Electromagnetic wave function and equation, Lorentz Force in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic wave function and equation in Rindler space-time. Specially, this article is that electromagnetic wave equation is corrected by the gauge fixing equation in Rindler space-time. We define the force in Rindler space-time We find Lorentz force (electromagnetic force) by electro-magnetic field transformations in Rindler space-time. In the inertial frame, Lorentz force is defined as 4-dimensional force. Hence, we had to obtain 4-dimensional force in Rindler space-time. We define energy-momentum in Rindler space-time.

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic wave equation and function and Lorentz force by electro-magnetic field transformations in Rindler space-time. This article correct the article "Electromagnetic Field Equation and Lorentz Gauge in Rindler space-time" about the existence proof of electromagnetic wave function and equation. We define energy-momentum in Rindler space-time.

The Rindler coordinate is

$$ct = \left(\frac{c^2}{a_0} + \xi^1\right)\sinh\left(\frac{a_0\xi^0}{c}\right)$$

$$x = \left(\frac{c^2}{a_0} + \xi^1\right)\cosh\left(\frac{a_0\xi^0}{c}\right) - \frac{c^2}{a_0} \quad , y = \xi^2 , z = \xi^3 \tag{1}$$

Therefore,

$$cdt = c\cosh\left(\frac{a_0\xi^0}{c}\right)d\xi^0\left(1 + \frac{a_0}{c^2}\xi^1\right) + \sinh\left(\frac{a_0\xi^0}{c}\right)d\xi^1$$

$$dx = c\sinh\left(\frac{a_0\xi^0}{c}\right)d\xi^0\left(1 + \frac{a_0}{c^2}\xi^1\right) + \cosh\left(\frac{a_0\xi^0}{c}\right)d\xi^1 , dy = d\xi^2 ,\ dz = d\xi^3 \tag{2}$$

Hence,

$$\frac{1}{c}\frac{\partial}{\partial t} = \frac{c\partial\xi^0}{c\partial t}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{c\partial t}\frac{\partial}{\partial\xi^1}$$

$$= \frac{\cosh\left(\frac{a_0\xi^0}{c}\right)}{\left(1 + \frac{a_0\xi^1}{c^2}\right)}\frac{\partial}{c\partial\xi^0} - \sinh\left(\frac{a_0\xi^0}{c}\right)\frac{\partial}{\partial\xi^1}$$

$$\frac{\partial}{\partial x} = \frac{c\partial\xi^0}{\partial x}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{\partial x}\frac{\partial}{\partial\xi^1}$$

$$= -\frac{\sinh\left(\frac{a_0\xi^0}{c}\right)}{\left(1 + \frac{a_0\xi^1}{c^2}\right)}\frac{\partial}{c\partial\xi^0} + \cosh\left(\frac{a_0\xi^0}{c}\right)\frac{\partial}{\partial\xi^1}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial\xi^2} , \frac{\partial}{\partial z} = \frac{\partial}{\partial\xi^3} \tag{3}$$

Hence,

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial \xi^0})^2 - \nabla_\xi{}^2$$

$$\vec{\nabla} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}), \quad \vec{\nabla}_\xi = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3})$$

(4)

2. Corrected electromagnetic wave equation in the Rindler space-time

The electro-magnetic field $(\vec{E}, \vec{B})$ is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{c\partial t}, \vec{B} = \vec{\nabla} \times \vec{A} \qquad (5)$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler space-time [1].

$$\vec{E}_\xi = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial \vec{A}_\xi}{c\partial \xi^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi \times \vec{A}_\xi$$

In this time, $\vec{\nabla}_\xi = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3}), \vec{A}_\xi = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$ (6)

Hence, Lorentz gauge condition is in Rindler space-time[1],

$$\phi_\xi \to \phi_\xi - \frac{1}{c}\frac{\partial \Lambda}{\partial \xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, \quad \vec{A}_\xi \to \vec{A}_\xi + \vec{\nabla}_\xi \Lambda, \quad \Lambda \text{ is a scalar function.} \qquad (7)$$

$$A^\mu{}_{;\mu} = \frac{\partial A^\mu}{\partial \xi^\mu} + \Gamma^\mu{}_{\mu\rho}A^\rho \to \partial_\mu(A^\mu + g^{\mu\upsilon}\partial_\upsilon \Lambda) + \Gamma^0{}_{01}(A^1 + \frac{\partial \Lambda}{\partial \xi^1}) \qquad (8)$$

