Turbulence in Fluid Flow Analyzed Simply

"5% of the people think; 10% of the people think that they think; and the other 85% would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

The fluid flow in the Navier-Stokes solution may be characterized as follows. The x–direction solution consists of linear, parabolic, and hyperbolic terms. The first three terms characterize polynomial parabolas. The characteristic curve for the integral of the x–nonlinear term is a radical parabola. The integral of the y–nonlinear term is similar parabolically to that of the x–nonlinear term. The integral of the z–nonlinear term is a combination of two radical parabolas and a hyperbola. The polynomial parabolas alone produce laminar flow. It is illustrated that the polynomial parabolas, the radical parabolas and the hyperbola branches working together produce turbulence, rotation, swirling, and chaos

Introduction

Solutions of the Navier-Stokes Equations (x–direction)

See also viXra:1706.0193 and viXra:1512.0334

N-S Equation: $$\boxed{-\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial p}{\partial x}+\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}=\rho g_x}$$

Solutions

$$V_x=-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt\pm\sqrt{2hg_xx}+\frac{ng_xy}{V_y}+\frac{qg_xz}{V_z}+\frac{\psi_y(V_y)}{V_y}+\frac{\psi_z(V_z)}{V_z}+C_9$$
$$P(x)=d\rho g_x x;\quad (a+b+c+d+h+n+q=1)\quad V_y\neq 0,\ V_z\neq 0$$

Summary for the fractional terms of the x–direction

$\frac{ng_xy}{V_y}$ and $\frac{qg_xz}{V_z}$ in terms of x, y, z and t

$$\frac{ng_xy}{V_y}=\frac{-(ng_x)(-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_1x+C_3y+C_5z+\beta_5g_zt\pm\sqrt{2\beta_8g_zz})}{\beta_7g_z}$$

$$\frac{qg_xz}{V_z}=\frac{-(qg_xz)\{[(\beta_7g_zy)(-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt\pm\sqrt{2hg_xx}]-[CE]\}}{(\beta_7g_zy)(qg_xz-\beta_6g_zx)}$$

$$(CE=-(ng_xy)(-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_{14}x+C_{15}y+C_{16}z+\beta_5g_zt\pm\sqrt{2\beta_8g_zz})$$

For communication purposes, each of the terms containing the even powers x^2, y^2 and z^2 will be called a polynomial parabola, and each of the terms containing the square roots $\pm\sqrt{x}$, $\pm\sqrt{y}$ and $\pm\sqrt{z}$ will be called a radical parabola. Also, each of the terms containing variables in the denominator will be called a hyperbola. The terms, polynomial parabola, radical parabola and hyperbola will be used interchangeably with what produces these profiles.

