the other, creating a twisting torque (Mansuripur 2012).

All relativistic paradoxes raised so far have been explained by the relativity of simultaneity or the existence of some hidden or overlooked momentum. The recent paradox raised by Mansuripur is considered by several physicists to fail to account for a hidden angular momentum possessed by the magnet (Vanzella 2013; Barnett 2013; Saldanha 2013; Khorrami 2013; Griffiths and Hnizdo 2013) or the linear and angular electromagnetic field momentum contained in the charge-magnetic dipole system (Redfern 2016) and therefore it is not a real paradox, but Mansuripur does not accept the hidden angular momentum explanation (Cho 2013; Mansuripur 2013).

Although the Einsteinian special relativity reaches into every corner of modern physics, a growing number of physicists are entertaining the possibility that special relativity is not quite right (Cho 2005). The Einsteinian relativistic 4-dimensional space-time is incompatible with quantum mechanics in describing point particles, which makes it impossible to develop a quantum theory of gravity. Kostelecký and colleagues have proposed a Standard Model Extension (SME), which adds myriad of background fields to the so-called Standard Model, the relativistic quantum field theory that explains all the particles seen so far (Colladay and Kostelecký 1997; Colladay and

Kostelecký 1998). The SME is an effective field theory that contains the Standard Model and general relativity (Kostelecký and Mewes 2009; Kostelecký and Potting 2009), but it clashes with special relativity because the background fields provide universal benchmarks for determining whether an object is moving or not. Many experiments have been designed to test the SME (Kostelecký and Russell 2011). A deviation from special relativity could lead to a quantum theory of gravity. Given that many physicists still consider it essential to have background fields to reconcile quantum mechanics and the theory of relativity, it might be worthwhile to reexamine how the Einsteinian special relativity and background field based theories such as the LET explain the experimental evidence of time dilation among others. Differences in their logic merits could provide some indication of their superiority in giving a more valid description of nature.

One of the most important experimental evidences of time dilation is the increased lifetime of high speed elementary particles. The first of such experimental studies was to estimate the mean lifetime of muons at high speed. Muon is an unstable subatomic particle with a mean lifetime of 2.197019±0.000021 μs (MuLan Collaboration. 2007). Most naturally occurring muons on the earth are decay products of pions created by collision between a cosmic ray proton and an atomic nucleus in the upper atmosphere. If there was no time dilation, even at the speed of light those muons would travel only around 0.66 km before decay in the upper regions of the atmosphere, and then very few muons would be detected at ground level. In early 1940s, however, Rossi and colleagues detected a large number of muons at ground level, which suggests that muons travelling at above 99.4% of the speed of light (0.994 c) in the atmosphere could survive much longer than the estimated proper lifetime of muons (Rossi, Hilberry and Hoag 1940; Rossi and Hall 1941; Rossi and Nereson 1942; Rossi and Nereson 1943). The speed of light c is 299792458 m/s. This finding has been considered as the first experimental evidence of relativistic time dilation.

A more precise experiment was conducted by Frisch and Smith to investigate the time dilation effect in high speed muons (Frisch and Smith 1963). They compared the flux of muons in Mount Washington and in Cambridge, Massachusetts. The height difference between the two sites is 1907 m, so it takes 6.4 μs for muons at 0.994 *c* to traverse this distance. They found that there were approximately 563 muons per hour in Mount Washington and 412 muons per hour in Cambridge, from which a time dilation factor of 8.8±0.8 can be calculated. This result is in good agreement with the predicted 8.4±2. Such time dilation of moving particles has also been confirmed by observations in particle accelerators using different types of particles, pion (Durbin, Loar and Havens 1952), kaon (Burrowes et al. 1959) and muon (Lundy 1962; Eckhause et al. 1963; Meyer et al. 1963).

Since the aforementioned explanation is generally described in textbooks as based on the Einsteinian special relativity, the aim of the present study is to examine in depth whether each step in this interpretation is logically consistent with the fundamental assumptions and conclusions of the Einsteinian special relativity. Comparisons will also be made with background field based theories such as the LET and the SME. It has been generally accepted that the LET, the Einsteinian special relativity, and Edwards' scheme with constant two-way speed of light and variant one-way speed of light give the same predictions which are testable with current technologies (Zhang 1995; Anderson, Vetharaniam and Stedman 1998). Therefore, a closer examination of their logic for arriving at their predictions might give some clues on the merits of their interpretations. It must be emphasized here that the present study is only intended to find out which version of these relativistic theories is better at explaining the experimental evidence of time dilation. It does not question the validity of the relativistic theories as a whole, the Lorentz transformation or time dilation per se.

The rest of the paper is organized as follows: section 2 examines the Einsteinian and Lorentzian interpretation of the increased lifetime of high speed particles; section 3 investigates the distance between two observers as measured by them in their own stationary frames; section 4 looks into the proper time of each frame as indicated by its own clocks; section 5 presents a velocity paradox; section 6 discusses the solutions to the paradox; and section 7 concludes.

2. The Einsteinian and Lorentzian Interpretation of the Lifetime of High Speed Particles

It is well established experimentally that the lifetime of unstable elementary particles is increased by their speed relative to the earth. According to the interpretation based on the Einsteinian special relativity, the increase in the lifetime of these particles such as muons represents the time dilation measured by clocks in the reference frame of observers at rest on the earth. In the reference frame where these high speed particles are stationary (i.e. the particle frame), the lifetime of muons measured by clocks moving with them is still their proper lifetime, 2.197019±0.000021 μs. If there are stationary muons on the earth, observers moving with the high speed muons will also find an increase in the lifetime of these muons stationary on the earth. The time dilation effect is reciprocal between the observers on the earth and the observers travelling with the high speed particles. The velocity that determines the extent of time dilation is that between the observed muons and the observers.