Lorentz gauge fix condition is in Rindler space-time[1],

$$0 = \frac{1}{c}\frac{\partial \phi_\xi}{\partial \xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi + \frac{A_{\xi^1}}{c^2}\frac{a_0}{(1+\frac{a_0\xi^1}{c^2})}$$

$$\to \frac{1}{c}\frac{\partial \phi_\xi}{\partial \xi^0} + \vec{\nabla}_\xi \cdot \vec{A}_\xi + \frac{A_{\xi^1}}{c^2}\frac{a_0}{(1+\frac{a_0\xi^1}{c^2})} - [\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial \xi^0})^2 - \nabla_\xi{}^2]\Lambda + \frac{\partial \Lambda}{\partial \xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}$$

$$= 0 \qquad (9)$$

Hence, the gauge equation is

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\Lambda-\frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0 \tag{10}$$

We can use Eq(10) as an electromagnetic wave equation because we can apply electromagnetic wave function instead of the gauge function Λ to Eq(10) in Rindler space-time. Hence,

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]E_{\xi^1}-\frac{\partial E_{\xi^1}}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]B_{\xi^1}-\frac{\partial B_{\xi^1}}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0 \tag{11}$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]E_y-\frac{\partial E_y}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]B_y-\frac{\partial B_y}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0 \tag{12}$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]E_z-\frac{\partial E_z}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]B_z-\frac{\partial B_z}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0 \tag{13}$$

The electro-magnetic wave function is

$$E_x=E_{x0}\sin\Phi, E_y=E_{y0}\sin\Phi, E_z=E_{z0}\sin\Phi$$

$$B_x=B_{x0}\sin\Phi, B_y=B_{y0}\sin\Phi, B_z=B_{z0}\sin\Phi \tag{14}$$

$$E_{\xi^1}=E_X=E_{X0}\sin\Phi, B_{\xi^1}=B_X=B_{X0}\sin\Phi$$

$$E_{\xi^2}=E_y\cosh(\frac{a_0\xi^0}{c})-B_z\sinh(\frac{a_0\xi^0}{c}),$$

$$=(E_{y0}\sin\Phi)\cosh(\frac{a_0\xi^0}{c})-(B_{z0}\sin\Phi)\sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^2} = B_y \cosh(\frac{a_0 \xi^0}{c}) + E_z \sinh(\frac{a_0 \xi^0}{c})$$

$$= (B_{y0} \sin\Phi)\cosh(\frac{a_0 \xi^0}{c}) + (E_{z0} \sin\Phi)\sinh(\frac{a_0 \xi^0}{c})$$

$$E_{\xi^3} = E_z \cosh(\frac{a_0 \xi^0}{c}) + B_y \sinh(\frac{a_0 \xi^0}{c})$$

$$= (E_{z0} \sin\Phi)\cosh(\frac{a_0 \xi^0}{c}) + (B_{y0} \sin\Phi)\sinh(\frac{a_0 \xi^0}{c})$$

$$B_{\xi^3} = B_z \cosh(\frac{a_0 \xi^0}{c}) - E_y \sinh(\frac{a_0 \xi^0}{c})$$

$$= (B_{z0} \sin\Phi)\cosh(\frac{a_0 \xi^0}{c}) - (E_{y0} \sin\Phi)\sinh(\frac{a_0 \xi^0}{c}) \qquad (15)$$

$$\Phi = \omega(t - l\frac{x}{c} - m\frac{y}{c} - n\frac{z}{c}) ,$$

$$= \omega\{(\frac{c}{a_0} + \frac{\xi^1}{c})(\sinh(\frac{a_0 \xi^0}{c}) - l\cosh(\frac{a_0 \xi^0}{c})) + \frac{c}{a_0} - m\frac{\xi^2}{c} - n\frac{\xi^3}{c}\},$$

$$l^2 + m^2 + n^2 = 1 \qquad (16)$$

3. Electro-magnetic Force in Rindler space-time

In inertial frame, Lorentz 4-force is

$$F^0 = m_0 \frac{d}{dt}(\frac{cdt}{d\tau}) = q\frac{\vec{u}}{c} \cdot \vec{E} \qquad (17)$$

$$\vec{F} = m_0 \frac{d}{dt}(\frac{d\vec{x}}{d\tau}) = q[\vec{E} + \frac{\vec{u}}{c} \times \vec{B}], \quad \vec{u} = \frac{d\vec{x}}{dt} \qquad (18)$$