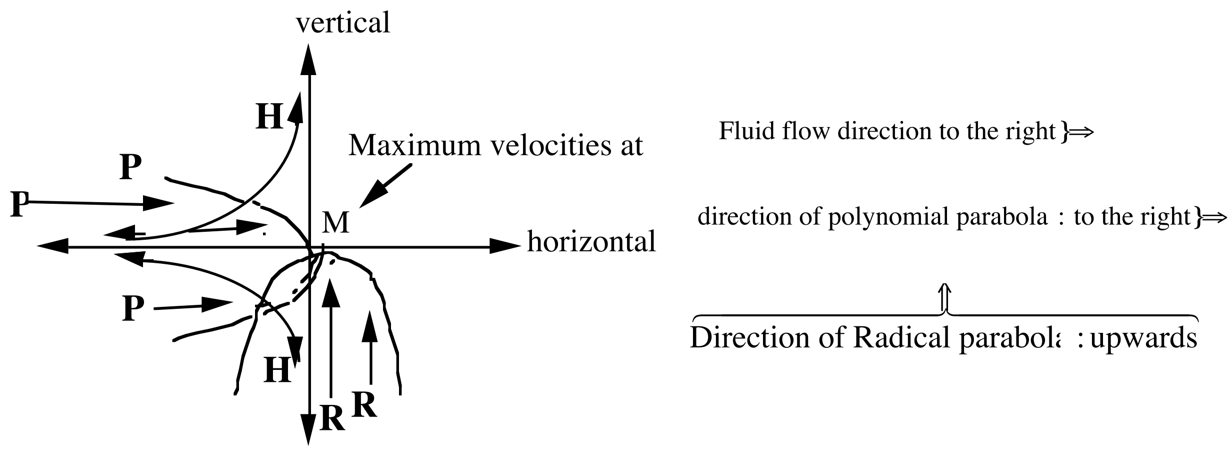
Turbulence Occurrence

We cover two approaches, namely Approach A and Approach B.
In Approach A, the fluid flow direction as well as that of the polynomial parabolas is horizontally to the right; while the direction of the radical parabolas is upwards. In Approach B, the fluid flow direction as well as that of the polynomial parabolas is vertically downwards in the page, while the direction of the radical polynomials is to the left.

Approach A

In the Navier-Stokes solutions, during fluid flow, the polynomial parabolas, the radical parabolas, and the hyperbolas are present at any speed. The polynomial parabolas are prominent and dominate flow while the radical parabolas are dormant at low speeds, and consequently, the flow is laminar. At a low speed, a radical parabola (or a polynomial parabola susceptible to radicalization) is not very active, since the radicand of the parabola is small and consequently, the square root is small. When the speed becomes large, and certain Reynolds numbers are reached, the "x" in $\sqrt{2hg_x x}$ becomes large and therefore the radical parabola becomes active. Note that the radical parabola will be moving at right angles to the direction of fluid flow, the direction of which is also that of the axis of symmetry of the dominating polynomial parabola. In the figure below, assume that flow is to the right. Then while the axis of symmetry of the polynomial parabola (**P**) is in the horizontal direction, the axis of symmetry of the radical parabola (**R**) would be in the vertical direction (that is, at right angles to fluid flow direction). Also, note the branches of the hyperbola (**H**) in the second and third quadrants. For each branch of the hyperbola, one end becomes asymptotic to the axis of symmetry of the polynomial parabola, while the other end becomes asymptotic to the axis of symmetry of the radical parabola, and thereby "interlocking" the polynomial parabola and the radical parabola together.

Velocity Profiles



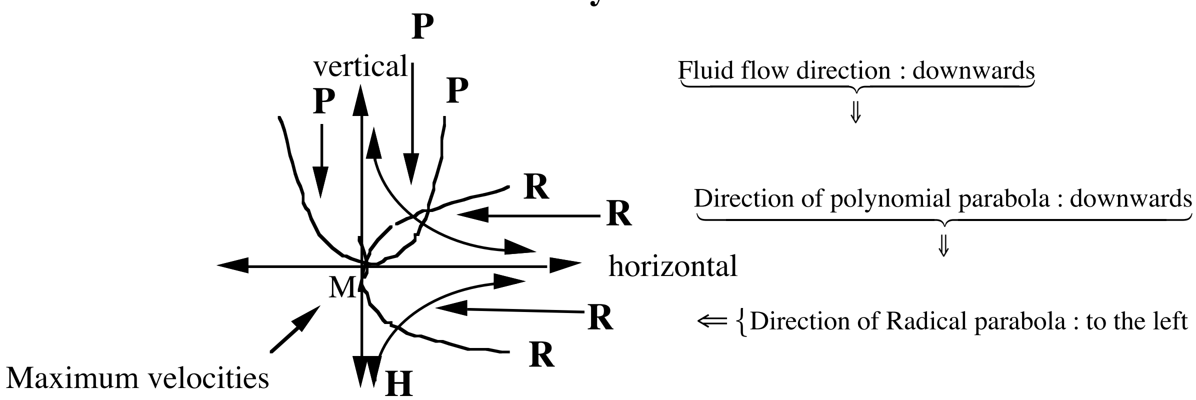
P--Polynomial parabola; R--Radical parabola; H--Hyperbola;

