NEUTROSOPHIC MAGDM BASED ON ENTROPIES OF DEGREES α, β AND R-NORM

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Abstract: Yager [1988] developed the ordered weighted averaging (OWA) operator and applied in decision making problems. Xu & Yager [2006] developed some geometric aggregation operators, such as the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy hybrid geometric (IFHG) operator and gave an application of the IFHG operator to multiple attribute group decision making with intuitionistic fuzzy information. Xu [2007e] and Xu & Chen [2007a, 2007b] also developed some arithmetic aggregation operators for decision making problems. One important issue in the theory of ordered weighted averaging (OWA) operators is the determination of the associated weights. One of the first approaches, suggested in the literature is a special class of OWA operators having maximal Shannon entropy of the OWA weights for a given level of orness; algorithmically it is based on the solution of a constrained optimization problem. The MAGDM problems have investigated under neutrosophic fuzzy environment, and proposed an approach to handling the situations where the attribute values are characterized by NFSs, and the information about attribute weights completely unknown. The proposed approach first fuses all individual neutrosophic fuzzy decision matrices into the collective neutrosophic fuzzy decision matrix by using the NFOWA operator. Then the obtained attribute weights and the NFHA operator have used to get the overall neutrosophic fuzzy values of alternatives   In this paper, the proposed approach in this work not only can comfort the influence of unjust arguments on the decision results, but also avoid losing or distorting the original decision information in the process of aggregation. Thus, the proposed approach provides us an effective and practical way to deal with multi-person multi-attribute decision making problems, where the attribute values are characterized by NFSs and the information about attribute weights is partially known. The suitable alternative is selected through the algorithm from the given neutrosophic information in which the unknown weights are derived based upon normal distribution.

Section 1 - Previous Literatures:

The interval-valued intuitionistic fuzzy sets (IVIFSs), introduced by Atanassov & Gargov[1989] which is characterized by a membership function and a non-membership function whose values are rather than exact numbers, are a very useful means to describe the decision information in the process of decision making. Wei & Wang [2007] respectively, developed some geometric aggregation operators and applied them to MAGDM with interval-valued intuitionistic fuzzy information. In this work, based on the one important issue in the theory of ordered weighted averaging (OWA) operators suggested by O'Hagan, (1988) a special class of OWA operators having maximal entropy of the OWA weights for a given level of orness is utilized. Using the method of Lagrange multipliers.Li [1999] solved the constrained optimization problem of OWA operators having maximal entropy analytically and derived a polynomial equation which is then solved to determine the optimal weighting vector. Also MAGDM problem is investigated in which all the information provided by the decision-makers is presented as interval valued intuitionistic fuzzy decision matrices where each of its elements is characterised by interval valued intuitionistic fuzzy number (IVIFN).

Park et al. [2009] proposed an ordered weighted geometric (OWG) model to aggregate all individual interval valued intuitionistic fuzzy decision matrices provided by the decision makers into the collective interval valued intuitionistic fuzzy decision matrix. In the proposed model, from the maximal entropy attribute weight information, an optimization model is established to determine the unknown weights. Then

the obtained attribute weights and the operators are used to fuse the interval valued intuitionistic fuzzy information in the collective interval valued intuitionistic fuzzy decision matrix to get the overall Interval Valued Intuitionistic Fuzzy values of the alternatives. A MAGDM model based on the maximal entropy weights [Li, 1999] is presented for computing the attributes weights, and a numerical illustration is given.

Some definitions are given first;

Definition 1.1: An OWA operator of dimension n is a mapping F: $R^n \rightarrow R$ that has an associated weighting vector W = $(w_1, w_2,…, w_n)^T$ with $w_1 + w_2 +…+ w_n = 1$; $0 \leq w_i \leq 1$ for all i varying from 1 to n, and such that F($a_1, a_2,…, a_n$) = $\sum_{i=1}^{n} w_i b_i$ where b_j is the jth largest element of the collection of the aggregated objects {a_1, $a_2,…, a_n$}. Yager [1988] introduced two characterizing measures associated with the weighting vector *W* of an OWA operator.