We want to obtain the Lorentz 4-force in Rindler space-time. Hence, we define the force in Rindler space-time.

$$F_\xi^{\ 0} = m_0 \frac{d}{d\xi^0}(\frac{cd\xi^0}{d\tau})$$

$$\vec{F}_\xi = m_0 \frac{d}{d\xi^0}(\frac{d\vec{\xi}}{d\tau}) \qquad (19)$$

Or,

$$F_\xi^{\ \mu} = m_0 \frac{d}{d\xi^0}(\frac{d\xi^\mu}{d\tau}) \qquad (20)$$

Hence, 4-force is in inertial frame

$$F^{\alpha} = m_0 \frac{d}{dt}(\frac{dx^{\alpha}}{d\tau}) \tag{21}$$

In this time, Minkowski force is in inertial frame or in Rindler space-time[13].

$$f^{\alpha} = m_0 \frac{d}{d\tau}(\frac{dx^{\alpha}}{d\tau}) = m_0 \frac{d^2 x^{\alpha}}{d\tau^2} \tag{22}$$

$$f_{\xi}{}^{\mu} = m_0 \frac{d}{d\tau}(\frac{d\xi^{\mu}}{d\tau}) = m_0 \frac{d^2 \xi^{\mu}}{d\tau^2} \tag{23}$$

Minkowski force is

$$f^{\alpha} = m_0 \frac{d^2 x^{\alpha}}{d\tau^2} = m_0 \frac{d}{d\xi^0}(\frac{\partial x^{\alpha}}{\partial \xi^{\mu}} \frac{d\xi^{\mu}}{d\tau}) \frac{d\xi^0}{d\tau}$$

$$= m_0 \frac{d}{d\xi^0}(\frac{\partial x^{\alpha}}{\partial \xi^{\mu}}) \frac{d\xi^{\mu}}{d\tau} \frac{d\xi^0}{d\tau} + m_0 \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} \frac{d}{d\xi^0}(\frac{d\xi^{\mu}}{d\tau}) \frac{d\xi^0}{d\tau}$$

$$= m_0 \frac{d}{d\xi^0}(\frac{\partial x^{\alpha}}{\partial \xi^{\mu}}) \frac{d\xi^{\mu}}{d\tau} \frac{d\xi^0}{d\tau} + F_{\xi}{}^{\mu} \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} \frac{d\xi^0}{d\tau}$$

$$F_{\xi}{}^{\mu} = m_0 \frac{d}{d\xi^0}(\frac{d\xi^{\mu}}{d\tau}) \tag{24}$$

Hence,

$$\begin{aligned} f^0 &= m_0 \frac{cd^2 t}{d\tau^2} = m_0 \frac{d}{d\xi^0}(\frac{\partial t}{\partial \xi^0}) \frac{cd\xi^0}{d\tau} \frac{d\xi^0}{d\tau} + F_{\xi}{}^0 \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} \\ &+ m_0 \frac{d}{d\xi^0}(\frac{c\partial t}{\partial \xi^1}) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} + F_{\xi}{}^1 \frac{c\partial t}{\partial \xi^1} \frac{d\xi^0}{d\tau} \end{aligned} \tag{25}$$

$$\begin{aligned} f^1 &= m_0 \frac{d^2 x}{d\tau^2} = m_0 \frac{d}{d\xi^0}(\frac{\partial x}{c\partial \xi^0}) \frac{cd\xi^0}{d\tau} \frac{d\xi^0}{d\tau} + F_{\xi}{}^0 \frac{\partial x}{c\partial \xi^0} \frac{d\xi^0}{d\tau} \\ &+ m_0 \frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^1}) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} + F_{\xi}{}^1 \frac{\partial x}{\partial \xi^1} \frac{d\xi^0}{d\tau} \end{aligned} \tag{26}$$

$$F_{\xi}{}^0 = m_0 \frac{d}{d\xi^0}(\frac{cd\xi^0}{d\tau}), \quad F_{\xi}{}^1 = m_0 \frac{d}{d\xi^0}(\frac{d\xi^1}{d\tau})$$