Using the interpretation based on the Einsteinian special relativity, we get a rough estimate of the average lifetime of those high speed muons that reached the ground detector after departing from the height of 1907 meters in the experiment by Frisch and Smith is at least

$$t_{BA,earth} = \frac{d_{BA,earth}}{v_{muon,earth}} = \frac{1907}{0.994 \times 299792458} = 6.39946 \times 10^{-6}\ (s) \qquad (1)$$

In Eq. (1), $t_{BA,earth}$ is the time measured by clocks on the earth for the muons to travel from their initial position (B) to the observers (A) on the earth; $d_{BA,earth}$ the distance between A and B as measured in the earth frame; and $v_{muon,earth}$ the velocity of the muons measured in the earth frame. What the current technologies could not let us do is to measure the lifetime of these muons with clocks and observers travelling with them and to measure the lifetime of muons stationary on the earth while we travel at 0.994c relative to them. Although the current technologies do not enable these measurements, we may still deduce some logical predictions from the Einsteinian framework.

From the principle of relativity, we know that in the muon frame the muons should also have reached the ground detector, because they have reached the ground detector in the earth frame. According to the Einsteinian special relativity, the lifetime of the muons measured by observers travelling with them should be their proper lifetime, i.e. their lifetime at rest, 2.197019 μs. So for those muons that have reached the ground detector, we should have

$$t_{AB,muon} = \frac{d_{AB,muon}}{v_{earth,muon}} < 2.197019 \times 10^{-6}\ \text{(s)} \qquad (2)$$

In Eq. (2), $t_{AB,muon}$ is the time measured by clocks travelling with the muons for the earth to travel from its initial position (A) to the observers (B) travelling with the muons; $d_{AB,muon}$ the distance between A and B as measured in the muon frame; and $v_{earth,muon}$ the velocity of the earth measured in the muon frame. From Einstein's derivation of the Lorentz transformation as well as the principle of relativity, we know that $v_{earth,muon}$ is equal to $v_{muon,earth}$.

Obviously, in order to be consistent with the prediction of special relativity, $d_{AB,muon}$ must be smaller than $d_{BA,earth}$. Many physicists believe that this is the case and

$$d_{AB,muon} = d_{BA,earth}\sqrt{1 - \frac{v^2}{c^2}} \tag{3}$$

If Eq. (3) is correct, there would be no logical inconsistency in the Einsteinian interpretation of time dilation. However, since the earth frame and the muon frame should have equal status as there is no privileged reference frame in the Einsteinian special relativity, whether Eq. (3) is in keeping with the principle of relativity is worth examining.

According to the Einsteinian special relativity, due to the principle of relativity and the reciprocity of the relativistic effects, the lifetime of muons stationary on the earth measured by observers travelling with the high speed moving muons at 0.994c should also be increased by time dilation in the same manner as the lifetime of those moving muons measured by observers on the earth. Then the time for the earth (and the muons stationary on the earth) to travel from its initial position (A) to the high speed moving muons (B) measured in the muon frame by clocks travelling with the muons is at least

$$t_{AB,muon} = \frac{d_{AB,muon}}{v_{earth,muon}} = 6.40282 \times 10^{-6} \quad \text{(s)} \tag{4}$$

That is, $d_{AB,muon}$ cannot be smaller than $d_{BA,earth}$. A paradox seems to arise here between Eqs. (2) and (4) on $t_{AB,muon}$ and $d_{AB,muon}$. It is easy to see that Eq. (4) contradicts Eqs. (2) and (3). Eq. (4) is a consequence of the principle of relativity, which implies

$$d_{AB,muon} = d_{BA,earth} \tag{5}$$

Many relativist physicists would not accept Eqs. (4) and (5). For them, the distance $d_{AB,muon}$ is the moving version of the distance $d_{BA,earth}$, therefore it should undergo length contraction in the muon frame. For correctly understanding the Einsteinian special relativity and the logic consistency of the time dilation interpretation, we need to know which equation is the correct one, Eq. (3) or Eq. (5). Here the relationship between $d_{AB,muon}$ and $d_{BA,earth}$ in the Einsteinian special relativity becomes the key issue. When $d_{AB,muon}$ contracts with Lorentz length contraction formula $d_{AB,muon} = d_{BA,earth}\sqrt{1-\frac{v^2}{c^2}}$, Eq. (2) would be met. This issue will be examined in detail in the following section.

The LET was displaced by the Einsteinian special relativity because of the preference of mainstream physicists rather than any critical experimental tests. The Einsteinian special relativity, the Edwards transformation and the LET share the same predictions for all currently feasible experimental tests. Due to this similarity in explaining existing experiments, people often forget the deeper difference between the Einsteinian special relativity and the LET. The Einsteinian definition of simultaneity requires the constancy of one-way speed of light, while the LET is only intended to insure the invariance of two-way speed of light. It has been well established that all testable results of special relativity can be derived from theories based on constancy of two-way speed of light, such as the Edwards transformation (Edwards 1963; Winnie 1970a; Winnie 1970b). The Einsteinian definition of simultaneity is not necessary for a version of relativity theory to explain all the experimental results.

In the LET, the velocity that determines the extent of time dilation is the velocity between the observed objects and the ether frame. If the velocity between the muons and the observers on the earth is 0.99c, they cannot both have a velocity of 0.99c relative to the ether frame. Although observers in one reference frame cannot measure the velocity of their own frame relative to the ether frame because of Lorentz invariance, comparison between the extents of time dilation (the rates of clocks) in two reference frames would give an indication of their velocities relative to the ether frame. For example, if the increase in the lifetime of high speed muons observed in the earth frame is similar to that predicted by Lorentz time dilation formula using velocity of muons relative to the earth, we can infer that the velocity of the earth in the ether frame is close to zero or far smaller than the speed of light. Observers travelling with the muons would not find time dilation in the muon frame because their clocks will also run slowly; but they should find that muons stationary on the earth have a decreased lifetime when measured with their travelling clocks, and clocks stationary on the earth running faster than their travelling clocks. In the LET there is no reciprocity between the ether frame and the moving frame for time dilation or length contraction. This is why the LET does not lead to paradoxes.

