ON THE GEOMETRIC STRUCTURE OF THE SPATIOTEMPORAL MANIFOLD

Vu B Ho

Advanced Study, 9 Adela Court, Mulgrave, Victoria 3170, Australia

Email: vubho@bigpond.net.au

Abstract: In this work we will discuss the geometric structure of the spatiotemporal manifold which appears apparently as a multiverse whose intrinsic geometric structures will be shown to be resulted from geometric interactions between space and time. The geometric and topological structures of the total spatiotemporal manifold are formed from the geometric interactions of the decomposed cells from the base space of the total spatiotemporal manifold which is considered as a fiber bundle. In particular we will discuss in details spacetime which has the mathematical structure of a 6-sphere bundle in which the dynamics of the fibers is resulted from the geometric interactions of different types of decomposed cells that give rise to various relationships between space and time. We also show that the concept of the spatiotemporal manifold being viewed as a multiverse endowed with the structure of a CW complex was in fact suggested by Newton himself in his book Opticks. In Newton's multiverse, the expansion of space can be seen as a geometric evolution which redistributes to smooth out irregularities of the spatiotemporal manifold.

1. Introduction

Even though it can be argued that it should be classified as a philosophical rather than scientific inquiry, the concept of the so-called multiverse that has been introduced into modern physics is worth investigating [1,2]. Despite the idea is regarded to be scientifically based but in fact identical to the idea of multi-worlds that had been proposed and widely taught in traditional religions, such as Buddhism [3], the modern conceptual formulation of the multiverse is vaguely hypothesised because the proposed idea is not uniquely defined but varies from one hypothesis to another depending on the assumed structure and nature of each universe within the multiverse. By definition, a multiverse is a collection of parallel universes that exist together and contain all forms of physical existence, regardless of what forms the physical existence may exist. It is obvious from this definition that the term multiverse is no more than the statement that there exist different forms of physical objects in different forms of spacetime structures in a total spatiotemporal continuum. The spacetime structures are classified according to how they can be perceived. On the other hand, as shown in our recent works that quantum mechanics can be formulated in line with Einstein's perception that physics should be formulated in terms of the intrinsic geometric structures of spacetime similar to the geometric formulation of his general relativity [4]. In such theories, particles are no longer simple objects but they are endowed with geometric and topological structures and the interactions between particles depend on the intrinsic mathematical structures, such as those of a CW complex [5,6,7], and the probabilistic character of quantum mechanics may be due to the fact that the mathematical objects that are used for their formulations may have a probabilistic characteristic [8]. Therefore, if quantum mechanics can be formulated in a deterministic manner within the framework of statistical mechanics in classical physics then the concept of multiverse emerged from quantum theories will remain but be given a different meaning with different perspective. Despite the fact that the formulation of multiverse can be classified into levels, in this work we will discuss only two cases when a multiverse is assumed to consist of infinitely many universes beyond the cosmic horizon of our observable universe and a multiverse which consists of simultaneous worlds as interpreted in quantum mechanics and we will show that both of these types of multiverse can be formulated in the form of a fiber bundle whose fiber structures will determine the appearance of an observable spacetime. Therefore, the amount of universes depends on the number of fiber structures can be endowed on the base space of the spatiotemporal manifold. Even though the amount could be infinite, however, in this work we will only discuss those fiber structures that can be formed from the spatial and temporal geometric interactions that are associated with the decompositions of n-cells for $n = 0, 1, 2$ and 3.

