Interference Model to Calculate the Lepton Masses

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In this paper, we extend the original idea of *de Broglie matter-waves* into a special physical interaction form which has been abbreviated as *mass interference*. We postulated that all particles could form a two-particle quantum state containing two standing matter-waves. The estimable mass of the particles is the result of this standing matter-wave interference. For the practical use of the newly introduced mass interference interaction model, we have created a generalized form of the quantized harmonic oscillator known from Quantum Mechanics. Utilizing this new theory, we are presenting two successful calculation methods for the theoretical determination of the lepton masses. This new simple theory has been strengthened by the direct presence of the physical relation between the leptons and the neutrons, which has long been recognized.

1. Introduction

Perhaps the greatest challenge of modern theoretical physics is the calculation of elementary particle masses. First, we need to clarify the concept of particle. There are several different definitions of particles. Among them, the most common, place a restrictive condition on the mass of the particles. For example, our simplest definition is that the mass of particles must be smaller than the famous Planck mass:

$$m < m_{Pl} = \sqrt{\frac{\hbar c}{G}} = 2.2 \times 10^{-8} kg. \qquad (1.1)$$

This requirement permits defined particles to have a relatively large mass [1].

In a broader sense, the particle mass calculation is a calculation, not only of the *ground-state*, but also the calculation of the *excitation energies* of the nuclei, atoms and molecules. Currently, we are still very far from achieving this goal, but in many subfields resounding successes have been achieved in the modern history of physics. Of course, in the past, many physical models have been used to calculate the ground and exited state energies of particles, and we cannot speak about a final universal theory for these basic tasks.

Currently, four fundamental physical forces are conventionally accepted: the gravitational, the electromagnetic, and the strong and weak forces (the latter two are only typical of the nuclear interaction). It is well known, that in the 1970's the electromagnetic and weak forces were traced back to a common origin. As a result, the four forces have been reduced to three basic physical forces; the strong, the *electro-weak* and the gravitational force.

To further simplify things, for example, the interference of light can be considered an ordinary physical interaction. The interference phenomena occur widely in physics, the best known are the *acoustic interference*, *radio interference* and *quantum interference*. In the present work two theoretical calculation methods are given based on a *special mass interference models*. First we present a simple interference model for the calculation of the *lepton masses* based on the work of an American physicist, *Gerald Rosen* [2]. The motivation is perfectly understandable; the physical behavior of the leptons, their common *charges* and *half-spins* show a close relationship between them. It is merely their masses that are different. After the presentation of G. Rosen's result, we turn to a new variation of the *mass interference concept*, which leads to an interesting connection between leptons and nucleons, specifically with respect to the neutron.

2. Empirical background of the lepton masses

In the literature there are two empirical relations for the three lepton masses. The one of them is the famous Koide formula [3]:

$$\frac{m_e + m_m + m_t}{\left(\sqrt{m_e} + \sqrt{m_m} + \sqrt{m_t}\right)^2} = 0.666659(10) \approx \frac{2}{3}, \qquad (2.1)$$

where m_e = electron mass, m_m = muon mass and finally m_t = tau mass. In relation to the present work there is a more important but a less well-known formula for the lepton masses:

$$m_k \cong C_0\left[1 + \sqrt{2}\cos\alpha_k\right]^2; \ (k = 1,2,3); \\ C_0 = 313.85773\, MeV; \ \alpha_k = 2k\pi/3 + 2/9, \qquad (2.2)$$

where m_k = electron, muon and tau mass for *k=1, k=2 and k=3*, respectively. This formula was published by Gerald Rosen with the theoretical basement [4]. The accuracy of this formula is very good (given by G. Rosen):

$$0.51099650\, MeV = m_e(1 - 4.7 \times 10^{-6}); \\ 105.65891\, MeV = m_m(1 + 5.09 \times 10^{-6}); \qquad (2.3) \\ 1776.9764\, MeV = m_t(1 - 7.63 \times 10^{-6}).$$

Our task now is to give an alternative theoretical interpretation of this lepton mass formula. Our starting point is that the formula (2.2) for the lepton masses contains two parts; the one part is the constant C_0 and the remaining part depends on the type of lepton. The smallest mass is the electron mass what can be associated with the ground state of a supposed *common origin particle* of the leptons. The muon and tau masses can be regarding as the excited states of their common origin particle. Of course, a wide variety of possible theoretical explanations could exist, but now we present a very simple physical model called "*Mass Interference Model*".

