

Electromagnetic wave function and equation, Lorentz Force in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic wave function and equation in Rindler space-time. Specially, this article is that electromagnetic wave equation is corrected by the gauge fixing equation in Rindler space-time. We define the force in Rindler space-time. We find Lorentz force (electromagnetic force) by electro-magnetic field transformations in Rindler space-time. In the inertial frame, Lorentz force is defined as 4-dimensional force. Hence, we had to obtain 4-dimensional force in Rindler space-time. We define energy-momentum in Rindler space-time.

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic wave equation and function and Lorentz force by electro-magnetic field transformations in Rindler space-time. This article correct the article "Electromagnetic Field Equation and Lorentz Gauge in Rindler space-time" about the existence proof of electromagnetic wave function and equation. We define energy-momentum in Rindler space-time.

The Rindler coordinate is

$$\begin{aligned} ct &= \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \\ x &= \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3 \end{aligned} \quad (1)$$

Therefore,

$$\begin{aligned} cdt &= c \cosh\left(\frac{a_0 \xi^0}{c}\right) d\xi^0 \left(1 + \frac{a_0}{c^2} \xi^1\right) + \sinh\left(\frac{a_0 \xi^0}{c}\right) d\xi^1 \\ dx &= c \sinh\left(\frac{a_0 \xi^0}{c}\right) d\xi^0 \left(1 + \frac{a_0}{c^2} \xi^1\right) + \cosh\left(\frac{a_0 \xi^0}{c}\right) d\xi^1, dy = d\xi^2, dz = d\xi^3 \end{aligned} \quad (2)$$

Hence,

$$\begin{aligned} \frac{1}{c} \frac{\partial}{\partial t} &= \frac{\partial \xi^0}{\partial t} \frac{1}{c} \frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial t} \frac{\partial}{\partial \xi^1} \\ &= \frac{\cosh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} - \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial x} &= \frac{\partial \xi^0}{\partial x} \frac{1}{c} \frac{\partial}{\partial \xi^0} + \frac{\partial \xi^1}{\partial x} \frac{\partial}{\partial \xi^1} \\ &= -\frac{\sinh\left(\frac{a_0 \xi^0}{c}\right)}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} \frac{\partial}{\partial \xi^0} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial y} &= \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial z} = \frac{\partial}{\partial \xi^3} \end{aligned} \quad (3)$$

Hence,

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2(1 + \frac{a_0}{c^2} \xi^1)^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_{\xi}^2$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right), \quad \vec{\nabla}_{\xi} = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3} \right)$$

(4)

2. Corrected electromagnetic wave equation in the Rindler space-time

The electro-magnetic field $(\vec{E}, \vec{B})$ is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{c\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

(5)

Hence, we can define the electro-magnetic field $(\vec{E}_{\xi}, \vec{B}_{\xi})$ in Rindler space-time [1].

$$\vec{E}_{\xi} = -\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \vec{\nabla}_{\xi} \left\{ \phi_{\xi} \left(1 + \frac{a_0 \xi^1}{c^2} \right)^2 \right\} - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial \vec{A}_{\xi}}{c\partial \xi^0}$$

$$\vec{B}_{\xi} = \vec{\nabla}_{\xi} \times \vec{A}_{\xi}$$

In this time, $\vec{\nabla}_{\xi} = \left(\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3} \right), \vec{A}_{\xi} = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$ (6)

Hence, Lorentz gauge condition is in Rindler space-time[1],

$$\phi_{\xi} \rightarrow \phi_{\xi} - \frac{1}{c} \frac{\partial \Lambda}{\partial \xi^0} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2}, \quad \vec{A}_{\xi} \rightarrow \vec{A}_{\xi} + \vec{\nabla}_{\xi} \Lambda, \quad \Lambda \text{ is a scalar function.} \quad (7)$$

$$A^{\mu}_{;\mu} = \frac{\partial A^{\mu}}{\partial \xi^{\mu}} + \Gamma^{\mu}_{\mu\rho} A^{\rho} \rightarrow \partial_{\mu} (A^{\mu} + g^{\mu\nu} \partial_{\nu} \Lambda) + \Gamma^0_{01} (A^1 + \frac{\partial \Lambda}{\partial \xi^1}) \quad (8)$$