Therefore, the polynomial parabola, the radical parabola and the branches of the hyperbola become connected together to form a system such that any changes in one of them affect the behavior of the others. Any action that increases the flow velocity such that certain Reynolds numbers are reached, increases the " x " in $\sqrt{2hg_x x}$ and consequently, increases the effect of the radical parabola which is in direction at right angles to the fluid flow direction. The radical parabolas would be moving, from various positions, to the left or to the right, at right angles to the direction of fluid flow, noting that the direction of fluid flow is the direction of the polynomial parabolas, and at the same time, the hyperbolas will be moving asymptotically to fluid flow direction and asymptotically to direction of the radical parabola as in the figure. Thus, while the dominating polynomial parabolas are moving to the right, and the radical parabolas are moving at right angles to direction of fluid flow, the hyperbolas would be moving asymptotically to the axes of symmetry of the polynomial and radical parabolas, resulting in deviation from laminar flow and producing flows such as vortex flow, swirling flow, and turbulent flow. Imagine the polynomial parabolas pulling to the right, while the radical parabolas are pushing upwards, with the hyperbola halves pressing against the axes of the parabolas and the resulting deviation from laminar flow to turbulence and chaos.

Approach B

In the Navier-Stokes solutions, during fluid flow, the polynomial parabolas, the radical parabolas, and the hyperbolas are present at any speed. The polynomial parabolas are prominent and dominate flow while the radical parabolas are dormant at low speeds, and consequently, the flow is laminar. At a low speed, a radical parabola (or a polynomial parabola susceptible to radicalization) is not very active, since the radicand of the parabola is small and consequently, the square root is small. When the speed becomes large, and certain Reynolds numbers are reached, the "x" in $\sqrt{2hg_x x}$ becomes large and therefore the radical parabola becomes active. Note that the radical parabola will be moving at right angles to the direction of fluid flow, the direction of which is also that of the axis of symmetry of the dominating polynomial parabola. In the figure below, assume that flow is downwards. Then while the axis of symmetry of the polynomial parabola (**P**) is in the vertical direction, the axis of symmetry of the radical parabola (**R**) would be in the horizontal direction (that is, at right angles to fluid flow direction). Also, note the branches of the hyperbola (**H**) in the first and fourth quadrants. For each branch of the hyperbola, one end becomes asymptotic to the axis of symmetry of the polynomial parabola, while the other end becomes asymptotic to the axis of symmetry of the radical parabola, and thereby "interlocking" the polynomial parabola and the radical parabola together.

Velocity Profiles



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Design Applications

With regards to the variables x, y, and z, the parabolicity of the even powers x^2, y^2 and z^2 and the parabolicity of $\pm\sqrt{x}$, $\pm\sqrt{y}$ and $\pm\sqrt{z}$ hint at inverse relations. For examples, $V_x = x^2$ and $V_x = \pm\sqrt{x}$ are inverse relations of each other, $V_x = y^2$ and $V_x = \pm\sqrt{y}$ are inverse relations of each other, $V_x = z^2$ and $V_x = \pm\sqrt{z}$ are inverse relations of each other. The implications of knowing these relationships is that if one knows the steps, rules or formulas for designing for laminar flow, one can deduce the steps, rules or formulas for designing for turbulent flow by reversing the steps and using opposite operations in each step of the corresponding laminar flow design. Thus for every method, or formula for laminar flow, there is a corresponding method, formula for turbulent flow design. Similarly, if one knows the steps, rules or formulas for designing for turbulent flow, one can deduce the steps, rules or formulas for designing for laminar flow by reversing the steps and using opposite operations in each step of the corresponding turbulent flow design. Thus for every method, or formula for turbulent flow design, there is a corresponding method, formula for laminar flow design. (see also, "Power of Ratios" book by A. A. Frempong, p. 28).

References:

For paper edition of the above paper, see p,259 of the book entitled "Power of Ratios" , Second Edition, by A. A. Frempong, published by Yellowtextbooks.com. Without using ratios or proportion, the author would never be able to split-up the Navier-Stokes equations into sub-equations which were readily integrable. The impediment to solving the Navier-Stokes equations for over 150 years (whether linearized or non-linearized) has been due to finding a way to split-up the equations. Since ratios were the key to splitting the Navier-Stokes equations, and solving them, the solutions have also been published in the " Power of Ratios" book which covers definition of ratio and applications of ratio in mathematics, science, engineering, economics and business fields.

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