Therefore,

$$\begin{aligned} &F_{\xi}{}^0 \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} + F_{\xi}{}^1 \frac{c\partial t}{\partial \xi^1} \frac{d\xi^0}{d\tau} \\ &= f^0 - m_0 \frac{d}{d\xi^0}(\frac{\partial t}{\partial \xi^0}) \frac{cd\xi^0}{d\tau} \frac{d\xi^0}{d\tau} - m_0 \frac{d}{d\xi^0}(\frac{c\partial t}{\partial \xi^1}) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} = A \end{aligned} \tag{27}$$

$$F_{\xi}^{\ 0}\frac{\partial x}{c\partial\xi^0}\frac{d\xi^0}{d\tau}+F_{\xi}^{\ 1}\frac{\partial x}{\partial\xi^1}\frac{d\xi^0}{d\tau}$$
$$=f^1-m_0\frac{d}{d\xi^0}\left(\frac{\partial x}{c\partial\xi^0}\right)\frac{cd\xi^0}{d\tau}\frac{d\xi^0}{d\tau}-m_0\frac{d}{d\xi^0}\left(\frac{\partial x}{\partial\xi^1}\right)\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}=B \tag{28}$$

If we represent by the matrix,

$$\begin{pmatrix}A\\B\end{pmatrix}=\begin{pmatrix}\frac{\partial t}{\partial\xi^0}\frac{d\xi^0}{d\tau} & \frac{c\partial t}{\partial\xi^1}\frac{d\xi^0}{d\tau}\\ \frac{\partial x}{c\partial\xi^0}\frac{d\xi^0}{d\tau} & \frac{\partial x}{\partial\xi^1}\frac{d\xi^0}{d\tau}\end{pmatrix}\begin{pmatrix}F_{\xi}^{\ 0}\\F_{\xi}^{\ 1}\end{pmatrix} \tag{29}$$

In this time, Rindler coordinate is

$$ct=\left(\frac{c^2}{a_0}+\xi^1\right)\sinh\left(\frac{a_0\xi^0}{c}\right),x=\left(\frac{c^2}{a_0}+\xi^1\right)\cosh\left(\frac{a_0\xi^0}{c}\right)-\frac{c^2}{a_0}\quad ,y=\xi^2,z=\xi^3 \tag{30}$$

So,

$$\frac{\partial t}{\partial\xi^0}=\left(\frac{c^2}{a_0}+\xi^1\right)\cosh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c^2},\ \frac{\partial x}{c\partial\xi^0}=\left(\frac{c^2}{a_0}+\xi^1\right)\sinh\left(\frac{a_0\xi^0}{c}\right)\frac{a_0}{c^2}$$

$$\frac{c\partial t}{\partial\xi^1}=\sinh\left(\frac{a_0\xi^0}{c}\right),\ \frac{\partial x}{\partial\xi^1}=\cosh\left(\frac{a_0\xi^0}{c}\right) \tag{31}$$

Hence,

$$Det=\left(\frac{\partial t}{\partial\xi^0}\frac{\partial x}{\partial\xi^1}-\frac{c\partial t}{\partial\xi^1}\frac{\partial x}{c\partial\xi^0}\right)\left(\frac{d\xi^0}{d\tau}\right)^2=\left(1+\frac{a_0}{c^2}\xi^1\right)\left(\frac{d\xi^0}{d\tau}\right)^2 \tag{32}$$

Therefore,

$$\begin{pmatrix}F_{\xi}^{\ 0}\\F_{\xi}^{\ 1}\end{pmatrix}=\frac{1}{\left(1+\frac{a_0}{c^2}\xi^1\right)\left(\frac{d\xi^0}{d\tau}\right)^2}\begin{pmatrix}\frac{\partial x}{\partial\xi^1}\frac{d\xi^0}{d\tau} & -\frac{c\partial t}{\partial\xi^1}\frac{d\xi^0}{d\tau}\\ -\frac{\partial x}{c\partial\xi^0}\frac{d\xi^0}{d\tau} & \frac{\partial t}{\partial\xi^0}\frac{d\xi^0}{d\tau}\end{pmatrix}\begin{pmatrix}A\\B\end{pmatrix} \tag{33}$$