Lorentz gauge fix condition is in Rindler space-time[1],

$$0 = \frac{1}{c} \frac{\partial \phi_{\xi}}{\partial \xi^0} + \vec{\nabla}_{\xi} \cdot \vec{A}_{\xi} + \frac{A_{\xi^1}}{c^2} \frac{a_0}{(1 + \frac{a_0 \xi^1}{c^2})}$$

$$\rightarrow \frac{1}{c} \frac{\partial \phi_{\xi}}{\partial \xi^0} + \vec{\nabla}_{\xi} \cdot \vec{A}_{\xi} + \frac{A_{\xi^1}}{c^2} \frac{a_0}{(1 + \frac{a_0 \xi^1}{c^2})} - \left[\frac{1}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2} \left(\frac{\partial}{\partial \xi^0} \right)^2 - \nabla_{\xi}^2 \right] \Lambda + \frac{\partial \Lambda}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})}$$

$$= 0 \quad (9)$$

Hence, the gauge equation is

$$\left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] \Lambda - \frac{\partial \Lambda}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0$$

(10)

We can use Eq(10) as an electromagnetic wave equation because we can apply electromagnetic wave function instead of the gauge function Λ to Eq(10) in Rindler space-time. Hence,

$$\begin{aligned} & \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] E_{\xi^1} - \frac{\partial E_{\xi^1}}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0 \\ & \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] B_{\xi^1} - \frac{\partial B_{\xi^1}}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0 \end{aligned} \quad (11)$$

$$\begin{aligned} & \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] E_y - \frac{\partial E_y}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0 \\ & \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] B_y - \frac{\partial B_y}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0 \end{aligned} \quad (12)$$

$$\begin{aligned} & \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] E_z - \frac{\partial E_z}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0 \\ & \left[\frac{1}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)^2} \left(\frac{\partial}{\partial \xi^0}\right)^2 - \nabla_{\xi^2}^2 \right] B_z - \frac{\partial B_z}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{\left(1 + \frac{a_0 \xi^1}{c^2}\right)} = 0 \end{aligned} \quad (13)$$

The electro-magnetic wave function is

$$E_x = E_{x0} \sin \Phi, E_y = E_{y0} \sin \Phi, E_z = E_{z0} \sin \Phi$$

$$B_x = B_{x0} \sin \Phi, B_y = B_{y0} \sin \Phi, B_z = B_{z0} \sin \Phi \quad (14)$$

$$E_{\xi^1} = E_x = E_{x0} \sin \Phi, B_{\xi^1} = B_x = B_{x0} \sin \Phi$$

$$E_{\xi^2} = E_y \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_z \sinh\left(\frac{a_0 \xi^0}{c}\right),$$

$$= (E_{y0} \sin \Phi) \cosh\left(\frac{a_0 \xi^0}{c}\right) - (B_{z0} \sin \Phi) \sinh\left(\frac{a_0 \xi^0}{c}\right)$$

$$\begin{aligned}
B_{\xi^2} &= B_y \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_z \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
&= (B_{y0} \sin \Phi) \cosh\left(\frac{a_0 \xi^0}{c}\right) + (E_{z0} \sin \Phi) \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
E_{\xi^3} &= E_z \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_y \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
&= (E_{z0} \sin \Phi) \cosh\left(\frac{a_0 \xi^0}{c}\right) + (B_{y0} \sin \Phi) \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
B_{\xi^3} &= B_z \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_y \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
&= (B_{z0} \sin \Phi) \cosh\left(\frac{a_0 \xi^0}{c}\right) - (E_{y0} \sin \Phi) \sinh\left(\frac{a_0 \xi^0}{c}\right)
\end{aligned} \tag{15}$$