2. Newton's multiverse

The idea that a multiverse consists of infinitely many universes beyond the cosmic horizon of our observable universe is probably best described by Newton himself: ... *since Space is divisible in infinitum, and Matter is not necessarily in all places, it may be also allow'd that God is able to create Particles of Matter of several Sizes and Figures, and in several Proportions to Space, and perhaps of different Densities and Forces, and thereby to vary the Laws of Nature, and make Worlds of several sorts in several Parts of the Universe* ... [1,9]. It is seen from Newton's suggestion of a non-uniform physical structure of the Universe through the statement *Worlds of several sorts in several Parts of the Universe* that the Universe is not a multiverse that consists of parallel universes as hypothesised from the probabilistic formulation of quantum mechanics but only different spacetime structures at different places of the Universe as a whole [10]. So, according to Newton, the initial Universe was inhomogeneous. In the mind of God, there should be some good reasons for such distribution of matter densities. To build a cosmic engine, for instance. Or, if the Almighty wants to create a cosmic biological system He simply creates a dynamics to smooth it out so that different sorts of biodiversity at every level of biological organisation can rightfully emerge from the evolutionary process. The redistributing procedure can be seen physically as a movement of matter from a dense place to somewhere less dense or mathematically as a geometric evolution by decompositions of n-cells from the base spacetime of the differentiable spatiotemporal manifold. For a local observation, this process can also be apparently seen as an expansion of the observable universe, which is only one of several Parts in Newton's total Universe. Mathematically and physically, this can be described as follows. As discussed in our previous works that all physical objects that possess the geometric and topological structures of a differentiable manifold are assumed to decompose submanifolds in the forms of 0-cells, 1-cells, 2-cells and 3-cells. We also suggested that there are forces that are associated with these decomposed submanifolds. The Universe as a whole can be assumed to be endowed with the structure of a CW complex M which is capable of decomposing n-cells. In particular, the forming and releasing of a 3-cell from a 3-dimensional manifold M can be expressed as a decomposition in the form $M = M\#S^3$. If we assume Newton's suggestion that worlds of several sorts in several parts of the Universe are made up of matter of different densities and forces then we can assume that the physical interactions associated with the forming and releasing of 3-cells are geometric processes that smooth out irregularities of the intrinsic geometric structure of the spatiotemporal manifold. The geometric irregularities can be viewed physically as an inhomogeneous distribution of matter in space and the forming and releasing of the S^3 cells as an expansion. A similar geometric process that smooths out an inhomogeneous distribution of a substance can be realised on the surface of a 2-dimensional sphere. In order to smooth out the irregularities, 1-cells in the form of circles can be formed and released from a position with dense substance and the geometric process is viewed as a local expansion. With this realisation, the geometric process of decomposition of 3-cells S^3 to smooth out irregularities of the distribution of matter in the observable universe can be formulated in terms of general relativity in which the change of intrinsic geometric structures of the manifold is due to the change of mathematical objects that define the manifold. These mathematical objects are perceived as physical entities like the energy-momentum tensor and the equations that describe the changes can be obtained from mathematical identities, such as Bianchi identities, the geometric Ricci flow or Einstein field equations of general relativity. Furthermore, based on observations from the observable universe, the cosmological evolution can be investigated by assuming the cosmological principle which states that at large scale the spatial component of the observable universe at any given cosmic time is homogeneous and isotropic. In this case we can apply Einstein field equations of general relativity given as follows [11,12]

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R - \Lambda g_{\alpha\beta} = \kappa T_{\alpha\beta} \tag{1}$$

With these assumptions, it is shown that the cosmological evolution can be described by the Robertson-Walker metric [13,14,15]

$$ds^2 = c^2dt^2 - S^2(t)\left(\frac{1}{1-kr^2}(dr)^2 + r^2(d\theta)^2 + r^2\sin^2\theta(d\phi)^2\right) \tag{2}$$

with the energy-momentum tensor of the form

$$T_{\alpha\beta} = (\rho + p)u_\alpha u_\beta - pg_{\alpha\beta} \tag{3}$$

Now, as shown in our works on the Ricci flow [16,17], the cosmological evolution can be described by the evolution equation of the Ricci flow [18]. It was shown that by applying the Lie differentiation with respect to a vector field X^μ, we may propose the following tensor equation to describe a covariant Ricci flow

$$L_X g_{\alpha\beta} = \kappa R_{\alpha\beta} \tag{4}$$

where κ is a dimensional constant. Applying Weyl's postulate by introducing comoving synchronous coordinate systems, the covariant Ricci flow given by Equation (4) is reduced to the evolution equation

$$\frac{\partial g_{\alpha\beta}}{c\partial t} = kR_{\alpha\beta} \tag{5}$$

where k is a dimensional constant. Furthermore, as shown in our work on cosmological evolution that if we also assume a cosmological evolution with a line element of the form

$$ds^2 = Dc^2dt^2 - A(x,y,z,t)(dx^2 + dy^2 + dz^2) \tag{6}$$

where D is constant then we obtain the following evolution equation

$$-\frac{3}{c^2D}\frac{\partial^2 A}{\partial t^2} + \frac{2}{A}\nabla^2 A + \frac{3}{ck}\frac{\partial A}{\partial t} + \frac{3}{2A^2}(\boldsymbol{\nabla} A)^2 = 0 \tag{7}$$