3. The Distance between Two Inertial Observers in Relative Motion

In the preceding section, we have shown that from the Einsteinian special relativity we can infer two conflicting results, Eqs. (3) and (5). The mainstream physicists support Eq. (3), so we will investigate in this section whether their support of Eq. (3) would lead to logic inconsistency with the fundamental assumptions and conclusions of the Einsteinian special relativity. If Eq. (5) is correct and the proper lifetime of muons measured by the observers co-moving with the muons is 2.2 μs, then the velocity of the earth measured in the muon frame is

$$v_{earth,muon} = \frac{d_{AB,muon}}{t_{AB,muon}} > \frac{1907}{2.2 \times 10^{-6}} = 8.668 \times 10^8. \quad (6)$$

In Eq. (6), $v_{earth,muon} > 8.668 \times 10^8$ because $t_{AB,muon} < 2.2 \times 10^{-6}$. The velocity of the earth relative to the muons ($v_{earth,muon}$) as measured by the muon frame is at least 8.668×10^8 m/s, which is nearly three times the speed of light c, violating the constancy of the speed of light, one of the two fundamental postulates of the Einsteinian special relativity. This result shows that if Eq. (5) is correct, special relativity leads to a self-contradiction for the interpretation of time dilation.

Probably because of the implication of Eq. (6), most relativity physicists would reject Eq. (5) and maintain that the distance between A and B measured in the earth frame is different from that measured in the muon frame. They choose Eq. (3) as the true relationship between the distance measured in the muon frame and the distance measured in the earth frame, which can keep the velocity of the earth smaller than *c* in the muon frame. Does Eq. (3) conform to the fundamental assumptions and conclusions of the Einsteinian special relativity such as non-existence of any privileged inertial reference frame?

The view that the distance measured in the earth frame should be different from the same distance measured in the muon frame might be just a bias that always tends to view the earth frame as the rest frame. To judge whether Eq. (3) is consistent with the fundamental assumptions and conclusions of the Einsteinian special relativity, we can use two inertial observers A and B with relative velocity *v* to replace the earth and muons in the scenario discussed so far. The distance between A and B measured by A is denoted as $d_{BA,A}$, and the distance between A and B measured by B denoted $d_{AB,B}$. The relative velocity between A and B can be further denoted as $v_{BA,A}$, which is the velocity of observer B toward A as measured in observer A's frame, and $v_{AB,B}$, which is the velocity of observer A toward B as measured in observer B's frame (Figure 1). In the Einsteinian special relativity, $v_{BA,A} = v_{AB,B}$. Should $d_{BA,A}$ be different from $d_{AB,B}$?

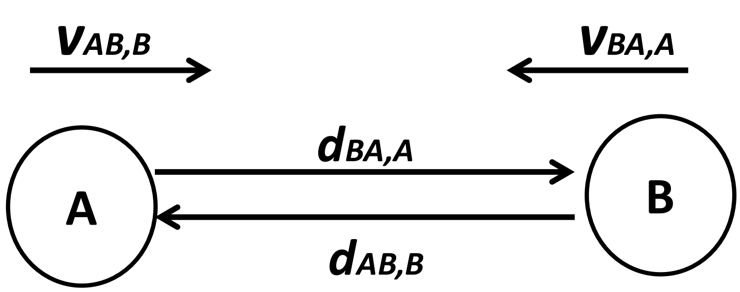

Fig.1 The distance between two inertial observers A and B in relative motion. The velocity $v_{BA,A}$ is that of observer B as measured by observer A, $v_{AB,B}$ is that of observer A as measured by observer B; $d_{BA,A}$ is the distance between A and B measured by A and $d_{AB,B}$ the distance between A and B measured by B.

In order to justify Eq. (3), some physicists use a Minkowski diagram similar to Fig. 2A to illustrate how the two observers obtain different values for the distance between A and B. The worldline of observer A is represented by the line AD and its line of simultaneity at A is represented by the line AB. The lines BD and BC represent the worldline of observer B and her line of simultaneity at B respectively. It can be shown with standard approaches in textbooks for deriving length contraction (Kittel, Knight and Ruderman 1973; Schutz 2009) that the distance $d_{AB,B}$ (i.e. AB measured in frame B) is shorter than $d_{BA,A}$ (i.e. AB measured in frame A) in Fig.2A and $d_{AB,B} = d_{BA,A}\sqrt{1 - v^2/c^2}$.

According to those physicists, that the values of the distance measured by the two observers are different is a consequence of Einstein's two postulates taken together, along with Einstein's redefinition of the concept of distant simultaneity, such that spatial distances are frame-relative quantities. However, the diagram in Fig.2A is drawn with observer A's frame being the rest frame. Since there is no privileged frame in special relativity, observer B's frame has the same right as observer A's frame to be the "rest" frame. Eq. (4) is about the velocity, time and distance in the muon frame, therefore we need to show $d_{AB,B} = d_{BA,A}\sqrt{1 - v^2/c^2}$ in observer B's frame as well when B's frame is the rest frame.