$$\begin{aligned}
\Phi &= \omega \left(t - l \frac{x}{c} - m \frac{y}{c} - n \frac{z}{c} \right) \\
&= \omega \left\{ \left(\frac{c}{a_0} + \frac{\xi^1}{c} \right) \left(\sinh\left(\frac{a_0 \xi^0}{c}\right) - l \cosh\left(\frac{a_0 \xi^0}{c}\right) \right) + \frac{c}{a_0} - m \frac{\xi^2}{c} - n \frac{\xi^3}{c} \right\},
\end{aligned}$$

$$l^2 + m^2 + n^2 = 1 \tag{16}$$

3. Electro-magnetic Force in Rindler space-time

In inertial frame, Lorentz 4-force is

$$F^0 = m_0 \frac{d}{dt} \left(\frac{cdt}{d\tau} \right) = q \frac{\vec{u}}{c} \cdot \vec{E} \tag{17}$$

$$\vec{F} = m_0 \frac{d}{dt} \left(\frac{d\vec{x}}{d\tau} \right) = q \left[\vec{E} + \frac{\vec{u}}{c} \times \vec{B} \right], \quad \vec{u} = \frac{d\vec{x}}{dt} \tag{18}$$

We want to obtain the Lorentz 4-force in Rindler space-time. Hence, we define the force in Rindler space-time.

$$\begin{aligned}
F_{\xi}^0 &= m_0 \frac{d}{d\xi^0} \left(\frac{cd\xi^0}{d\tau} \right) \\
\vec{F}_{\xi} &= m_0 \frac{d}{d\xi^0} \left(\frac{d\vec{\xi}}{d\tau} \right)
\end{aligned} \tag{19}$$

Or,

$$F_{\xi}^{\mu} = m_0 \frac{d}{d\xi^0} \left(\frac{d\xi^{\mu}}{d\tau} \right) \tag{20}$$

Hence, 4-force is in inertial frame

$$F^\alpha = m_0 \frac{d}{dt} \left(\frac{dx^\alpha}{d\tau} \right) \quad (21)$$

In this time, Minkowski force is in inertial frame or in Rindler space-time[13].

$$f^\alpha = m_0 \frac{d}{d\tau} \left(\frac{dx^\alpha}{d\tau} \right) = m_0 \frac{d^2 x^\alpha}{d\tau^2} \quad (22)$$

$$f_\xi^\mu = m_0 \frac{d}{d\tau} \left(\frac{d\xi^\mu}{d\tau} \right) = m_0 \frac{d^2 \xi^\mu}{d\tau^2} \quad (23)$$

Minkowski force is

$$\begin{aligned} f^\alpha &= m_0 \frac{d^2 x^\alpha}{d\tau^2} = m_0 \frac{d}{d\xi^0} \left(\frac{\partial x^\alpha}{\partial \xi^\mu} \frac{d\xi^\mu}{d\tau} \right) \frac{d\xi^0}{d\tau} \\ &= m_0 \frac{d}{d\xi^0} \left(\frac{\partial x^\alpha}{\partial \xi^\mu} \right) \frac{d\xi^\mu}{d\tau} \frac{d\xi^0}{d\tau} + m_0 \frac{\partial x^\alpha}{\partial \xi^\mu} \frac{d}{d\xi^0} \left(\frac{d\xi^\mu}{d\tau} \right) \frac{d\xi^0}{d\tau} \\ &= m_0 \frac{d}{d\xi^0} \left(\frac{\partial x^\alpha}{\partial \xi^\mu} \right) \frac{d\xi^\mu}{d\tau} \frac{d\xi^0}{d\tau} + F_\xi^\mu \frac{\partial x^\alpha}{\partial \xi^\mu} \frac{d\xi^0}{d\tau} \\ F_\xi^\mu &= m_0 \frac{d}{d\xi^0} \left(\frac{d\xi^\mu}{d\tau} \right) \end{aligned} \quad (24)$$

