S- Anti Fuzzy M-Semigroup

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Abstract

In this paper , we define the concept of a smarandache anti fuzzy M-semigroup (S-Anti Fuzzy M-Semigroup) and some elementary properties about this concept are discussed.

Key words: Fuzzy sets, semigroup, M-semigroup

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Introduction

In 1965 Zadeh introduced the concept of fuzzy set[1], in 1971 Rosenfeld formulated the term of fuzzy subgroup[2]. In 1994 W.X.Gu , S.Y.Li and D.G.Chen studied fuzzy groups and gave some new concepts as M- fuzzy groups [3] . In 2002 W.B.Vasantha introduced the concepts of smarandache semigroups[4]. Smarandache fuzzy semigroups are studied in 2003 by W.B.Vasantha[5] . In 2011 H.R.Yassein and M.O.Karim introduced the concept of a smarandache M – semigroup (S-M-semigroup) and studied some basic properties [6].

In this paper, the concept of Smarandache anti-fuzzy M semigroup are given and its some elementary properties are discussed

1- Preliminaries

Definition (1.1): Let G be a group. A fuzzy subset μ of a group G is called anti fuzzy subgroup of the group G if:

1- $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$ for every $x, y \in G$.

2. $\mu(x)=\mu(x^{-1})$ for every $x \in G$. [7]

Definition (1.2): A semigroup H with operators is an algebraic system consisting of a semigroup H , set M , and a function defined in the product $M \times H$ and having values in H such that , if ma denotes the element in H determined by the element a in H and the element m in M , then $m(ab)=(ma)(mb)$, $a,b \in H$ and $m \in M$ then H is M – semigroup [3].

We shall usually use the phrase "G is an M-group" to a group with operators.

Definition (1.3): If μ is a fuzzy set of G and $t \in [0,1]$ then $\mu_t = \{ x \in G \mid \mu(x) \leq t \}$ is called a t-level set μ [6].

Definition (1.4): Let G and G' both be M – groups , f be a homomorphism from G onto G', if $f(mx)= mf(x)$ for every $m \in M$, $x \in X$, then f is called a M – homomorphism [5].

Definition (1.5): Let S be a semigroup , S is said to be a smarandache semigroup (S – semigroup) if S has a proper subset P such that P is a group under the operation of G [5].

Definition (1.6): Let G be any group. A mapping $\mu: G \to [0, 1]$ is a fuzzy group if (1) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ (2) $\mu(x-1) = \mu(x)$ for all $x, y \in G$[1].

Definition (1.7): Let H be M- semigroup . H is said to be a smarandache M – semigroup (S-M-semigroup) if H has a proper subset K such that K is M- group under the operation of H [6].

this S- fuzzy semigroup is denoted by $\mu_p: P \to [0,1]$ is fuzzy group .

Definition(1.8) : A group with operators is an algebraic system consisting of a group G , set M and a function defined in the product $M \times G$ and having value in G such that , if ma denotes the elements in G determined by the element m of M , then $m(ab)=(ma)(mb)$ hold for all a,b in G ,m in M [3].

Definition (1.9):Let H be a S - M –semigroup. A fuzzy subset $\mu: H \to [0,1]$ is said be smarandache fuzzy M-semigroup if μ restricted to at least one subset K of H which is subgroup is fuzzy subgroup[2].

Definition (1.10): Let S and S' be any two S-semigroups . A map φ from S to S' is said to be S- semigroup homomorphism if φ restricted to a subgroup $A\subset S \rightarrow A'\subset S'$ is a group homomorphism [2].

Definition (1.11): Let H and K be any two S-M- semigroup . A map φ from H to K' is said to be S-M- semigroup homomorphism if φ restricted to a M- subgroup $A\subset H \rightarrow A'\subset K'$ is M- homomorphism [6].

Definition (1.12): Let f be a function from a set X to a set Y while μ is fuzzy set of X then the image f(μ) of μ is the fuzzy set $f(\mu):Y\rightarrow [0,1]$ defined by : [7]

$$f(\mu(y)) = \begin{cases} \sup\limits_{x\in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{if } f^{-1}(y) = \phi \end{cases}$$

Definition (1.13): Let f be a function from a set X to a set Y while μ is fuzzy set of Y then the inverse image $f^{-1}(\mu)$ of μ under f is the fuzzy set $f^{-1}(\mu):X \rightarrow [0,1]$ defined by $f^{-1}(\mu)(x)= \mu(f(x))$ [7].

2-The Main Results

In this section we shall define Smarandache anti fuzzy M-semigroup and give some its results.

Definition (2.1): Let G be M – group and μ be anti fuzzy subgroup of G if $\mu(mx) \leq \mu(x)$ for every $x \in G$, $m\in M$,then μ is said to be anti fuzzy subgroup with operators of G , we use the pharse μ is an M – anti fuzzy subgroup of G instead of a fuzzy subgroup with operators of G .