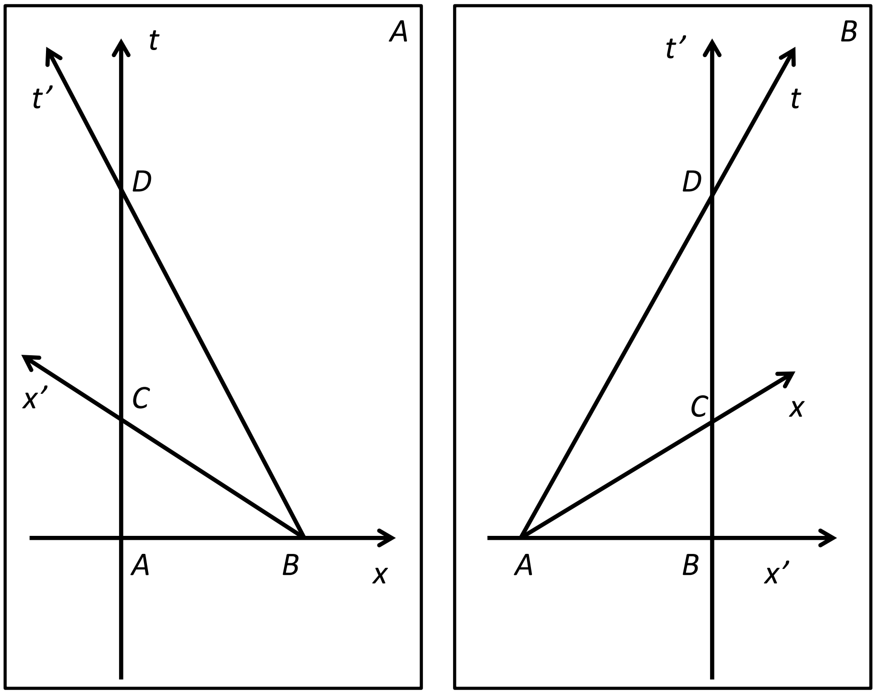

Fig.2 Minkowski diagrams representing the worldlines of observers A and B. Observer A's worldline is along the *t*-axis and its line of simultaneity at its time 0 (origin) is line AB in Fig.2A and line AC in Fig.2B. Observer B's wordline is along the *t′*-axis and its line of simultaneity at its time 0 (origin) is line BC in Fig.2A and line AB in Fig.2B.

Fig.2B is drawn with observer B's frame being the rest frame. Given that Fig.2A and Fig.2B are symmetric, $d_{BA,A}$ must be shorter than $d_{AB,B}$ in Fig.2B. With the same reasoning of those relativistic physicists, it is easy to show $d_{BA,A} = d_{AB,B}\sqrt{1 - v^2/c^2}$ in Fig.2B with standard approaches in textbooks for deriving length contraction. If $d_{BA,A}$ in Fig.2A has the same value as $d_{BA,A}$ in Fig.2B, the velocity of the earth in the muon frame would be

$$v_{earth,muon} = \frac{d_{AB,muon}}{t_{AB,muon}} = \frac{d_{BA,earth}}{t_{AB,muon}\sqrt{1-v^2/c^2}} > \frac{1907}{2.2\times10^{-6}\cdot\sqrt{1-0.994^2}} = 7.92466\times10^9 \quad \text{(m/s)}$$

This is more than 20 times faster than the speed of light. Using the reasoning and logic of those relativistic physicists, we have obtained an even more un-relativistic superluminal velocity than Eq. (6). Those physicists who insist on only using observer A's frame as the rest frame are not Einsteinian relativists. They are Lorentzian relativists, even though they think they are Einsteinian relativists. For true Einsteinian relativists, observer B's frame has the same right to be the rest frame as observer A's frame, because there is no privileged frame in the Einsteinian special relativity.

In the preceding analysis there are still three issues to be verified: 1) whether a distance measured in one reference frame can be calculated directly from the distance measured in another reference frame by using Lorentz length contraction formula or not; 2) whether $d_{AB,B}$ measured when observer A's frame is the rest frame has the same value as $d_{AB,B}$ measured when observer B's frame is the rest frame; and 3) what the true relationship between $d_{BA,A}$ measured in observer A's frame and $d_{AB,B}$ measured in observer B's frame is. As we have shown in the preceding paragraphs that calculating a distance in a reference frame from the distance measured in another reference frame could lead to conflicting results, the answer to the first question is probably negative. We will come back to the first issue again later and for now we examine the second and the third question.

To avoid any confusion caused by ambiguity in notation, we will rewrite $d_{BA,A}$ and $d_{AB,B}$ in Fig.2A as $d_{BA,A}^{A}$ and $d_{AB,B}^{A}$ respectively, and rewrite $d_{BA,A}$ and $d_{AB,B}$ in Fig.2B as $d_{BA,A}^{B}$ and $d_{AB,B}^{B}$ respectively (Ma 2014). The distance $d_{BA,A}^{A}$ is the distance between A and B measured by A when A's frame is the rest frame. The distance $d_{AB,B}^{A}$ is the distance between A and B measured by B when A's frame is the rest frame, so it might be considered to be observer A's observation of observer B's measurement of the distance between A and B. The distance $d_{AB,B}^{B}$ is the distance between A and B measured by B when B's frame is the rest frame. The distance $d_{BA,A}^{B}$ is the distance between A and B measured by A when B's frame is the rest frame, so it might be considered to be observer B's observation of observer A's measurement of the distance between A and B.

For the second question, if $d_{AB,B}^{A}$ has the same value as $d_{AB,B}^{B}$, by symmetry of the two diagrams in Fig.2A and Fig.2B and non-existence of privileged frames in the Einsteinian special relativity, the distance $d_{BA,A}^{A}$ should have the same value as $d_{BA,A}^{B}$,

$$d_{AB,B}^{A} = d_{AB,B}^{B} , \tag{7}$$

$$d_{BA,A}^{A} = d_{BA,A}^{B} . \tag{8}$$

From Fig.2A we know $d_{BA,A}^{A} > d_{AB,B}^{A}$, if $d_{AB,B}^{A} = d_{AB,B}^{B}$, we must have

$$d_{BA,A}^{A} > d_{AB,B}^{B} . \tag{9}$$

From Fig.2B, however, we know $d_{AB,B}^{B} > d_{BA,A}^{B}$, if $d_{BA,A}^{B} = d_{BA,A}^{A}$, we must have

$$d_{AB,B}^{B} > d_{BA,A}^{A} . \tag{10}$$

The two inequalities (9) and (10) contradict to each other, so for logical consistency $d_{BA,A}^{A} > d_{AB,B}^{B}$ and $d_{BA,A}^{A} < d_{AB,B}^{B}$ cannot be both true. Therefore, we cannot assume $d_{AB,B}^{B} = d_{AB,B}^{A}$ for the distance between two inertial observers A and B in relative motion, and for logical consistency we must have

$$d_{AB,B}^{B} \neq d_{AB,B}^{A} . \tag{11}$$

Fig.2A can only show that in the Minkowski diagram drawn from the perspective of observer A the distance $d^A_{BA,A} > d^A_{AB,B}$, which does not mean that $d^A_{BA,A} > d^B_{AB,B}$. Then we need to answer the third question, what the true relationship between $d^A_{BA,A}$ and $d^B_{AB,B}$ is.