Hence,

$$\begin{aligned} f^0 &= m_0 \frac{cd^2 t}{d\tau^2} = m_0 \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^0} \right) \frac{cd\xi^0}{d\tau} \frac{d\xi^0}{d\tau} + F_\xi^0 \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} \\ &\quad + m_0 \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^1} \right) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} + F_\xi^1 \frac{\partial t}{\partial \xi^1} \frac{d\xi^0}{d\tau} \end{aligned} \quad (25)$$

$$\begin{aligned} f^1 &= m_0 \frac{d^2 x}{d\tau^2} = m_0 \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^0} \right) \frac{cd\xi^0}{d\tau} \frac{d\xi^0}{d\tau} + F_\xi^0 \frac{\partial x}{\partial \xi^0} \frac{d\xi^0}{d\tau} \\ &\quad + m_0 \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^1} \right) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} + F_\xi^1 \frac{\partial x}{\partial \xi^1} \frac{d\xi^0}{d\tau} \end{aligned} \quad (26)$$

$$F_\xi^0 = m_0 \frac{d}{d\xi^0} \left(\frac{cd\xi^0}{d\tau} \right), \quad F_\xi^1 = m_0 \frac{d}{d\xi^0} \left(\frac{d\xi^1}{d\tau} \right)$$

Therefore,

$$\begin{aligned} &F_\xi^0 \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} + F_\xi^1 \frac{\partial t}{\partial \xi^1} \frac{d\xi^0}{d\tau} \\ &= f^0 - m_0 \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^0} \right) \frac{cd\xi^0}{d\tau} \frac{d\xi^0}{d\tau} - m_0 \frac{d}{d\xi^0} \left(\frac{\partial t}{\partial \xi^1} \right) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} = A \end{aligned} \quad (27)$$

$$\begin{aligned}
& F_{\xi}^0 \frac{\partial x}{\partial \xi^0} \frac{d\xi^0}{d\tau} + F_{\xi}^1 \frac{\partial x}{\partial \xi^1} \frac{d\xi^0}{d\tau} \\
& = f^1 - m_0 \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^0} \right) \frac{d\xi^0}{d\tau} - m_0 \frac{d}{d\xi^0} \left(\frac{\partial x}{\partial \xi^1} \right) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} = B
\end{aligned} \tag{28}$$

If we represent by the matrix,

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} & \frac{\partial t}{\partial \xi^1} \frac{d\xi^0}{d\tau} \\ \frac{\partial x}{\partial \xi^0} \frac{d\xi^0}{d\tau} & \frac{\partial x}{\partial \xi^1} \frac{d\xi^0}{d\tau} \end{pmatrix} \begin{pmatrix} F_{\xi}^0 \\ F_{\xi}^1 \end{pmatrix} \tag{29}$$

In this time, Rindler coordinate is

$$ct = \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c}\right), x = \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c}\right) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3 \tag{30}$$

So,

$$\begin{aligned}
\frac{\partial t}{\partial \xi^0} &= \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2}, \quad \frac{\partial x}{\partial \xi^0} = \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} \\
\frac{\partial t}{\partial \xi^1} &= \sinh\left(\frac{a_0 \xi^0}{c}\right), \quad \frac{\partial x}{\partial \xi^1} = \cosh\left(\frac{a_0 \xi^0}{c}\right)
\end{aligned} \tag{31}$$

Hence,

$$Det = \left(\frac{\partial t}{\partial \xi^0} \frac{\partial x}{\partial \xi^1} - \frac{\partial t}{\partial \xi^1} \frac{\partial x}{\partial \xi^0} \right) \left(\frac{d\xi^0}{d\tau} \right)^2 = \left(1 + \frac{a_0}{c^2} \xi^1 \right) \left(\frac{d\xi^0}{d\tau} \right)^2 \tag{32}$$