Definition (2.2): Let S be an S-semigroup. A fuzzy subset $\mu :S \rightarrow [0,1]$ is said to be Smarandache anti fuzzy semigroup (S- anti fuzzy semigroup) if μ restricted to at least one subset P of S which is a subgroup is anti fuzzy subgroup .

that is for all $x,y \in P \subset S$, $\mu(xy^{-1}) \leq \max \{ \mu(x) , \mu(y) \}$.

Definition (2.3): Let H be a S-M- semigroup. A fuzzy subset $\mu :H \rightarrow [0,1]$ is said to be Smarandache anti fuzzy M-semigroup if restricted to at least one subset K of H which is anti fuzzy M- subgroup

Proposition (2.4): If μ is S-anti fuzzy M-semigroup of S-M- semigroup then:

1) $\mu_K(m(xy)) \leq \max \{ \mu_K(mx) , \mu_k(my) \}$
2) $\mu_k(mx^{-1}) \leq \mu_k(x)$

For all $m \in M$, $x,y \in K$

Proof: μ is S- fuzzy M-semigroup

Then there exist subset K of H which is M-subgroup such μ restricted of K which is anti-fuzzy

i.e . $\mu_K:K \rightarrow [0,1]$, M- anti fuzzy subgroup

for all $x,y \in K$, $m \in M$, it is clear that

1) $\mu_K(m(xy)) \leq \mu_K((mx)(my))$
$\leq \max \{ \mu_K(mx) , \mu_k(my) \}$
2) $\mu_k(mx^{-1}) = \mu_k(mx)^{-1}$
$\leq \mu_K(mx)$

Proposition (2.5): Let G be S- semigroup , μ fuzzy set of G. Then μ is an S- anti fuzzy M-semigroup of G if and only if f $\forall\, t\in [0,1]$, μ_t is an S-M- semigroup $\mu_t \neq \varnothing$.

Proof: It is clear μ_t is semigroup of G while $\mu_t \neq \varnothing$ holds.

for any $x \in \mu_t$, $m \in M$

$\mu(mx) \leq \mu(x) \leq t$

hence mx in μ_t , hence μ_t is an M- semigroup of G .

Since μ S-anti fuzzy M- semigroup $\exists$ $K \subset G$ subgroup $\ni$ $\mu_t : K \to [0,1]$

fuzzy M- subgroup.

$\mu_{K_t} = \{ x \in K \mid \mu_K(x) \leq t \}$.

It is clear μ_{K_t} is group. Hence μ_t S-M-semigroup .

Conversely, Since μ_t S-M- semigroup then there exists a proper subset K of G such that K is M- subgroup .

If there exists $x \in K$, $m \in M$ such that $\mu_K(mx) > \mu_K(x)$.

let $t = \frac{1}{2}(\mu_K(mx) + \mu_K(x))$ then $\mu_K(x) < t < \mu_K(mx)$ $mx \notin \mu_{K_t}$ so here emerges a contradiction .

$\mu_K(mx) \leq \mu_K(x)$ always holds for any $x \in K$, $m \in M$.

μ_K is M- fuzzy subgroup hence μ is S –anti fuzzy M- subgroup. ■

Proposition (2.6): Let H and K both be S-M-semigroup and f as S-M- semigroup homomorphism from H onto K . if μ' is an S-anti fuzzy M- semigroup

of H' then $f^{-1}(\mu')$ is an S-anti fuzzy M-semigroup of H.

Proof:

Since $f : H \to K$ is as S-M- semigroup homomorphism then f restricted to M-subgroup .

$A \subset H \to B \subset K$ is M- homomorphism,

$f^{-1}(\mu)_A : A \to [0,1]$ such that A M-subgroup ,

For any $m \in M$, $x \in A$

$$f^{-1}(\mu)_A(mx) = \mu'_A(f(mx))$$

$$= \mu'_A(m(f(x)) \leq \mu'_A(f(x))$$

$$= f^{-1}(\mu')(x)$$

$f^{-1}(\mu')$ is S-anti fuzzy M- semigroup ■

Proposition (2.7): Let H and K both be S-M-semigroups and f as S-M- semigroup homomorphism from H onto K . if μ is an S-anti fuzzy M- semigroup of H then f(μ) is an S-anti fuzzy M-semigroup of K .

Proof:

Since $f : H \to K$ is as S-M- semigroup homomorphism then f restricted to M-subgroup .

$A \subset H \to B \subset K$ is M- homomorphism

$f(\mu)_{B'} : B \to [0,1]$ such that B M-subgroup ,

For any $m \in M$, $y \in B$

$$f(\mu)(my) = \sup \mu(x) \quad , x \in f^{-1}(my)$$

$$= \{\sup \mu(x) \quad , f(x) = my$$

$$\leq \sup \mu(mx') , f(mx') = mx , mx' \in H$$

$= \sup \mu(x')$, $mf(x')=my$, $mx' \in H$

$\leq \sup \mu(x')$, $f(x')=y$, $x' \in H$

$= f(\mu)(y)$

hence $f^{-1}(\mu')$ is S- anti fuzzy M- semigroup ■

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