According to the Einsteinian special relativity, the space-time interval in the Minkowski space is invariant, that is, the space-time interval is independent of the inertial reference frame chosen (Schutz 2009; Minkowski 1909; Landau and Lifshitz 1980). Therefore, the space-time interval between A and B is constant in all the inertial reference frames.

The space-time interval between A and B is

$$s_{AB}{}^2 = \left(x^A_{B,A} - x^A_{A,A}\right)^2 + \left(y^A_{B,A} - y^A_{A,A}\right)^2 + \left(z^A_{B,A} - z^A_{A,A}\right)^2 - c^2\left(t^A_{B,A} - t^A_{A,A}\right)^2$$

$$= (x'^B_{B,B} - x'^B_{A,B})^2 + (y'^B_{B,B} - y'^B_{A,B})^2 + (z'^B_{B,B} - z'^B_{A,B})^2 - c^2(t'^B_{B,B} - t'^B_{A,B})^2 \ . \tag{12}$$

In Eq. (12), $x^A_{*,A}$, $y^A_{*,A}$, $z^A_{*,A}$ and $t^A_{*,A}$ are space-time coordinates in the frame where observer A is stationary and A's frame is the rest frame in the Minkowski diagram (Fig.2A); $x'^B_{*,B}$, $y'^B_{*,B}$, $z'^B_{*,B}$ and $t'^B_{*,B}$ are space-time coordinates in the frame where observer B is stationary and B's frame is the rest frame in the Minkowski diagram (Fig.2B). The stars in the subscripts represent positions A or B, and from now on *t* will represent time coordinates (time points) rather than time intervals. A time interval will be indicated by Δ*t*.

Since $y^A_{*,A}$, $z^A_{*,A}$, $y'^B_{*,B}$ and $z'^B_{*,B}$ are all zero in the setup of Fig.2A and Fig.2B, Eq. (12) can be simplified to

$$s_{AB}{}^2 = \left(x^A_{B,A} - x^A_{A,A}\right)^2 - c^2\left(t^A_{B,A} - t^A_{A,A}\right)^2 = (x'^B_{B,B} - x'^B_{A,B})^2 - c^2(t'^B_{B,B} - t'^B_{A,B})^2 \tag{13}$$

In Fig.2A and Fig.2B, both $t^A_{*,A}$ and $t'^B_{*,B}$ are all zero, so Eq. (13) leads to

$$\left(x^A_{B,A} - x^A_{A,A}\right)^2 = (x'^B_{B,B} - x'^B_{A,B})^2 \, . \tag{14}$$

Since

$$x^A_{B,A} - x^A_{A,A} = d^A_{BA,A} \, ,$$

$$x'^B_{B,B} - x'^B_{A,B} = d^B_{AB,B} \, , \tag{15}$$

therefore,

$$d^B_{AB,B} = d^A_{BA,A} \, . \tag{16}$$

This proves that according to the Einsteinian special relativity, the distance between A and B measured by observer B is equal to the distance between A and B measured by observer A. We must point it out here that this result does not depend on the answer to the second question. It is derived from only the invariance of space-time intervals in the Minkowski space. It is a direct consequence of the invariance of space-time intervals.

Coming back to the first question, we can see that the distance between A and B measured by B when B's frame is the rest frame cannot be calculated from that measured by A when A's frame is the rest frame. The relationship between the two is determined to be equal by the invariance of space-time intervals in the Minkowski space. We can also think that the principle of relativity should also insure the equality between the two measurements; otherwise, there would be a privileged reference frame.

Our analysis in this section demonstrates that there should be four values of the distance between observers A and B in the Einsteinian special relativity: the value measured by observer A, $d_{BA,A}^{A}$; the value measured by observer B, $d_{AB,B}^{B}$; observer A's observation of observer B's measurement, $d_{AB,B}^{A}$; and observer B's observation of observer A's measurement, $d_{BA,A}^{B}$. Because of the lack of adequate notation to clearly define what a symbol represents, most discussions so far that allegedly use Minkowski diagram have failed to differentiate $d_{AB,B}^{B}$ from $d_{AB,B}^{A}$, i.e. observer B's measurement of the distance between A and B from observer A's observation of observer B's measurement. Many relativity physicists have mixed up $d_{AB,B}^{B}$ with $d_{AB,B}^{A}$, claiming that $d_{AB,B}^{B}$ is $d_{AB,B}^{A}$.

In the LET, time dilation and length contraction are physical or materialistic effects; for any one clock, the observers moving relative to the ether frame and the observers stationary in the ether frame have the same reading on that clock after taking signal transmission into account. In Einsteinian special relativity, time dilation and length contraction are observational effects due to the constancy of the speed of light. The time reading by observer A of a clock stationary in observer A's frame is different from the observation of that same clock by observer B who is moving relative to observer A's frame and considering herself as the rest frame. Since the Einsteinian relativistic effects are observational rather than materialistic, it is not really paradoxical that observer A finds observer B's measurement being shorter than observer A's, $d_{BA,A}^{A} > d_{AB,B}^{A}$; while observer B finds observer A's measurement being shorter than observer B's, $d_{AB,B}^{B} > d_{BA,A}^{B}$.

4. Time Readings by the Muon Observers and the Earth Observers

The standard explanation based on the Einsteinian special relativity for the increased lifetime of high speed unstable elementary particles is that, the mean lifetime of muons at high speed measured by the clocks co-moving with the muons is still 2.2 μs, which is the proper lifetime of muons,

$$\Delta t_{muon,muon} = \Delta t_{earth,earth} \,. \tag{17}$$

In Eq. (17), $\Delta t_{muon,muon}$ is the lifetime of the high speed moving muons measured by the clocks co-moving with the muons; $\Delta t_{earth,earth}$ is the lifetime of muons "at rest" on the earth measured by the clocks stationary on the earth.

The clocks stationary on the earth will record an increased lifetime of the high speed muons due to time dilation,

$$\Delta t_{muon,earth} = \frac{\Delta t_{muon,muon}}{\sqrt{1-v^2/c^2}} \,. \tag{18}$$

In Eq. (18), $\Delta t_{muon,earth}$ is the lifetime of high speed moving muons measured by the clocks stationary on the earth.