Therefore,

$$\begin{pmatrix} F_{\xi}^0 \\ F_{\xi}^1 \end{pmatrix} = \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1 \right) \left(\frac{d\xi^0}{d\tau} \right)^2} \begin{pmatrix} \frac{\partial x}{\partial \xi^1} \frac{d\xi^0}{d\tau} & - \frac{\partial t}{\partial \xi^1} \frac{d\xi^0}{d\tau} \\ - \frac{\partial x}{\partial \xi^0} \frac{d\xi^0}{d\tau} & \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} \tag{33}$$

Hence,

$$F_{\xi}^0 = \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1 \right) \left(\frac{d\xi^0}{d\tau} \right)^2} \left[A \frac{\partial x}{\partial \xi^1} \frac{d\xi^0}{d\tau} - B \frac{\partial t}{\partial \xi^1} \frac{d\xi^0}{d\tau} \right]$$

$$\begin{aligned}
&= \frac{1}{(1 + \frac{a_0}{c^2} \xi^1) (\frac{d\xi^0}{d\tau})} \frac{\partial x}{\partial \xi^1} [f^0 - m_0 \frac{d}{d\xi^0} (\frac{\partial t}{\partial \xi^0}) c (\frac{d\xi^0}{d\tau})^2 - m_0 \frac{d}{d\xi^0} (\frac{\partial t}{\partial \xi^1}) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau}] \\
&- \frac{1}{(1 + \frac{a_0}{c^2} \xi^1) (\frac{d\xi^0}{d\tau})} \frac{\partial t}{\partial \xi^1} [f^1 - m_0 \frac{d}{d\xi^0} (\frac{\partial x}{\partial \xi^0}) c (\frac{d\xi^0}{d\tau})^2 - m_0 \frac{d}{d\xi^0} (\frac{\partial x}{\partial \xi^1}) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau}] \quad (34)
\end{aligned}$$

As,

$$\begin{aligned}
F_\xi^1 &= \frac{1}{(1 + \frac{a_0}{c^2} \xi^1) (\frac{d\xi^0}{d\tau})^2} [-A \frac{\partial x}{\partial \xi^0} \frac{d\xi^0}{d\tau} + B \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau}] \\
&= - \frac{1}{(1 + \frac{a_0}{c^2} \xi^1) (\frac{d\xi^0}{d\tau})} \frac{\partial x}{\partial \xi^0} [f^0 - m_0 \frac{d}{d\xi^0} (\frac{\partial t}{\partial \xi^0}) c (\frac{d\xi^0}{d\tau})^2 - m_0 \frac{d}{d\xi^0} (\frac{\partial t}{\partial \xi^1}) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau}] \\
&+ \frac{1}{(1 + \frac{a_0}{c^2} \xi^1) (\frac{d\xi^0}{d\tau})} \frac{\partial t}{\partial \xi^0} [f^1 - m_0 \frac{d}{d\xi^0} (\frac{\partial x}{\partial \xi^0}) c (\frac{d\xi^0}{d\tau})^2 - m_0 \frac{d}{d\xi^0} (\frac{\partial x}{\partial \xi^1}) \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau}] \quad (35)
\end{aligned}$$

In this time,

$$\begin{aligned}
\frac{d}{d\xi^0} (\frac{\partial t}{\partial \xi^0}) &= \frac{d\xi^1}{d\xi^0} \cosh(\frac{a_0 \xi^0}{c}) \frac{a_0}{c^2} + (\frac{c^2}{a_0} + \xi^1) \sinh(\frac{a_0 \xi^0}{c}) \frac{a_0^2}{c^3} \\
\frac{d}{d\xi^0} (\frac{\partial x}{\partial \xi^0}) &= \frac{d\xi^1}{d\xi^0} \sinh(\frac{a_0 \xi^0}{c}) \frac{a_0}{c^2} + (\frac{c^2}{a_0} + \xi^1) \cosh(\frac{a_0 \xi^0}{c}) \frac{a_0^2}{c^3} \\
\frac{d}{d\xi^0} (\frac{\partial t}{\partial \xi^1}) &= \cosh(\frac{a_0 \xi^0}{c}) \frac{a_0}{c}, \quad \frac{d}{d\xi^0} (\frac{\partial x}{\partial \xi^1}) = \sinh(\frac{a_0 \xi^0}{c}) \frac{a_0}{c} \quad (36)
\end{aligned}$$