Definition 1.2: The first one, the measure of orness of the aggregation, is defined as:Orness (W) = (1 / (n-1)) $\sum_{i=1}^{n}(n - i)w_i$ and it characterizes the degree to which the aggregation is like an **or** operation. It is clear that *orness(W)* $\in$ [0,1] holds for any weighting vector.

Definition 1.3: The second one, the measure of dispersion of the aggregation, is defined as disp (W) = (-) $\sum_{i=1}^{n} w_i (in \, w_i)$, and it measures the degree to which *W* takes into account all information in the aggregation.

Definition 1.4: The classical measure of uncertainty has been dominating the literature of information theory since its appearance. It is the same as the measure of dispersion up to a positive constant multiplier, and then H_S(W) = (-) $\sum_{i=1}^{n} w_i (log_2 w_i)$. This is called the Shannon entropy. In the literature there have been described several classes of entropies each including the Shannon entropy as a special case. They include:

Definition 1.5: (Entropy of degree α):H_α(also called entropies of degree α) is defined as follows: H_α(W) = (1/ (1- α)) $log_2 (\sum_{i=1}^{n} w_i^{\alpha})$for all real numbers α ≠ 1

Definition 1.6: (Entropy of order β): Then H_β is introduced in the following form H_α(W) = $\frac{1}{2^{1-\beta}}(\sum_{i=1}^{n} w_i^{\beta} - 1)$for all β ≠ 1.

Definition 1.7: (Entropy of R-norm):H_R defined by the following formula:H_R(W) = (R / (R-1)) (1- $(\sum_{i=1}^{n} w_i^{R})^{\frac{1}{R}}$)for all $R \neq 1$.It is well known that H_R(W) = $\lim_{\alpha \to 1} H_\alpha(w)$ = $\lim_{\beta \to 1} H_\beta(w)$ = $\lim_{R \to 1} H_R(w)$.Hence it is clear that the actual type of aggregation performed by an OWA operator depends upon the form of the weighting vector. In this paper, using the method of the α-entropy (with parameter value 2), β-entropy (when parameter β is 2) and the 2-norm entropy (from entropy class R-norm entropies), weights are determined for the aggregation process in MAGDM problems.

Section 2-Maximalα-entropy, β-entropy and *R*-norm - entropies weights:

Introduction 2.1: In this section the maximal α-entropy and *R*-norm entropy weights are derived when their parameter values equal to 2.

$$H_\alpha(W) = (-) \, log_2 (\textstyle\sum_{i=1}^{n} w_i^{2}) \text{ if } \alpha = 2;$$

$$H_\beta(W) = (-2) \, (\textstyle\sum_{i=1}^{n} w_i^{2} - 1) \text{ if } \beta = 2; \text{ and}$$

$$H_R(W) = (-2) \, (1 - (\textstyle\sum_{i=1}^{n} w_i^{2})^{\frac{1}{2}}) \text{ if } R = 2.$$

Problem 2.2: Therefore determining a special class of OWA operators having maximal entropy of the OWA weights for a given level of orness is based on the solution of the following mathematical programming problem

Minimize $\sum_{i=1}^{n} w_i^{2}$

subject to $(1 / (n-1)) \sum_{i=1}^{n}(n-i)w_i = \alpha, \ 0 \le \alpha \le 1$;

$$w_1 + w_2 + \ldots + w_n = 1;$$

$$0 \le w_i \le 1 \text{ for all i varying from 1 to n } \ldots(1).$$

The solution is based on the use of the method Kuhn-Tucker multipliers and is rather complicated; hence we only refer to it. Notice that the objective function of this problem is strictly convex, which implies the unicity of the optimal solution. Solving this problem it could be found that the optimal solution is a window-type OWA operator. Thus there exists $1 \le k \le n$ such that $k \le i \le l \Leftrightarrow w_i \ne 0$.

