

Primes of the form $p = 1 + n! \sum n$, for some $n \in \mathbb{N}^+$

By
Maheswara Rao Valluri

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Abstract

The purpose of this note is to report on the discovery of the primes of the form $p = 1 + n! \sum n$, for some natural numbers $n > 0$. The number of digits in the prime p are approximately equal to $\lfloor \log_{10}(1 + n! \sum n) \rfloor + 1$.

Key Words: Primes, factorials, sum of the first n natural numbers

AMS Subject Classification: 11A41,05A10,97F40

Introduction

A natural number is a prime if it has only factors of 1 and itself. There are, by Euclidean theorem (about 350BC) infinitely many primes. There are many patterns of primes amongst which the classical known primes are the Mersenne primes of the form $2^p - 1$, where p is a prime [Da2011], and the Fermat primes of the form $2^{2^n} + 1$ for a natural number $n \geq 0$ [Da2011]. We omit the other classes of primes except the factorial primes.

There was a known fact that early 12th century Indian scholars knew about factorials. In 1677, a British mathematician, Fabian Stedman described factorials for music. In 1808, a French mathematician, Christian Kramp introduced notation $!$ for factorials. The factorial of n can be described as product of all positive integers less than or equal to n . In the Christian Kramp's notation, $n! = n(n-1)(n-2)\dots$. 3.2.1. Factorials of 0 and 1 can be written as $0! = 1$ and $1! = 1$, respectively. There are dozens of prime factorials. However, we recall only few of them, particularly, factorial primes of the form $(p! \pm 1)$ [Bor1972, BCP1982], double factorial primes of the form $n!! \pm 1$ for some natural number n [Mes1948], Wilson primes: p for which p^2 divides $(p-1)! + 1$ [Bee1920], and primorial primes of the form $(p\# \pm 1)$ which means as the product of all primes up to and including the prime [Dub1987, Dub1989]. Further, a class of the Smarandache prime is of the form $n! \times S_n(n) + 1$, where $S_n(n)$ is smarandache consecutive sequence [Ear2005].

The purpose of this note is to report on the discovery of the primes of the form $p = 1 + n! \sum n$ for some natural numbers $n > 0$.

Primes of the form $p = 1 + n! \sum n$ for some $n \in \mathbb{N}^+$

We list in the table 1, the primes of the form $p = 1 + n! \sum n$ for some natural numbers $n > 0$. They are verified up-to $n = 950$ and primes are found when $n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 19, 24, 251, 374$. The above primes can also be expressed as $p = 1 + \frac{(n+1)!n}{2}$, for some natural numbers $n > 0$. The author has used python software to search and verify the above form primes and could verify up-to $n = 950$. The author conjectures that there are infinitely many primes of the form $p = 1 + n! \sum n$ for some $n \in \mathbb{N}^+$.

Table 1: Primes List

S.No.	n	Prime, p	Number of digits in p
1	1	2	1
2	2	7	1
3	3	37	2
4	4	241	3
5	5	1801	4
6	6	15121	5
7	7	141151	6
8	8	1451521	7
9	9	16329601	8
10	10	199584001	9
11	12	37362124801	11
12	13	566658892801	12
13	14	9153720576001	13
14	19	23112569077678080001	20
15	24	186134520519971831808000001	27
16	251	25662820338985371726..Omitted..000000000001	500
17	374	22873802587990440054..Omitted..0000000000001	807

Size of the prime of the form $p = 1 + n! \sum n$ for some $n \in \mathbb{N}^+$

To compute the size of primes of above form, we use the Stirling's formula [Sec 2.2, CG2001]: $\log n! = (n + \frac{1}{2})\log n - n + \frac{1}{2}\log 2\pi + O(\frac{1}{n})$. Simply, we can also write $\log n! \sim n(\log n - 1)$, if necessary. The size of the prime, $p = 1 + n! \sum n$ is approximately equal to $\lfloor \log_{10}(1 + n! \sum n) \rfloor + 1$.

Conclusion

In this note, the author conjectures that there are infinitely many primes of the form $p = 1 + n! \sum n$ for some natural numbers $n > 0$. The author has found the primes for $n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 19, 24, 251, 374$, when they are searched and verified up-to $n = 950$. The number of digits in the primes of the form $(1 + n! \sum n)$ are approximately equal to $\lfloor \log_{10}(1 + n! \sum n) \rfloor + 1$. Furthermore, an investigation will be required for finding such primes.

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School of Mathematical and Computing Sciences
Fiji National University
P.O.Box:7222, Derrick Campus, Suva, Fiji
E-mail: maheswara.valluri@fnu.ac.fj