Therefore, Eq(34) is by Eq(31),Eq(36). Lorentz force F_ξ^0 is in Rindler space-time.

$$\begin{aligned}
F_\xi^0 &= \frac{1}{(1 + \frac{a_0}{c^2} \xi^1) (\frac{d\xi^0}{d\tau})} \cosh(\frac{a_0 \xi^0}{c}) \\
&\times [f^0 - m_0 \{ \frac{d\xi^1}{d\xi^0} \cosh(\frac{a_0 \xi^0}{c}) \frac{a_0}{c^2} + (\frac{c^2}{a_0} + \xi^1) \sinh(\frac{a_0 \xi^0}{c}) \frac{a_0^2}{c^3} \} c (\frac{d\xi^0}{d\tau})^2]
\end{aligned}$$

$$\begin{aligned}
& -m_0 \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} \\
& - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)} \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
& \times [f^1 - m_0 \left\{ \frac{d\xi^1}{d\xi^0} \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0^2}{c^3} \right\} c \left(\frac{d\xi^0}{d\tau}\right)^2 \\
& - m_0 \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau}] \tag{37}
\end{aligned}$$

Therefore, Eq(35) is by Eq(31),Eq(36).Lorentz force F_ξ^1 is in Rindler space-time.

$$\begin{aligned}
& F_\xi^1 \\
& = - \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)} \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} \\
& \times [f^0 - m_0 \left\{ \frac{d\xi^1}{d\xi^0} \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0^2}{c^3} \right\} c \left(\frac{d\xi^0}{d\tau}\right)^2 \\
& - m_0 \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau} \\
& + \frac{1}{\left(1 + \frac{a_0}{c^2} \xi^1\right) \left(\frac{d\xi^0}{d\tau}\right)} \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \\
& \times [f^1 - m_0 \left\{ \frac{d\xi^1}{d\xi^0} \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0^2}{c^3} \right\} c \left(\frac{d\xi^0}{d\tau}\right)^2 \\
& - m_0 \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} \frac{d\xi^1}{d\tau} \frac{d\xi^0}{d\tau}] \tag{38}
\end{aligned}$$

In this time, the transformation of electromagnetic field is[1]

$$E_x = E_{\xi^1} ,$$

$$E_y = E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right),$$

$$\begin{aligned}
E_z &= E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
B_x &= B_{\xi^1}, \\
B_y &= B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \\
B_z &= B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)
\end{aligned} \tag{39}$$

Hence,

$$\begin{aligned}
f^0 &= F^0 \frac{dt}{d\tau} = q \frac{\vec{u}}{c} \cdot \vec{E} \frac{dt}{d\tau} = q \frac{\vec{u}^1}{c} \cdot \vec{E}, \vec{u} = \frac{d\vec{x}}{dt}, \vec{u}^1 = \frac{d\vec{x}}{d\tau} \\
&= q \frac{1}{c} \left[\frac{dx}{d\tau} E_x + \frac{dy}{d\tau} E_y + \frac{dz}{d\tau} E_z \right], \frac{dx}{d\tau} = \frac{\partial x}{\partial \xi^0} \frac{d\xi^0}{d\tau} + \frac{\partial x}{\partial \xi^1} \frac{d\xi^1}{d\tau} \\
&= \frac{q}{c} \left[\left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} \frac{d\xi^0}{d\tau} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{d\xi^1}{d\tau} \right] E_{\xi^1} \\
&\quad + \frac{d\xi^2}{d\tau} \{ E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \} \\
&\quad + \frac{d\xi^3}{d\tau} \{ E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \}
\end{aligned} \tag{40}$$