According to this standard explanation, the clocks co-moving with the high speed muons will measure a much longer lifetime of the muons "at rest" in the earth frame, even though the mean lifetime of the muons "at rest" in the earth frame is still 2.2 μs as measured by the clocks stationary on the earth.

$$\Delta t_{earth,muon} = \frac{\Delta t_{earth,earth}}{\sqrt{1-v^2/c^2}} \,. \tag{19}$$

In Eq. (19), $\Delta t_{earth,muon}$ is the lifetime of muons "at rest" on the earth measured by the clocks co-moving with the high speed moving muons. In the preceding paragraphs, the phrase "muons at rest on the earth" is used for simplicity of description, as muons are not really at rest on the earth.

The aforementioned standard explanation on time readings of observers in the muon frame and the earth frame is consistent with the Einsteinian special relativity. According to special relativity, there is no medium of light in vacuum, nor is there privileged reference frame. Using the unambiguous notation in the preceding section, we can prove Eqs. (18) and (19) with invariance of space-time intervals in the Minkowski space. The time used for observers A and B to meet in Fig.2 is AD in observer A's frame and BD in observer B's frame. The invariance of space-time intervals implies

$$s_{AD}{}^2 = \left(x_{D,A}^A - x_{A,A}^A\right)^2 + \left(y_{D,A}^A - y_{A,A}^A\right)^2 + \left(z_{D,A}^A - z_{A,A}^A\right)^2 - c^2\left(t_{D,A}^A - t_{A,A}^A\right)^2$$

$$= (x'^B_{D,B} - x'^B_{A,B})^2 + (y'^B_{D,B} - y'^B_{A,B})^2 + (z'^B_{D,B} - z'^B_{A,B})^2 - c^2(t'^B_{D,B} - t'^B_{A,B})^2 \ , \tag{20}$$

$$s_{BD}{}^2 = \left(x_{D,A}^A - x_{B,A}^A\right)^2 + \left(y_{D,A}^A - y_{B,A}^A\right)^2 + \left(z_{D,A}^A - z_{B,A}^A\right)^2 - c^2\left(t_{D,A}^A - t_{B,A}^A\right)^2$$

$$= (x'^B_{D,B} - x'^B_{B,B})^2 + (y'^B_{D,B} - y'^B_{B,B})^2 + (z'^B_{D,B} - z'^B_{B,B})^2 - c^2(t'^B_{D,B} - t'^B_{B,B})^2 \ . \tag{21}$$

Since $y_{*,A}^A$, $z_{*,A}^A$, $y'^B_{*,B}$ and $z'^B_{*,B}$ are all zero in the setup of Fig.2A and Fig.2B, Eqs. (20) and (21) can be simplified to

$$\left(x_{D,A}^A - x_{A,A}^A\right)^2 - c^2\left(t_{D,A}^A - t_{A,A}^A\right)^2 = (x'^B_{D,B} - x'^B_{A,B})^2 - c^2(t'^B_{D,B} - t'^B_{A,B})^2 \ , \tag{22}$$

$$\left(x_{D,A}^A - x_{B,A}^A\right)^2 - c^2\left(t_{D,A}^A - t_{B,A}^A\right)^2 = (x'^B_{D,B} - x'^B_{B,B})^2 - c^2(t'^B_{D,B} - t'^B_{B,B})^2 \ . \tag{23}$$

Since $x_{D,A}^A = x_{A,A}^A$ and $x'^B_{D,B} = x'^B_{B,B}$,

$$-c^2\left(t_{D,A}^A - t_{A,A}^A\right)^2 = (x'^B_{B,B} - x'^B_{A,B})^2 - c^2(t'^B_{D,B} - t'^B_{A,B})^2 \ , \tag{24}$$

$$\left(x_{A,A}^{A}-x_{B,A}^{A}\right)^{2}-c^{2}\left(t_{D,A}^{A}-t_{B,A}^{A}\right)^{2}=-c^{2}(t'^{B}_{D,B}-t'^{B}_{B,B})^{2} \quad . \tag{25}$$

For A and B to meet, $(x'^{B}_{B,B}-x'^{B}_{A,B})^{2}=(d_{AB,B}^{B})^{2}=v^{2}(t'^{B}_{D,B}-t'^{B}_{A,B})^{2}=v^{2}(\Delta t'^{B}_{DA,B})^{2}$,

$$-c^{2}\left(\Delta t_{DA,A}^{A}\right)^{2}=v^{2}(\Delta t'^{B}_{DA,B})^{2}-c^{2}(\Delta t'^{B}_{DA,B})^{2} \quad ,$$

$$\Delta t'^{B}_{DA,B}=\frac{\Delta t_{DA,A}^{A}}{\sqrt{1-v^{2}/c^{2}}} \tag{26}$$

In Eq. (26), $\Delta t'^{B}_{DA,B}$ is the time interval DA in frame A measured by the clocks in frame B; $\Delta t_{DA,A}^{A}$ the time interval in frame A measured by the clocks in frame A. Because of time dilation, $\Delta t'^{B}_{DA,B}$ is longer than $\Delta t_{DA,A}^{A}$.

Similarly, $\left(x_{A,A}^{A}-x_{B,A}^{A}\right)^{2}=(d_{BA,A}^{A})^{2}=v^{2}\left(t_{D,A}^{A}-t_{B,A}^{A}\right)^{2}=v^{2}(\Delta t_{DB,A}^{A})^{2}$,

$$v^{2}(\Delta t_{DB,A}^{A})^{2}-c^{2}\left(\Delta t_{DB,A}^{A}\right)^{2}=-c^{2}(\Delta t'^{B}_{DB,B})^{2} \quad .$$

$$\Delta t_{DB,A}^{A}=\frac{\Delta t'^{B}_{DB,B}}{\sqrt{1-v^{2}/c^{2}}} \quad . \tag{27}$$

In Eq. (27), $\Delta t_{DB,A}^{A}$ is the time interval DB in frame B measured by the clocks in frame A; $\Delta t'^{B}_{DB,B}$ the time interval in frame B measured by the clocks in frame B. Because of time dilation, $\Delta t_{DB,A}^{A}$ is longer than $\Delta t'^{B}_{DB,B}$.