Without loss of generality we can assume that $n \ge 3$ and $0 < \alpha \le \frac{1}{2}$.

If $\alpha = 1/2$, then $w_1 = \ldots = w_n = 1/n$ is the optimal solution to (1). Furthermore this is the global optimal solution of all OWA operators of dimension n. To obtain the optimal solution for arbitrary α in (0, 1/2).

Consider the following disjoint union of intervals of (0, 1/2),

$$(0, 1/2) = \bigcup_{j=1}^{n-1} I_j \text{ where } I_j = (\frac{j-1}{3(n-1)}, \frac{j}{3(n-1)}), \text{ j varies from 1 to (n-2)}$$

$$I_{n-1} = (\frac{n-2}{3(n-2)}, \frac{1}{2}).$$

Now, considering α, there uniquely exists $1 \le p \le (n-1)$ such that α is in I_P.

Let $r = (n-p)$. Then the optimal solution to (1) can be obtained as:

$w_i^* = 0, \ 1 \le i < r$;

$$w_r^* = \frac{6(n-1)\alpha - 2(n-r-1)}{(n-r+1)(n-r+2)} \quad \ldots(2).; \quad W_n^* = \frac{2(2n-2r+1)-6(n-1)\alpha}{(n-r+1)(n-r+2)} \quad \ldots(3).$$

$$w_i^* = \frac{(n-i)}{(n-r)} w_r^* + \frac{(i-r)}{(n-r)} w_n^* \qquad r < i < n. \qquad \ldots(4).$$

Example 2.3: Let us suppose that $n=5$ and $\alpha = 0.4$, then obtaining the maximal Shannon entropy weights we have to solve

$$w_i [4(0.6) + 1 - 5w_1]^5 = (4(0.6))^4 [4(0.6) -5)w_1 +1]$$

It finds that $\quad w_1^* = 0.1278; \ w_5^* = \frac{((4(0.4)-5)w_1^*+1)}{(4(0.4)+1-5)w_1^*} = 0.2884$

$$w_2^* = \sqrt[4]{(w_1^*)^3 w_5^*} = 0.1566; \ w_3^* = \sqrt[4]{(w_1^*)^2 (w_5^*)^2} = 0.1920.$$

$$w_4^* = \sqrt[4]{w_1^*(w_5^*)^3} = 0.2353 \text{ and } \text{disp}(W^*) = 1.5692.$$

Obtaining the maximal entropy weights of α-entropy, β-entropy and *R*-norm, it could be found that $(0, 1/2) = \bigcup_{j=1}^{4} I_j$ where $I_j = \left[\frac{j-1}{12}, \frac{j}{12}\right], j=1,2,3$ and $I_4 = \{1/4, \frac{1}{2})$.

Since α is in I_4, then *r* =1.

$w_1^* = [24 (0.4) -6] / 30 = 0.1200; \ w_5^* = [18 - 24 (0.4)] / 30 = 0.2800;$

$w_2^* = (3/4)w_1^* + (1/4) w_5^* = 0.1600; \ w_3^* = (1/2)w_1^* + (1/2) w_5^* = 0.2000;$

$w_4^* = (1/4)w_1^* + (3/4) w_5^* = 0.2400$; and the corresponding α-entropy, β-entropy and R-norm are 2.2109, 1.5680 and 1.0705, respectively.

Section 3 - Algorithm for MAGDM problem:

Definition 3.1: A neutrosophic fuzzy set A on the universe of discourse X characterized by a truth membership function $T_A(x)$, an indeterminacy function $I_A(x)$ and a falsity membership function $F_A(x)$ is defined as A = { < x, $T_A(x)$, $I_A(x)$, $F_A(x)$ > : x ∈ X }, where T_A , I_A ,F_A :X→ [0. 1] and 0 ≤$T_A(x)$≤1; 0 ≤$I_A(x)$ ≤1;0 ≤$F_A(x)$ ≤1, for all x ∈ X.