$$\begin{aligned}
f^1 &= F^1 \frac{dt}{d\tau}, \\
F^1 &= q \left[E_x + \frac{1}{c} (u_y B_z - u_z B_y) \right], \vec{u} = \frac{d\vec{x}}{dt}, \vec{u}^1 = \frac{d\vec{x}}{d\tau} \\
f^1 &= F^1 \frac{dt}{d\tau} = q \left[E_x \frac{dt}{d\tau} + \frac{1}{c} \left(\frac{dy}{d\tau} B_z - \frac{dz}{d\tau} B_y \right) \right], \frac{dt}{d\tau} = \frac{\partial t}{\partial \xi^0} \frac{d\xi^0}{d\tau} + \frac{\partial t}{\partial \xi^1} \frac{d\xi^1}{d\tau} \\
&= q \left[\left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} \frac{d\xi^0}{d\tau} + \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{1}{c} \frac{d\xi^1}{d\tau} \right] E_{\xi^1} \\
&\quad + \frac{1}{c} \left\{ \frac{d\xi^2}{d\tau} (B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right)) \right. \\
&\quad \left. - \frac{d\xi^3}{d\tau} (B_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) - E_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right)) \right\}
\end{aligned} \tag{41}$$

In Eq(24), f^2 is

$$f^2 = m_0 \frac{d^2 y}{d\tau^2} = F_{\xi}^2 \frac{d\xi^0}{d\tau}, \quad F_{\xi}^2 = m_0 \frac{d}{dt} \left(\frac{dy}{d\tau} \right) \frac{dt}{d\xi^0} = F^2 \frac{dt}{d\xi^0} \quad (42)$$

Therefore, Lorentz force F_{ξ}^2 is in Rindler space-time.

$$\begin{aligned} F_{\xi}^2 &= m_0 \frac{d}{d\xi^0} \left(\frac{d\xi^2}{d\tau} \right) = F^2 \frac{dt}{d\xi^0} \\ &= q \left[E_y + \frac{1}{c} (u_z B_x - u_x B_z) \right] \frac{dt}{d\xi^0} \\ &= q \left\{ E_{\xi^2} \cosh\left(\frac{a_0 \xi^0}{c}\right) + B_{\xi^3} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \\ &\quad \times \left\{ \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \frac{1}{c} \frac{d\xi^1}{d\xi^0} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} + \frac{1}{c} \left\{ \frac{d\xi^3}{d\xi^0} B_{\xi^1} \right. \\ &\quad \left. - \left\{ \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{d\xi^1}{d\xi^0} \right\} \right. \\ &\quad \left. \times \left\{ B_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) + E_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \right\} \end{aligned} \quad (43)$$

In Eq(24), f^3 is

$$f^3 = m_0 \frac{d^2 z}{d\tau^2} = F_{\xi}^3 \frac{d\xi^0}{d\tau}, \quad F_{\xi}^3 = m_0 \frac{d}{dt} \left(\frac{dz}{d\tau} \right) \frac{dt}{d\xi^0} = F^3 \frac{dt}{d\xi^0} \quad (44)$$

Therefore, Lorentz force F_{ξ}^3 is in Rindler space-time.

$$\begin{aligned} F_{\xi}^3 &= m_0 \frac{d}{d\xi^0} \left(\frac{d\xi^3}{d\tau} \right) = F^3 \frac{dt}{d\xi^0} \\ &= q \left[E_z + \frac{1}{c} (u_x B_y - u_y B_x) \right] \frac{dt}{d\xi^0} \\ &= q \left\{ E_{\xi^3} \cosh\left(\frac{a_0 \xi^0}{c}\right) - B_{\xi^2} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \\ &\quad \times \left\{ \left(\frac{c^2}{a_0} + \xi^1 \right) \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c^2} + \frac{1}{c} \frac{d\xi^1}{d\xi^0} \sinh\left(\frac{a_0 \xi^0}{c}\right) \right\} \\ &\quad + \frac{1}{c} \left\{ \left\{ \left(\frac{c^2}{a_0} + \xi^1 \right) \sinh\left(\frac{a_0 \xi^0}{c}\right) \frac{a_0}{c} + \cosh\left(\frac{a_0 \xi^0}{c}\right) \frac{d\xi^1}{d\xi^0} \right\} \right. \end{aligned}$$

$$\times \{B_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) + E_{\xi^3} \sinh(\frac{a_0 \xi^0}{c})\} - \frac{d\xi^2}{d\xi^0} B_{\xi^1}\} \} \quad (45)$$