In particular, if the Ricci scalar R is constant then the quantity $A(x,y,z,t)$ for the line element given in Equation (6) can be found as

$$A(x,y,z,t) = A_0(x,y,z)e^{\frac{ckR}{3}t} \tag{8}$$

where A_0 is an initial spacelike hypersurface of the spatiotemporal manifold [19].

3. Multiverse as a fiber bundle

The concept of a multiverse that consists of a parallel existence of universes emerged during the formulating period of quantum mechanics, in particular with the work of Hugh Everett III in his thesis on the many-worlds interpretation of quantum mechanics [10], even though there is a claim that the idea was originated by Schrödinger. Anyway, the fundamental idea is that of Schrödinger's cat, with the assumption that quantum mechanics can only describe the physical existence probabilistically but not in the framework of statistical mechanics in classical physics. We now show that it is possible to describe the concept of a multiverse in the sense given by the many-worlds interpretation of quantum mechanics by considering the total spatiotemporal continuum as a fiber bundle which admits different types of fibers for a single base space of spacetime manifold. We will discuss the structure of spacetime structures that are resulted from different relationships between space and time. The apparent geometric and topological structures of the total spatiotemporal manifold are due to the dynamics and the geometric interactions of the decomposed cells from the base space of the total spatiotemporal manifold. The decomposed cells form different types of fibers which may also geometrically interact with each other. In particular we will discuss in details spacetime which has the mathematical structure of a 6-sphere bundle in which the dynamics of the fibers is resulted from the geometric interactions of different types of decomposed cells that give rise to various relationships between space and time. In this case it is assumed that we can only perceive within our physical ability the appearance of the grown intrinsic geometric structures on the base space of the total spatiotemporal manifold and the base space itself may not be observable with a reasonable assumption that a physical object is not observable if it does not have any form of geometric interactions. It could be that the base space of the spatiotemporal manifold at the beginning was only a six-dimensional Euclidean spatiotemporal continuum R^6 which had no non-trivial geometric structures therefore contained no physical objects. So how were the physical entities formed from such a plain spacetime continuum? Even though we could suggest that physical objects could be formed as three-dimensional differentiable manifolds from mass points with contact forces associated with the decomposed 0-cells, it is hard to imagine how they can be formed from a plain continuum without assuming that there must be some form of spontaneous symmetry breaking of the vacuum. Anyway, since the apparent spacetime structures are due to decomposed cells from the base spacetime and since there are many different relationships that arise from the geometric interactions of the decomposed cells of different dimensions, therefore there are different spactime structures each of which can represent a particular spacetime structure and all apparent spacetime structures can be viewed as parallel universes of a multiverse. If we assume that the spatiotemporal manifold is described by a six-dimensional differentiable manifold M which is composed of a three-dimensional spatial manifold and a three-dimensional temporal manifold, in which all physical objects are embedded, then the manifold M can be decomposed in the form $M = M\#S_S^3\#S_T^3$, where S_S^3 and S_T^3 are spatial and temporal 3-spheres, respectively. As discussed in our work on the spacetime structures of the electromagnetic and matter waves [20], despite this form of decomposition can be used to describe gravity as a global structure it cannot be used as a medium for any other physical fields which possess a wave character. Therefore we would need to devise different types of decomposition to account for these physical fields. For example, we may assume that *n*-cells can be decomposed from the spatiotemporal manifold at each point of the spatiotemporal continuum. This is equivalent to considering the spatiotemporal manifold as a fiber bundle $E = B \times F$, where B is the base space, which is the spatiotemporal continuum, and the fiber F, which is the *n*-cells. In the following we will only consider an *n*-cell as an *n*-sphere S^n and the total spatiotemporal manifold M will be regarded as an *n*-sphere bundle. It is reasonable to suggest that there may exist physical fields that are associated with different dimensions of the *n*-spheres, however, we will consider only the case with $n = 6$ so that S^6 is homeomorphic to $S_S^3 \times S_T^3$, hence the medium for a physical field will be assumed to be composed of $S_S^3 \times S_T^3$ cells at each point of the spatiotemporal manifold. It is expected that the formulation of possible fibers of the spatiotemporal manifold should be derived from a general line element $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$. For the purposes of illustration, we will show in the following that these fibers can be described by assuming the general line element to take the form of a centrally symmetric metric in both pseudo-Euclidean relativity and Euclidean relativity. In pseudo-Euclidean relativity, a general six-dimensional centrally symmetric line element can be written as