Hence,

$$F_{\xi}^{\ 0}=\frac{1}{\left(1+\frac{a_0}{c^2}\xi^1\right)\left(\frac{d\xi^0}{d\tau}\right)^2}\left[A\frac{\partial x}{\partial\xi^1}\frac{d\xi^0}{d\tau}-B\frac{c\partial t}{\partial\xi^1}\frac{d\xi^0}{d\tau}\right]$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})} \frac{\partial x}{\partial \xi^1} [f^0 - m_0 \frac{d}{d\xi^0}(\frac{\partial t}{\partial \xi^0})c(\frac{d\xi^0}{d\tau})^2 - m_0 \frac{d}{d\xi^0}(\frac{c\partial t}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}]$$

$$- \frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})} \frac{c\partial t}{\partial \xi^1} [f^1 - m_0 \frac{d}{d\xi^0}(\frac{\partial x}{c\partial \xi^0})c(\frac{d\xi^0}{d\tau})^2 - m_0 \frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}] \quad (34)$$

As,

$$F_\xi^{\ 1} = \frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})^2}[-A\frac{\partial x}{c\partial \xi^0}\frac{d\xi^0}{d\tau} + B\frac{c\partial t}{\partial \xi^0}\frac{d\xi^0}{d\tau}]$$

$$= - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})} \frac{\partial x}{c\partial \xi^0} [f^0 - m_0 \frac{d}{d\xi^0}(\frac{\partial t}{\partial \xi^0})c(\frac{d\xi^0}{d\tau})^2 - m_0 \frac{d}{d\xi^0}(\frac{c\partial t}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}]$$

$$+ \frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})} \frac{\partial t}{\partial \xi^0} [f^1 - m_0 \frac{d}{d\xi^0}(\frac{\partial x}{c\partial \xi^0})c(\frac{d\xi^0}{d\tau})^2 - m_0 \frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}] \quad (35)$$

In this time,

$$\frac{d}{d\xi^0}(\frac{\partial t}{\partial \xi^0}) = \frac{d\xi^1}{d\xi^0}\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2} + (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0^{\ 2}}{c^3} \quad ,$$

$$\frac{d}{d\xi^0}(\frac{\partial x}{c\partial \xi^0}) = \frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2} + (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0^{\ 2}}{c^3}$$

$$\frac{d}{d\xi^0}(\frac{c\partial t}{\partial \xi^1}) = \cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c}, \quad \frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^1}) = \sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c} \quad (36)$$

Therefore, Eq(34) is by Eq(31),Eq(36). Lorentz force $F_\xi^{\ 0}$ is in Rindler space-time.

$$F_\xi^{\ 0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\cosh(\frac{a_0\xi^0}{c})$$

$$\times [f^0 - m_0\{\frac{d\xi^1}{d\xi^0}\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2} + (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0^{\ 2}}{c^3}\}c(\frac{d\xi^0}{d\tau})^2$$

$$-m_0\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c}\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}]$$

$$-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\sinh(\frac{a_0\xi^0}{c})$$

$$\times[f^1-m_0\{\frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}+(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0{}^2}{c^3}\}c(\frac{d\xi^0}{d\tau})^2$$

$$-m_0\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}] \tag{37}$$

Therefore, Eq(35) is by Eq(31),Eq(36).Lorentz force F_{ξ}^{1} is in Rindler space-time.

$$F_{\xi}^{1}$$

$$=-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}(\frac{c^2}{a_0}+\xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}$$

$$\times[f^0-m_0\{\frac{d\xi^1}{d\xi^0}\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}+(\frac{c^2}{a_0}+\xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0{}^2}{c^3}\}c(\frac{d\xi^0}{d\tau})^2$$

$$-m_0\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c}\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}]$$

$$+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}$$

$$\times[f^1-m_0\{\frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}+(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0{}^2}{c^3}\}c(\frac{d\xi^0}{d\tau})^2$$

$$-m_0\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}] \tag{38}$$

In this time, the transformation of electromagnetic field is[1]

$$E_x=E_{\xi^1}\ ,$$

$$E_y=E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$B_x = B_{\xi^1},$$

$$B_y = B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z = B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}) \qquad (39)$$