Using symmetry between DA in Fig.2A and DB in Fig.2B as well as $d_{AB,B}^{B}=d_{BA,A}^{A}$, we can see that

$$\Delta t_{DA,A}^{A}=\Delta t'^{B}_{DB,B} \tag{28}$$

That is, when there is no privileged frame, the time interval of events in frame A measured by the clocks in frame A is equal to the time interval of events in frame B measured by the clocks in frame B. Eq. (28) is a more precise illustration of Eq. (17), which is a fundamental conclusion of the Einsteinian special relativity. Since $v_{BA,A} = v_{AB,B}$ according to the Einsteinian special relativity as well as the principle of relativity, Eq. (28) can also be derived directly from $d_{AB,B}^{B} = d_{BA,A}^{A}$.

5. Superluminal Velocity and a Velocity Paradox

Now we have clarified two key issues on determining the velocity of the earth measured in the muon frame: 1) the distance between observers A and B measure by A in A's frame is the same as the distance between them measure by B in B's frame; 2) the lifetime of high speed muons measured by clocks co-moving with them is their proper lifetime, which is about 2.2 μs. With these two results, it is easy to see that Eq. (6) is the logical outcome of the relativistic time dilation and the invariance of the space-time interval in the Minkowski space,

$$v_{earth,muon} = \frac{d_{AB,muon}}{\Delta t_{AB,muon}} \geq \frac{d_{AB,muon}}{\Delta t_{muon,muon}} = \frac{1907}{2.2\times10^{-6}} = 8.668 \times 10^{8}\ m/s\ .$$

The velocity of the earth relative to the muons as measured by the muon frame is nearly three times the speed of light *c*. From this result, the standard interpretation on the increased lifetime of high speed elementary particles violates the constancy of the speed of light in the muon frame. We have a superluminal velocity of the earth in the muon frame. This result suggests that the Einsteinian special relativity, which requires the constancy of the one-way speed of light, is not logically consistent, at least in the interpretation of time dilation in high speed muons. The LET, which does not require the constancy of the one-way speed of light, is therefore superior to the Einsteinian special relativity in terms of logical consistency.

The present study also reveals a velocity paradox in the Einsteinian special relativity. The Einsteinian special relativity maintains that velocity between two inertial frames A and B is the same for A and B,

$$v_{BA,A} = v_{AB,B} \; . \tag{29}$$

In Eq. (29), $v_{BA,A}$ is the velocity of B relative to A measured by A, and $v_{AB,B}$ the velocity of A relative to B measured by B. This basic assumption of special relativity is a logical consequence of the principle of relativity when there is no privileged reference frame. Applying Eq. (29) to the scenario of the earth and high speed muons, we must have for the earth and the muons

$$v_{earth,muon} = v_{muon,earth} \; . \tag{30}$$

If the velocity of muons measured by the observers on the earth is 0.994 c, the velocity of the earth measured by the observers co-moving with the muons should also be 0.994 c. Eq. (30) has been taken for granted in all expositions of Einsteinian special relativity.

Eqs. (6) and (30) give different values of the velocity between the two reference frames. The clocks co-moving with the muons indicate a mean lifetime of 2.2 μs, which is assumed by the standard explanation based on the Einsteinian special relativity. The distance between the initial positions of the earth and muons measured in the muon frame is the same as that measured in the earth frame, i.e. 1907 m, because of the invariance of the space-time interval in the Minkowski space. The velocity of the earth calculated with the distance divided by the time measured in the muon frame, $v > 2.891c$, is different from the velocity of earth in the muon frame derived from the principle of relativity, Eq. (30), $v = 0.994c$. Therefore, the Einsteinian special relativity will lead to paradoxical predictions of the velocity of the earth in the muon frame. This shows again that Einsteinian special relativity is not logically consistent, at least in the interpretation of time dilation. The LET is superior to the Einsteinian special relativity, at least in explaining time dilation.

Some relativity physicists might use the constancy of the speed of light to dismiss Eq. (6) and conclude that Eq. (30) is the correct description of the velocity measured by the observers co-moving with the muons and that the distance between observers A and B measured in B's frame is different from that measured in A's frame. Since the Einsteinian special relativity requires to maintain the proper mean lifetime of muons and the constancy of the speed of light, the distance between the earth and the muons measured by the observers co-moving with the muons has to contract according to the length contraction formula,

$$d_{AB,muon}^{muon} = d_{BA,earth}^{earth}\sqrt{1 - v^2/c^2}\ , \tag{31}$$

whereas the distance measured by the observers on the earth does not contract,

$$d_{BA,earth}^{earth} = \frac{d_{AB,muon}^{muon}}{\sqrt{1-v^2/c^2}}\ . \tag{32}$$

In Eqs. (31) and (32), the superscripts indicate the "rest" frame.

We have shown already in Section 3 that Eq. (31) cannot be true if invariance of the space-time interval in the Minkowski space is not violated. In Eqs. (31) and (32), the earth frame and the muon frame are obviously not equal, and one is more privileged than the other. The earth frame or the muon frame has become a more privileged frame in the standard interpretation of the increased lifetime of high speed elementary particles, which is supposedly the Einsteinian relativistic explanation. Because there is no privileged reference frame in the Einsteinian special relativity, the standard explanation alleging Eqs. (31) and (32) is not consistent with the Einsteinian special relativity.