3. The Mass Interference Model of the Leptons

The Mass Interference Model is a very simple theoretical model for the calculation of particle masses. The basic idea of mass interference is tightly connected to the famous *matter-wave axiom* of the French physicist *de Broglie* (1924). As we know from the history of quantum physics, de Broglie supposed that matter must not differ essentially from light, so matter must also have a wave as the light has. In 1926 this idea lead to the *Schrödinger equation* which is probably the most important equation of modern Quantum Physics. However, it appears that physicists did not find all the important consequences of the de Broglie axiom. Indeed, if we can perceive matter to be a wave, then there must be amplitude of the matter-wave. In classical physics, the energy of the wave is proportional to the amplitude-squared, so the energy (or mass) of the particle must be proportional to its matter-wave amplitude squared:

$$m_a \propto a^2. \tag{3.1}$$

Here "*a*" is the amplitude of the matter wave. Of course, from this expression, we cannot reach further conclusions, so we have to give further assumptions. In Section 1, we have expanded the definition of particles to include not only *ground state energy* (i.e. the *rest mass*), but also *excited state energies*. From this fact, we have to suppose that the particles contain at least two interactive sub-particles, namely two matter-waves, ensuring the existence of the excitation states. In a normal situation, these matter-waves form an *interfered standing wave* which we can observe as a particle. The mass of this particle will be proportional to the next expression:

$$m_a \propto a^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta \le (a_1 + a_2)^2. \tag{3.2}$$

The interaction (interference) of these two matter-waves causes *mass defect*; this is the *supplementary condition* for the *stability* of the resulting particle. This mass expression reminds us of the interference of light:

$$I(\mathbf{r}) \propto A^2 = A_1^2(\mathbf{r}) + A_2^2(\mathbf{r}) + 2A_1A_2\cos\delta(\mathbf{r}) \le \left[A_1(\mathbf{r}) + A_2(\mathbf{r})\right]^2, \tag{3.3}$$

where $I(\mathbf{r})$ is the intensity of light in the position $\mathbf{r}$ and *A1*, *A2* are the interfering light amplitudes. In the case of the special "mass interference" there is no position dependence, and, of course, no time dependence. The above expression of light interference can be written into an unusual form:

$$\begin{aligned} I(\mathbf{r}) \propto A^2 &= \left[A_1(\mathbf{r},\delta_1) + A_2(\mathbf{r},\delta_2)\right]^2 \le \\ &\le \left[A_1(\mathbf{r},\delta_1) + A_2(\mathbf{r},\delta_2)\right]^2_{\delta_1=\delta_2}. \end{aligned} \tag{3.4}$$

Similarly to this interference expression of the light we can also write the mass interference expression (3.2) to an alternative form:

$$m_a \propto a^2 = \left[a_1(\delta_1) + a_2(\delta_2)\right]^2 \le \left[a_1(\delta_1) + a_2(\delta_2)\right]^2_{\delta_1=\delta_2}. \tag{3.5}$$

This form of the mass interference expression directly leads to the G. Rosen formula (2.2):

$$m_a \propto a^2 = C_0\left[1 + a_2(\delta_2)/a_1(\delta_1)\right]^2 = C_0\left[1 + \sqrt{2}\cos\alpha_k\right]^2, \tag{3.6}$$

where

$$\begin{aligned} &C_0 = a_1(\delta_1)^2;\ 2C_0 = a_2(\delta_1)^2; \\ &a_2(\delta_2)/a_1(\delta_1) = \sqrt{2}\cos\alpha_k;\ (k = 1,2,3). \end{aligned} \tag{3.7}$$

This result shows that the masses of the two sub-particles are:

$$\begin{aligned} m_1 &= a_1^2 = C_0 = 313.85773\ MeV; \\ m_2 &= a_2^2 = 2C_0 = 627.71546\ MeV. \end{aligned} \tag{3.8}$$

Important to note, that the sum of these two sub-particle masses is approximately equal to the nucleon mass.

4. Update of Lepton Mass Calculations

We have repeated the calculations of G. Rosen using the CODATA 2010 mass data for the three leptons:

$$\begin{aligned} &Mass\ electron:\ m_e = 0.510998928\ MeV; \\ &Mass\ muon:\ m_m = 105.6583715\ MeV; \\ &Tau\ particle\ mass:\ m_t = 1776.82\ MeV. \end{aligned} \tag{4.1}$$

With the help of a careful fitting procedure, we have obtained for the value C_0 in (2.2):

$$C_0 = 313.848479...\ MeV. \tag{4.2}$$

The recalculated lepton masses are the next:

$$\begin{aligned} 0.51098144\ MeV &= m_e(1 - 3.42\times10^{-5}); \\ 105.655795\ MeV &= m_m(1 - 2.44\times10^{-5}); \\ 1776.92410\ MeV &= m_t(1 + 5.86\times10^{-5}). \end{aligned} \tag{4.3}$$

5. The Mass Oscillator Model (MOM)

There is another physical possibility, with respect to the definition of the interference of de Broglie-type matter-waves. In the *classical theory of gravity* there is a remarkable fact that the *gravitational self-energy* is proportional to the square of the selected mass. We can suppose that in this special case, the energy of the matter-wave is proportional to the gravitational energy of the mass, i.e. to the square of it. In this situation the interference equation (3.2) will be modified into the next form:

$$m_a^2 \propto a^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta \le (a_1 + a_2)^2. \tag{5.2}$$