$$ds^2 = e^\psi c^2dt^2 + c^2t^2\left(d\theta_T{}^2 + \sin^2\theta_T d\phi_T{}^2\right) - e^\chi dr^2 - r^2\left(d\theta_S{}^2 + \sin^2\theta_S d\phi_S{}^2\right) \tag{9}$$

If we rearrange the (θ, ϕ) directions of both the spatial and temporal cells so that they coincide, i.e., $\theta_S = \theta_T = \theta$ and $\phi_S = \phi_T = \phi$, then we have

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - (r^2 - c^2t^2)(d\theta^2 + \sin^2\theta d\phi^2) \tag{10}$$

There are profound differences in the structure of the spatiotemporal manifold that arise from the line element given in Equation (10). This line element can be re-written in the forms

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2\left(1 - \frac{c^2}{v^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{11}$$

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 + c^2t^2\left(1 - \frac{v^2}{c^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{12}$$

It is seen from Equation (11) that when $v \to \infty$ then the spatiotemporal cells will appear as the spacetime that we assume to perceive. On the other hand, from Equation (12), if $v \to 0$ then the spatiotemporal cells will appear as a temporal manifold in which time has three dimensions and space has one. These are two spacetime structures that seem to dominate the appearance of physical existence. A more special case that is also worth mentioning when $v = c$ and in this case the line elements given in Equations (11) and (12) are both reduced to

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 \tag{13}$$

This line element is similar to the line element of the 2-dimensional space-time manifold (t, r) that we considered in our previous work on the wave-particle duality in quantum physics that can be formulated by applying the principle of least action. The field equations of general relativity given in Equation (1) can be derived using the principle of least action $\delta S = 0$, where the action S is defined as $S = \int \left(\frac{1}{2\kappa}(R - 2\Lambda) + \mathcal{L}_M\right)\sqrt{-g}\, d^4x$, where $\mathcal{L}_M$ characterises matter fields. Since $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \equiv 0$ for $= 2$, we obtain $T_{\mu\nu} = \left(\frac{\Lambda}{\kappa}\right)g_{\mu\nu}$. In particular, if we assume further that $\psi(r, t) \equiv \chi(r, t)$, then $\psi(r, t)$ satisfies the equations

$$\frac{\partial\psi}{c\partial t} - \frac{\partial\psi}{\partial r} = 0 \tag{14}$$

$$\frac{\partial^2\psi}{c^2\partial t^2} - \frac{\partial^2\psi}{\partial r^2} = 0 \tag{15}$$

with the general solution of the form

$$\psi(t, r) = f(r + ct) \tag{16}$$

where $f(t, r)$ is an arbitrary function of t and r. Equation (14) is the transport equation, or the equation of continuity, in classical dynamics which describes a flow or movement of mass, charge, energy or momentum at a constant rate c in the negative direction of r. Even though Equation (14) does not have the status of Newton's dynamical equation to describe the dynamics of a particle, it represents a particle motion in a deterministic manner. On the other hand, the general solution given in Equation (16) is also a solution of the wave equation given in Equation (15). From these results it can be concluded that the function $\psi(t, r) = f(r + ct)$ can be used to describe the dynamics of either a particle or a wave.