Procedures of steps for an algorithm V using IIFOWA operator 3.2:

Step 1: Utilize the NFOWA operator to aggregate all individual neutrosophic fuzzy decision matrices $R^{(k)}$ = < $(T_{ij}(k)$, $I_{ij}(k)$, $F_{ij}(k)$ > = ($r_{ij}^{(k)}$) (k varies from 1, 2,3, and 4) into a collective neutrosophic fuzzy decision matrix $R = (r_{ij})_{m \times n}$.

Step 2: Derive the weights by the entropy weights of α-entropy, β-entropy and R-norm by using $w_r^* = \frac{6(n-1)\alpha - 2(n-r-1)}{(n-r+1)(n-r+2)}$; $W_n^* = \frac{2(2n-2r+1)-6(n-1)\alpha}{(n-r+1)(n-r+2)}$; .

$w_i^* = \frac{(n-i)}{(n-r)} w_r^* + \frac{(i-r)}{(n-r)} w_n^*$; r ≤ i < n;

Step 3: Use the NFHA operator to get the overall values r_j of the alternatives O_j(j=1, 2,…..n).

Step: 4 Using r* = (1,0,0) = (T_A*, I_A*, F_A*), find d(r*, r_j) =

$\sqrt{\left(T_A^* - T_{j_A}\right)^2 + \left(I_A^* - I_{j_A}\right)^2 + \left(F_A^* - F_{j_A}\right)^2}$ to calculate the distances between informational neutrosophic values r_j= (TjA, IjA, FjA) = (T_{j_A}, I_{j_A}, F_{j_A}) (j = 1, 2 ,...., n).

Step 5: Rank the alternatives based on distances.

Step 6: Select the best alternative.

Example 3.3: Steps for the given problem.

STEP 1:   To derive a weight vector *w* by using entropy weights of α-entropy, β-entropy, and R-norm $w_i^* = 0$, 1 ≤ i < r; $w_r^* = \frac{6(n-1)\alpha - 2(n-r-1)}{(n-r+1)(n-r+2)}$; $W_n^* = \frac{2(2n-2r+1)-6(n-1)\alpha}{(n-r+1)(n-r+2)}$ $w_i^* = \frac{(n-i)}{(n-r)} w_r^* + \frac{(i-r)}{(n-r)} w_n^*$ r < i < n.

Assumption: n = 5; r = 1; α = 0.4;

$w_1^* = \frac{6(5-1)(0.4)-2(5-1-1)}{(5-1+1)(5-1+2)} = 0.12$; $w_5^* = \frac{2(2x5)-(2x1)+1)-6(5-1)(0.4)}{(5-1+1)(5-1+2)} = 0.28.$

$w_2^* = \frac{(5-2)}{(5-1)}w_1^* + \frac{(2-1)}{(5-1)}w_5^* = (3/4) (0.12) + (1/4)(0.28) = 0.16.$

$w_2^* = \frac{(5-3)}{(5-1)}w_1^* + \frac{(3-1)}{(5-1)}w_5^* = (2/4) (0.12) + (2/4)(0.28) = 0.20.$

$w_4^* = \frac{(5-4)}{(5-1)}w_1^* + \frac{(4-1)}{(5-1)}w_5^* = (1/4) (0.12) + (3/4)(0.28) = 0.24.$

Hence $w_1^* = 0.12$; $w_2^* = 0.16$; $w_3^* = 0.20$; $w_4^* = 0.24$; $w_5^* = 0.28$;