Simplifying this expression we introduce the next variables:

$$m_r^2 = a_1^2 + a_2^2;\ m_s^2 = 2a_1a_2\cos\delta, \tag{5.3}$$

where m_r is a particle-specific constant and m_s is the variable part of the particle mass determining the ground and exited states of the particle. Important to note, that both subparts of the particle mass remain different from zero as we can check in the Rosen's formula. Finally, the new mass interference formula can be written into a general form (involving the proportionality factor):

$$\begin{aligned} &m^2 = m_r^2 + m_s^2; \\ &\left(m_r = cst.;\ m_s = m_s(\delta)\right). \end{aligned} \tag{5.4}$$

The recently elaborated studies have shown that this new mass interference formula is suitable for the calculation of the mass of some types of particles. For the first step, we must give the simplest expression for the particle mass spectrum. For this purpose we have introduced a recursive formula for the above defined interference expression:

$$\begin{aligned} &\left(m - \mu_{n+1}\right)^2 = m_r^2 + \left(m_s - \mu_n\right)^2; \\ &\left(\mu_0 = m_s,\ n = 0,1,2,...\right). \end{aligned} \tag{5.5}$$

When $m_r = 0$, this formula can be written:

$$\left(m - \mu_{n+1}\right)^2 = \left(m - \mu_n\right)^2;\ \left(n = 0,1,2,...\right). \tag{5.6}$$

Supposing that the elements of the μ_n series are different from zero and each other, the unique solution of (5.6) is the next:

$$m = (\mu_n + \mu_{n+1})/2;\ (n = 0,1,2,...). \quad (5.7)$$

This result reminds us to the energy levels of the *quantized harmonic oscillator:*

$$\begin{gathered}(E - a_{n+1})^2 = (E - a_n)^2;\ (n = 0,1,2,...);\\ E = mc^2;\ a_n = \mu_n c^2 = n\hbar\omega;\ c = speed\ of\ light.\end{gathered} \quad (5.8)$$

Regarding this interesting consequence of our considerations, we have named the recursive mass equation (5.5) the Mass Oscillator Model (MOM). This result can be characterizing as the simplest generalization of the well-known quantized harmonic oscillator model from the Quantum Mechanics.

6. MOM Calculations for the Lepton Masses

The new mass calculation model has enabled us to explore the relations between the lepton masses and the neutron mass. For the determination of the lepton masses we have used the recursive formula (5.5) in which, for the mass *m* we have chosen the mass of the neutron. The detailed investigation has lead us to the next coupled equations:

$$\begin{gathered} m = m_n,\ (m_n = neutron\ mass);\\ m_s = 2m_m,\ (m_m = muon\ mass);\\ m_e = 2\mu_4,\ (m_e = electron\ mass);\\ m_t = 2m_n - 4\mu_1 - \mu_2,\ (m_t = tau\ mass),\end{gathered} \quad (6.1)$$

where the *sub-masses* μ-s were determined by the introduced recursive formula (5.5). In the calculation we have chosen these entire four particle masses to be unknown variables. In the fitting procedure these variables were fitted to their own experimental values. The good results, which we obtained, confirmed us in the physical reality of our Mass Oscillator Model:

$$\begin{gathered} 939.350979\ MeV = m_n(1 - 2.28\times10^{-4});\\ 105.653483\ MeV = m_m(1 - 4.63\times10^{-5});\\ 0.511007\ MeV = m_e(1 + 1.56\times10^{-5});\\ 1777.280\ MeV = m_t(1 + 2.59\times10^{-4}).\end{gathered} \quad (6.2)$$

Summary

In this paper we have shown two lepton mass calculation methods using a new hypothesis for the physical interactions. Our initial motivation was a possible reinterpretation of de Broglie matter-wave theory. We added a new meaning for the matter-wave amplitudes and at the same time examined the interference of these mystic matter-waves. The common point of the introduced two theoretical models for the lepton mass calculation is the supposed interference between the component sub-masses behaving as matter-waves. The estimable mass of a particle is the consequence of the interference between these two matter-waves. Our unusual idea has been confirmed with the successful calculations of the lepton masses as has been shown in the above. It is a remarkable fact that we have also found the direct connection between the lepton and neutron masses in the frame of the newly introduced mass interference model named the Mass Oscillator Model (MOM.)

References

[1] http://en.wikipedia.org/wiki/List_of_particles

[2] http://home.comcast.net/~gerald-rosen/heuristicmpla.pdf

[3] http://en.wikipedia.org/wiki/Koide_formula

[4] G. Rosen, "Heuristic Development of a Dirac-Goldhaber Model for Lepton and Quark Structure", *Modern Physics Letters* A, Vol. 22, No. 4, (2007) pp. 283-288.

[5] D. Sarkadi, http://www.scribd.com/doc/64984275/The-Structure-of-the-Physical-Mass-Introduction-to-the-Self-Momentum-Theory