6. Discussions

In the present study we have shown that because of the invariance of space-time intervals across inertial reference frames, the distance between observers A and B measured in observer A's frame ($d_{BA,A}^{A}$) must be equal to the distance between A and B measured in observer B's frame ($d_{AB,B}^{B}$). Given $d_{AB,B}^{B} = d_{BA,A}^{A}$, the conclusion by the Einsteinian special relativity that the lifetime of high speed muons measured by observers co-moving with these muons is the same as that of "rest muons" on the earth would lead to a superluminal velocity of the earth measured in the muon frame. The usual solution for the superluminal dilemma in explaining the increased lifetime of high speed particles is to insist that for the muons and their co-moving observers the distance between the earth and the muons is not 1907 m; instead the distance is shorter than 660 m because of the length contraction effect in the Einsteinian special relativity. The present study shows that such a view contradicts the invariance of space-time intervals in the Minkowski space which is fundamental in the Einsteinian special relativity. It also contradicts another basic conclusion of the Einsteinian special relativity, the non-existence of privileged inertial reference frames. When there is no privileged frame, if the distance measured in the muon frame is shorter than 660 m, so should be the distance measured in the earth frame. Therefore, making the two reference frames unequal will violate the principle of relativity and the invariance of space-time intervals in the Minkowski frame.

The following argument might be used by some relativity physicists to justify Eqs. (31) and (32) and $d_{earth-muon,muon}^{muon} \neq d_{muon-earth,earth}^{earth}$: "The distance from the muons to the observers on the ground can be viewed as a giant stationary measuring rod of 1907 m to the observers on the earth. Because the muons and their co-moving observers travel at a speed close to the speed of light (0.994c), the giant measuring rod will have length contraction according to special relativity, becoming shorter than 660 m. Therefore, the average lifetime of those muons as measured by their co-moving observers is still 2.2 μs, and the speed of the earth relative to the muons measured by the co-moving observers is still smaller than c."

This explanation contradicts the Einsteinian special relativity by creating a privileged reference frame and violating the invariance of space-time intervals in the Minkowski space. As velocity is relative, the distance from the earth to the muons can also be viewed as a giant measuring rod stationary to the muons, and the observers on the earth travel at 0.994*c* towards the muons. The observers on the earth will find that the giant measuring rod contracts according to the Einsteinian special relativity, so the distance of 1907 m they measured is also an outcome of length contraction. When the observers on the earth measure a contracted distance of 1907m, the observers co-moving with the muons should also measure a contracted distance of 1907m. Therefore, if muons and observers on the earth obtain different values for the distance between them, the principle of relativity and the invariance of the space-time interval in the Minkowski space would be violated.

The present study has proved that two inertial observers A and B in relative motion will obtain the same value in measuring the distance between them. Given that Einsteinian special relativity rejects any privileged inertial reference frame and that the space-time interval is frame-independent, the present result is logically obvious and natural. From the non-existence of privileged frame, the Minkowski diagram depicted in Fig.2B has the same status as that depicted in Fig.2A. With the two Minkowski diagrams, the frame-independence of space-time intervals in the Minkowski space leads directly to the conclusion that the distance between A and B measured by observer B when B's frame is the "rest" frame has the same value as the distance between A and B measured by observer A when A's frame is the "rest" frame.

The view that the distance measured by observer B is shorter than that measured by observer A, $d_{AB,B}^{B} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$, is obviously contradictory to the Einsteinian special relativity that rejects the existence of privileged inertial reference frames. If the distance between A and B measured by observer B, $d_{AB,B}^{B}$, is shorter than that measured by observer A, $d_{BA,A}^{A}$, the two reference frames would be unequal. Therefore, people supporting $d_{AB,B}^{B} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$ actually advocate an interpretation that designates one frame being more privileged than the other.

The present study shows that it is not possible to have two reference frames unequal within the framework of the Einsteinian special relativity. The equality between the two frames and the frame-independence of space-time intervals in the Minkowski space ensure the equality between the values of the distance measured by A and B. Since Lorentz ether theory can also explain relativistic phenomena and there is a privileged ether frame in Lorentz ether theory, it might provide a mechanism for $d_{AB,B}^{B} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$ to be true. People insisting on $d_{AB,B}^{B} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$ are unwitting supporters of Lorentz ether theory, despite their stated support for the Einsteinian special relativity.

The present study suggests that the physics community should adopt the Lorentz ether theory because of its superiority in explaining the increased lifetime of high speed elementary particles. Replacing the Einsteinian special relativity with the Lorentz ether theory will result in no substantial changes in physics except removing real or apparent paradoxes in physics, because many perceived Einsteinian interpretations of physical phenomena such as time dilation are implicitly and unwittingly Lorentzian. The present study also shows that philosophy of science can play an important role in the development of modern physics by investigating the logical consistency in the reasoning of physics studies.

7. Conclusions

From the present study we can draw following conclusions:

1) From the non-existence of privileged inertial reference frames in the Einsteinian special relativity and the frame-independence of space-time intervals in the Minkowski space, the distance between two inertial observers A and B measured by A is the same as that measured by B, $d_{AB,B}^{B} = d_{BA,A}^{A}$.

2) Given $d_{AB,B}^{B} = d_{BA,A}^{A}$, the Einsteinian view that the lifetime of high speed particles measured by observers co-moving with those particles is the same as their proper lifetime at rest would leads to superluminal velocities of the earth in the muon frame, violating the constancy of the speed of light, which is one of the two fundamental postulates of special relativity. Therefore, the Einsteinian special relativity is logically inconsistent in its interpretation of the increased lifetime of high speed elementary particles.

3) Many physicists might try to avoid the superluminal velocity in the Einsteinian explanation of time dilation by insisting that the distance between two inertial observers A and B measured by B is shorter than that measured by A with a relationship $d_{AB,B}^{B} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$. This view is wrong, because it violates invariance of space-time intervals in the Minkowski space.

4) The view $d_{AB,B}^{B} = d_{BA,A}^{A}\sqrt{1 - v^2/c^2}$ does not conform to the Einsteinian special relativity neither, because it treats A and B unequally so that one of them is more privileged than the other.

5) The LET would not lead to this velocity paradox and other real or apparent paradoxes suffered by the Einsteinian special relativity, therefore, it is superior to the Einsteinian special relativity at least in the interpretation of the increased lifetime of high speed elementary particles.

6) Philosophy of science can have an important role in the development of modern physics by investigating the logical consistency in the reasoning of physics studies.

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