4. Energy-momentum in Rindler spacetime

In initial frame, energy-momentum is

$$\vec{p} = m_0 \frac{d\vec{x}}{d\tau}, \quad E = m_0 c^2 \frac{dt}{d\tau} \quad (46)$$

$$\begin{aligned} p_x &= m_0 \frac{dx}{d\tau} = m_0 \frac{\partial x}{\partial \xi^\alpha} \frac{d\xi^\alpha}{d\tau} \\ &= m_0 \frac{\partial x}{\partial \xi^0} \frac{cd\xi^0}{d\tau} + m_0 \frac{\partial x}{\partial \xi^1} \frac{d\xi^1}{d\tau} \\ &= m_0 (1 + \frac{a_0}{c^2} \xi^1) \sinh(\frac{a_0 \xi^0}{c}) \frac{cd\xi^0}{d\tau} + m_0 \cosh(\frac{a_0 \xi^0}{c}) \frac{d\xi^1}{d\tau} \end{aligned} \quad (47)$$

$$\begin{aligned} E &= m_0 c^2 \frac{dt}{d\tau} = m_0 c \frac{cdt}{d\xi^\alpha} \frac{d\xi^\alpha}{d\tau} \\ &= m_0 c \frac{\partial t}{\partial \xi^0} \frac{cd\xi^0}{d\tau} + m_0 c \frac{\partial t}{\partial \xi^1} \frac{d\xi^1}{d\tau} \\ &= m_0 c (1 + \frac{a_0}{c^2} \xi^1) \cosh(\frac{a_0 \xi^0}{c}) \frac{cd\xi^0}{d\tau} + m_0 c \sinh(\frac{a_0 \xi^0}{c}) \frac{d\xi^1}{d\tau} \end{aligned} \quad (48)$$

Hence, we can define energy-momentum in Rindler spacetime.

$$\begin{aligned} \vec{p}_\xi &= m_0 \frac{d\vec{\xi}}{d\tau}, \quad E_\xi = m_0 c^2 (1 + \frac{a_0}{c^2} \xi^1) \frac{d\xi^0}{d\tau} \\ p_x &= m_0 \frac{dx}{d\tau} = \sinh(\frac{a_0 \xi^0}{c}) \frac{E_\xi}{c} + \cosh(\frac{a_0 \xi^0}{c}) p_{\xi^1} \\ E &= m_0 c^2 \frac{dt}{d\tau} = \cosh(\frac{a_0 \xi^0}{c}) E_\xi + \sinh(\frac{a_0 \xi^0}{c}) p_{\xi^1} c \\ p_y &= m_0 \frac{dy}{d\tau} = m_0 \frac{d\xi^2}{d\tau} = p_{\xi^2}, \quad p_z = m_0 \frac{dz}{d\tau} = m_0 \frac{d\xi^3}{d\tau} = p_{\xi^3} \end{aligned} \quad (50)$$

Therefore, general case is

$$E_\xi^2 - p_\xi^2 c^2 = m_0^2 c^4 [(1 + \frac{a_0 \xi^1}{c^2})^2 \frac{(d\xi^0)^2}{d\tau^2} - \frac{1}{c^2} \frac{d\vec{\xi} \cdot d\vec{\xi}}{d\tau^2}] = m_0^2 c^4 \quad (51)$$

In special case, light is

$$E_\xi = p_\xi c \quad (52)$$

5. Conclusion

We find the electro-magnetic wave equation and function and the electro-magnetic force in uniformly accelerated frame. We define energy-momentum in Rindler space-time.

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