Hence,

$$f^0 = F^0\frac{dt}{d\tau} = q\frac{\vec{u}}{c}\cdot\vec{E}\frac{dt}{d\tau} = q\frac{\vec{u}'}{c}\cdot\vec{E}, \vec{u} = \frac{d\vec{x}}{dt}, \vec{u}' = \frac{d\vec{x}}{d\tau}$$

$$= q\frac{1}{c}[\frac{dx}{d\tau}E_x + \frac{dy}{d\tau}E_y + \frac{dz}{d\tau}E_z],\ \frac{dx}{d\tau} = \frac{\partial x}{\partial\xi^0}\frac{d\xi^0}{d\tau} + \frac{\partial x}{\partial\xi^1}\frac{d\xi^1}{d\tau}$$

$$= \frac{q}{c}[\{(\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}\frac{d\xi^0}{d\tau} + \cosh(\frac{a_0\xi^0}{c})\frac{d\xi^1}{d\tau}\}E_{\xi^1}$$

$$+ \frac{d\xi^2}{d\tau}\{E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})\}$$

$$+ \frac{d\xi^3}{d\tau}\{E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})\}] \qquad (40)$$

$$f^1 = F^1\frac{dt}{d\tau},$$

$$F^1 = q[E_x + \frac{1}{c}(u_y B_z - u_z B_y)] \qquad , \vec{u} = \frac{d\vec{x}}{dt}, \vec{u}' = \frac{d\vec{x}}{d\tau}$$

$$f^1 = F^1\frac{dt}{d\tau} \quad = q[E_x\frac{dt}{d\tau} + \frac{1}{c}(\frac{dy}{d\tau}B_z - \frac{dz}{d\tau}B_y)],\ \frac{dt}{d\tau} = \frac{\partial t}{\partial\xi^0}\frac{d\xi^0}{d\tau} + \frac{\partial t}{\partial\xi^1}\frac{d\xi^1}{d\tau}$$

$$= q[\{(\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}\frac{d\xi^0}{d\tau} + \sinh(\frac{a_0\xi^0}{c})\frac{1}{c}\frac{d\xi^1}{d\tau}\}E_{\xi^1}$$

$$+ \frac{1}{c}\{\frac{d\xi^2}{d\tau}(B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}))$$

$$- \frac{d\xi^3}{d\tau}(B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c}))\}] \qquad (41)$$

In Eq(24), f^2 is

$$f^2 = m_0 \frac{d^2 y}{d\tau^2} = F_\xi{}^2 \frac{d\xi^0}{d\tau}, \quad F_\xi{}^2 = m_0 \frac{d}{dt}(\frac{dy}{d\tau})\frac{dt}{d\xi^0} = F^2 \frac{dt}{d\xi^0} \tag{42}$$

Therefore, Lorentz force $F_\xi{}^2$ is in Rindler space-time.

$$\begin{aligned}
F_\xi{}^2 &= m_0 \frac{d}{d\xi^0}(\frac{d\xi^2}{d\tau}) = F^2 \frac{dt}{d\xi^0} \\
&= q[E_y + \frac{1}{c}(u_z B_x - u_x B_z)]\frac{dt}{d\xi^0} \\
&= q[\{E_{\xi^2} \cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3} \sinh(\frac{a_0\xi^0}{c})\} \\
&\quad \times \{(\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2} + \frac{1}{c}\frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\} + \frac{1}{c}\{\frac{d\xi^3}{d\xi^0} B_{\xi^1} \\
&- \{(\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c} + \cosh(\frac{a_0\xi^0}{c})\frac{d\xi^1}{d\xi^0}\} \\
&\quad \times \{B_{\xi^3} \cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2} \sinh(\frac{a_0\xi^0}{c})\}\}]
\end{aligned} \tag{43}$$

In Eq(24), f^3 is

$$f^3 = m_0 \frac{d^2 z}{d\tau^2} = F_\xi{}^3 \frac{d\xi^0}{d\tau}, \quad F_\xi{}^3 = m_0 \frac{d}{dt}(\frac{dz}{d\tau})\frac{dt}{d\xi^0} = F^3 \frac{dt}{d\xi^0} \tag{44}$$