Step 2: Assume that the information in decision making are in neutrosophic fuzzy matrices as follows:

$$R^1 = \begin{bmatrix} <0.25,0.54,0.8> & <0.3,0.4,0.9> & <0.7,0.35,0.5> & <0.9,0.2,0.8> \\ <0.6,0.5,0.5> & <0.6,0.2,0.3> & <0.2,0.4,0.9> & <0.6,0.23,0.7> \\ <0.3,0.45,0.9> & <0.7,0.1,0.4> & <0.6,0.5,0.5> & <0.4,0.2,0.9> \\ <0.45,0.38,0.27> & <0.37,0.68,0.16> & <0.6,0.25,0.3> & <0.1,0.4,0.8> \end{bmatrix}$$

$$R^2 = \begin{bmatrix} <0.1,0.3,0.7> & <0.6,0.6,0.5> & <0.4,0.2,0.1> & <0.3,0.7,0.6> \\ <0.3,0.55,0.37> & <0.75,0.42,0.1> & <0.32,0.67,0.56> & <0.35,0.56,0.72> \\ <0.5,0.4,0.32> & <0.65,0.25,0.32> & <0.6,0.3,0.1> & <0.75,0.25,0.55> \\ <0.27,0.9,0.81> & <0.31,0.4,0.6> & <0.75,0.65,0.55> & <0.3,0.7,0.9> \end{bmatrix}$$

$$R^3 = \begin{bmatrix} <0.32,0.47,0.6> & <0.9,0.1,0.3> & <0.6,0.4,0.5> & <0.3,0.5,0.7> \\ <0.12,0.32,0.52> & <0.17,0.81,0.9> & <0.5,0.3,0.1> & <0.45,0.65,0.27> \\ <0.50,0.6,0.23> & <0.56,0.52,0.23> & <0.3,0.6,0.1> & <0.57,0.52,0.55> \\ <0.54,0.83,0.72> & <0.73,0.86,0.61> & <0.5,0.52,0.4> & <0.6,0.4,0.2> \end{bmatrix}$$

$$R^4 = \begin{bmatrix} <0.7,0.3,0.1> & <0.5,0.4,0.4> & <0.2,0.1,0.6> & <0.7,0.9,0.6> \\ <0.3,0.56,0.73> & <0.57,0.24,0.1> & <0.23,0.76,0.65> & <0.53,0.65,0.27> \\ <0.32,0.32,0.6> & <0.56,0.52,0.32> & <0.1,0.3,0.9> & <0.57,0.52,0.55> \\ <0.72,0.5,0.18> & <0.13,0.6,0.4> & <0.55,0.56,0.78> & <0.7,0.1,0.6> \end{bmatrix}$$

$$R^5 = \begin{bmatrix} <0.52,0.45,0.1> & <0.57,0.37,0.1> & <0.76,0.65,0.23> & <0.57,0.52,0.55> \\ <0.3,0.6,0.7> & <0.7,0.4,0.1> & <0.3,0.7,0.6> & <0.5,0.4,0.6> \\ <0.2,0.3,0.2> & <0.6,0.2,0.5> & <0.1,0.6,0.65> & <0.3,0.9,0.7> \\ <0.27,0.5,0.81> & <0.75,0.25,0.32> & <0.32,0.67,0.56> & <0.35,0.56,0.72> \end{bmatrix}$$

Step 3: Using the weights $w_1^* = 0.12$; $w_2^* = 0.16$; $w_3^* = 0.20$; $w_4^* = 0.24$; $w_5^* = 0.28$, the above fuzzy neutrosophic matrices are converted into a single matrix is as follows:

Step 4: Using the weights w = {0.2717, 0.2608, 0.2254, 0.2421} obtained from Poisson distribution.

New reduced row matrix R is [(0.4115, 0.5054, 0.5805), (0.6048, 0.4811, 0.4611),

(0.4538, 0.4676, 0.5207), (0.5332, 0.5570, 0.6153)]

Step 5: $d = \sqrt{\frac{1}{2}[\sum[(1-T)^2 + (0-I)^2 + (0-F)^2]]}$

d(r , r_1) = 0.6851 = A_1; d(r , r_2) = 0.5478 = A_2.

d(r , r_3) = 0.6455 = A_3; d(r , r_4) = 0.6737 = A_4..

Step 6: $A_1 > A_4 > A_3 > A_2$.

Step 7: A_1 is best alternative

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