Now, for the values of v in the range $0 < v < c$ and $c < v < \infty$ there may be other forms of spacetime structures that depend on possible relationships between space and time. It is also observed that for $0 < v < c$ then from Equation (12) the spatiotemporal manifold appears as a temporal manifold and for $c < v < \infty$ then from Equation (11) the spatiotemporal manifold appears as a spatial manifold. We also showed in our previous works that it is possible to classify geometric interactions associated with the decomposition of submanifolds from a three-dimensional differentiable manifold. In general, we may consider physical objects of any scale as dimensional differentiable manifolds of dimension n which can emit submanifolds of dimension $m \leq n$ by decomposition. However, in order to formulate a physical theory we would need to devise a mathematical framework that allows us to account for the amount of subspaces that are emitted or absorbed by a differentiable manifold. This assumption leads to the visualisation of physical objects as CW complexes. In order to describe the evolution of a geometric process as a physical interaction we assume that an assembly of cells of a specified dimension will give rise to a certain form of physical interactions and the intermediate particles, which are the force carriers of physical fields decomposed during a geometric evolution, may possess the geometric structures of the *n*-spheres and the n-tori. Therefore, for observable physical phenomena, the study of physical dynamics reduces to the study of the geometric evolution of differentiable manifolds. In particular, if a physical object is considered to be a three-dimensional manifold then there are four different types of physical interactions that are resulted from the decomposition of 0-cells, 1-cells, 2-cells and 3-cells and these cells can be associated with the corresponding spatial forces $F_n = k_n r^n$ and temporal forces $F_n = h_n t^n$, respectively. Therefore, we can assume that a general spatiotemporal force which is a combination of the spatial and temporal forces resulted from the decomposition of spatiotemporal *n*-cells of all dimensions to take the form

$$F = \sum_{n=-3}^{3} (k_n r^n + h_n t^n) \tag{17}$$

where k_n and h_n are constants which can be determined from physical considerations. We will discuss this situation further in the following using equations of motion from both the spatial and temporal Newton's second laws of motion

$$m\frac{d^2\mathbf{r}}{d\tau^2} = \mathbf{F} \tag{18}$$

$$D\frac{d^2\mathbf{t}}{ds^2} = \mathbf{F} \tag{19}$$

From Equations (17), (18) and (19), it is seen that a complete geometric structure would be a structure that is resulted from the relationship between space and time that satisfies the most general equation in the form

$$m\frac{d^2\mathbf{r}}{d\tau^2}+D\frac{d^2\mathbf{t}}{ds^2}=\sum_{n=-3}^{3}(k_n r^{n-1}\mathbf{r}+h_n t^{n-1}\mathbf{t}) \tag{20}$$

Whether such a general equation can be solved rigorously requires further investigation, in the mean time in the following we will consider simple cases in which a relationship between space and time can be found. The following presentation is rather suggestive therefore we will only consider radial motion in both formulations of spatial and temporal dynamics. Even though the calculations are simple, repeated and tedious we will give in details a few cases that involve the decomposition of *n*-cells to show that the appearance of the decomposed cells $S_S^3\times S_T^3$ that form the fibers of the total spatiotemporal manifold in fact can be viewed as parallel universes of a multiverse. In particular, we will show that there is a conversion between spatial and temporal submanifolds that may form a medium for a physical field, such as the electromagnetic field.

3.1 Spacetime structures due to spatial geometric interactions

As suggested in our work on a classification of geometric interactions that decomposed 1-cells can manifest either as a linear force $\mathbf{F}=k_1\mathbf{r}$ or a force of inverse law $\mathbf{F}=k_2\mathbf{r}/r^2$ or a combination of the two. Applying the spatial Newton's second law for radial motion for a linear force we obtain

$$m\frac{d^2r}{dt^2}=k_1 r \tag{21}$$

$$r=c_1e^{\sqrt{k_1/mt}}+c_2e^{-\sqrt{k_1/mt}} \tag{22}$$

If we consider the case $m>0$ and $k_1<0$ then we can obtain a simple solution

$$r=A\sin(\omega t) \tag{23}$$

where $\omega=\sqrt{-k_1/m}$. By differentiation, we obtain

$$\frac{dr}{dt}=A\omega\cos(\omega t) \tag{24}$$

If we assume a linear approximation between space and time for the values of $v{\sim}c$, i.e., $dr/dt\sim r/t=v$, then the line elements given in Equations (11) and (12) become