Therefore, Lorentz force $F_\xi{}^3$ is in Rindler space-time.

$$\begin{aligned}
F_\xi{}^3 &= m_0 \frac{d}{d\xi^0}(\frac{d\xi^3}{d\tau}) = F^3 \frac{dt}{d\xi^0} \\
&= q[E_z + \frac{1}{c}(u_x B_y - u_y B_x)]\frac{dt}{d\xi^0} \\
&= q[\{E_{\xi^3} \cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0\xi^0}{c})\} \\
&\quad \times \{(\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2} + \frac{1}{c}\frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\} \\
&+ \frac{1}{c}\{\{(\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c} + \cosh(\frac{a_0\xi^0}{c})\frac{d\xi^1}{d\xi^0}\}
\end{aligned}$$

$$\times\{B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})\} - \frac{d\xi^2}{d\xi^0}B_{\xi^1}\}] \quad (45)$$

4. Energy-momentum in Rindler spacetime

In initial frame, energy-momentum is

$$\vec{p} = m_0\frac{d\vec{x}}{d\tau} \quad , \quad E = m_0c^2\frac{dt}{d\tau} \quad (46)$$

$$p_x = m_0\frac{dx}{d\tau} = m_0\frac{\partial x}{\partial\xi^\alpha}\frac{d\xi^\alpha}{d\tau}$$

$$= m_0\frac{\partial x}{c\partial\xi^0}\frac{cd\xi^0}{d\tau} + m_0\frac{\partial x}{\partial\xi^1}\frac{d\xi^1}{d\tau}$$

$$= m_0(1+\frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{cd\xi^0}{d\tau} + m_0\cosh(\frac{a_0\xi^0}{c})\frac{d\xi^1}{d\tau} \quad (47)$$

$$E = m_0c^2\frac{dt}{d\tau} = m_0c\frac{cdt}{\partial\xi^\alpha}\frac{d\xi^\alpha}{d\tau}$$

$$= m_0c\frac{\partial t}{\partial\xi^0}\frac{cd\xi^0}{d\tau} + m_0c\frac{c\partial t}{\partial\xi^1}\frac{d\xi^1}{d\tau}$$

$$= m_0c(1+\frac{a_0}{c^2}\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{cd\xi^0}{d\tau} + m_0c\sinh(\frac{a_0\xi^0}{c})\frac{d\xi^1}{d\tau} \quad (48)$$

Hence, we can define energy-momentum in Rindler spacetimee.

$$\vec{p}_\xi = m_0\frac{d\vec{\xi}}{d\tau} \quad , \quad E_\xi = m_0c^2(1+\frac{a_0}{c^2}\xi^1)\frac{d\xi^0}{d\tau} \quad (49)$$

$$p_x = m_0\frac{dx}{d\tau} = \sinh(\frac{a_0\xi^0}{c})\frac{E_\xi}{c} + \cosh(\frac{a_0\xi^0}{c})p_{\xi^1}$$

$$E = m_0c^2\frac{dt}{d\tau} = \cosh(\frac{a_0\xi^0}{c})E_\xi + \sinh(\frac{a_0\xi^0}{c})p_{\xi^1}c$$

$$p_y = m_0\frac{dy}{d\tau} = m_0\frac{d\xi^2}{d\tau} = p_{\xi^2}\,, p_z = m_0\frac{dz}{d\tau} = m_0\frac{d\xi^3}{d\tau} = p_{\xi^3} \quad (50)$$

Therefore, general case is

$$E_\xi{}^2 - p_\xi{}^2c^2 = m_0{}^2c^4[(1+\frac{a_0\xi^1}{c^2})^2\frac{(d\xi^0)^2}{d\tau^2} - \frac{1}{c^2}\frac{d\vec{\xi}\cdot d\vec{\xi}}{d\tau^2}] = m_0{}^2c^4 \quad (51)$$

In special case, light is

$$E_\xi = p_\xi c \quad (52)$$

5. Conclusion

We find the electro-magnetic wave equation and function and the electro-magnetic force in uniformly accelerated frame. We define energy-momentum in Rindler space-time.

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