$$ds^2=e^{\psi}c^2dt^2-e^{\chi}dr^2-r^2\left(1-\frac{c^2}{A^2\omega^2\cos^2(\omega t)}\right)(d\theta^2+\sin^2\theta d\phi^2) \tag{25}$$

$$ds^2=e^{\psi}c^2dt^2-e^{\chi}dr^2+c^2t^2\left(1-\frac{A^2\omega^2\cos^2(\omega t)}{c^2}\right)(d\theta^2+\sin^2\theta d\phi^2) \tag{26}$$

There is a conversion between the spatial and temporal submanifolds of the 6-spherical cells that are decomposed from the total spatiotemporal manifold. On the other hand, by applying the temporal Newton's second law for radial motion for the linear force we obtain

$$D\frac{d^2t}{dr^2} = k_1 r \tag{27}$$

A general solution is given as

$$\frac{dt}{dr} = \frac{k_1}{2D}r^2 + \frac{1}{v_0} \tag{28}$$

$$t = \frac{k_1}{6D}r^3 + \frac{1}{v_0}r + t_0 \tag{29}$$

If we assume a linear approximation between space and time for the values of $v{\sim}c$, i.e., $dr/dt \sim r/t = v$, then the line elements given in Equations (11) and (12) become

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2\left(1 - c^2\left(\frac{k_1}{2D}r^2 + \frac{1}{v_0}\right)^2\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{30}$$

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 + c^2t^2\left(1 - \frac{1}{c^2\left(\frac{k_1}{2D}r^2 + \frac{1}{v_0}\right)^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{31}$$

In this case the apparent spacetime formed by the 6-spherical cells appears as spatial for $\left|c\left(\frac{k_1}{2D}r^2 + \frac{1}{v_0}\right)\right| < 1$ and temporal for $\left|c\left(\frac{k_1}{2D}r^2 + \frac{1}{v_0}\right)\right| > 1$. Now if we apply the spatial Newton's second law to the inverse law for radial motion then we obtain

$$m\frac{d^2r}{dt^2} = \frac{k_2}{r} \tag{32}$$

Solutions to Equation (32) can be found in terms of the inverse of the Gauss error function $\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-x^2}\,dx$ as follows [21]

$$r = \exp\left(\frac{-c_1 - 2a\,\mathrm{erf}^{-1}\left(\pm i\sqrt{\frac{2}{\pi}}\sqrt{ae^{\frac{c_1}{a}}(c_2+t)^2}\right)^2}{2a}\right) \tag{33}$$

Even though it is straightforward to calculate dr/dt from Equation (33) and apply to the line elements given in Equations (11) and (12), the results are rather cumbersome. On the other hand, if we apply the temporal Newton's second law to the inverse law for radial motion then we obtain

$$D\frac{d^2t}{dr^2} = \frac{k_2}{r} \quad (34)$$

Solutions can be found as

$$\frac{dt}{dr} = \frac{k_2}{D}\ln(r) + c_1 \quad (35)$$

$$t = \frac{k_2}{D}r\ln(r) + c_1 r + c_2 \quad (36)$$

If we assume a linear approximation between space and time for the values of $v \sim c$, i.e., $dr/dt \sim r/t = v$, then the line elements given in Equations (11) and (12) become

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2\left(1 - c^2\left(\frac{k_2}{D}\ln(r) + c_1\right)^2\right)(d\theta^2 + \sin^2\theta d\phi^2) \quad (37)$$

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 + c^2t^2\left(1 - \frac{1}{c^2\left(\frac{k_2}{D}\ln(r) + c_1\right)^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \quad (38)$$

In this case the apparent spacetime formed by the 6-spherical cells appears as spatial for $\left|c\left(\frac{k_2}{D}r\ln(r) + c_1\right)\right| < 1$ and temporal for $\left|c\left(\frac{k_2}{D}r\ln(r) + c_1\right)\right| > 1$.

The decomposed 2-cells from an elementary particle can manifest either as a square force $\mathbf{F} = k_3 r\mathbf{r}$ or a force of inverse square law $\mathbf{F} = k_4\mathbf{r}/r^3$ or a combination of the two. Applying the spatial Newton's second law to the square law for radial motion we obtain

$$m\frac{d^2r}{dt^2} = k_3 r^2 \quad (39)$$

Solutions to Equation (39) can be found in terms of the Weierstrass elliptic function $\wp$ as follows [21]

$$r = \left(\frac{6m}{k_3}\right)^{\frac{1}{3}}\wp\left(\left(\frac{k_3}{6m}\right)^{\frac{1}{3}}(t + c_1); 0, c_2\right) \quad (40)$$

We can also calculate dr/dt for Equation (40) and apply to the line elements given in Equations (11) and (12), the results are also rather cumbersome. On the other hand, by applying the temporal Newton's second law to the square force for radial motion we have

$$D\frac{d^2t}{dr^2} = k_3 r^2 \quad (41)$$

Solutions are found as

$$\frac{dt}{dr} = \frac{k_3}{3D} r^3 + \frac{1}{v_0} \tag{42}$$

$$t = \frac{k_3}{12D} r^4 + \frac{1}{v_0} r + t_0 \tag{43}$$

If we assume a linear approximation between space and time for the values of $v \sim c$, i.e., $dr/dt \sim r/t = v$, then the line elements given in Equations (11) and (12) become

$$ds^2 = e^{\psi} c^2 dt^2 - e^{\chi} dr^2 - r^2 \left(1 - c^2 \left(\frac{k_3}{3D} r^3 + \frac{1}{v_0}\right)^2\right) (d\theta^2 + \sin^2\theta d\phi^2) \tag{44}$$

$$ds^2 = e^{\psi} c^2 dt^2 - e^{\chi} dr^2 + c^2 t^2 \left(1 - \frac{1}{c^2 \left(\frac{k_3}{3D} r^3 + \frac{1}{v_0}\right)^2}\right) (d\theta^2 + \sin^2\theta d\phi^2) \tag{45}$$

In this case the apparent spacetime appears as spatial for $\left|c\left(\frac{k_3}{3D} r^3 + \frac{1}{v_0}\right)\right| < 1$ and temporal for $\left|c\left(\frac{k_3}{3D} r^3 + \frac{1}{v_0}\right)\right| > 1$. If we apply the spatial Newton's second law to the inverse square law for radial motion then we obtain

$$m \frac{d^2 r}{dt^2} = \frac{k_4}{r^2} \tag{46}$$

$$r = \pm \left(\frac{1}{c_1} t \sqrt{c_1 - \frac{2k_4}{mt}} + \frac{k_4}{m c_1^{\frac{3}{2}}} \ln\left(\sqrt{c_1} t \sqrt{c_1 - \frac{2k_4}{mt}} - \frac{k_4}{m} + c_1 t\right)\right) + c_2 \tag{47}$$

By applying the temporal Newton's second law to the inverse square law for radial motion we obtain

$$D \frac{d^2 t}{dr^2} = \frac{k_4}{r^2} \tag{48}$$

Solutions to Equation (48) can be found as

$$\frac{dt}{dr} = -\frac{k_4}{D}\frac{1}{r} + \frac{1}{v_0} \tag{49}$$

$$t = -\frac{k_4}{D} \ln(r) + \frac{1}{v_0} r + t_0 \tag{50}$$

If we assume a linear approximation between space and time for the values of $v \sim c$, i.e., $dr/dt \sim r/t = v$, then the line elements given in Equations (11) and (12) become

$$ds^2 = e^{\psi} c^2 dt^2 - e^{\chi} dr^2 - r^2 \left(1 - c^2 \left(-\frac{k_4}{D}\frac{1}{r} + \frac{1}{v_0}\right)^2\right) (d\theta^2 + \sin^2\theta d\phi^2) \tag{51}$$

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 + c^2t^2\left(1 - \frac{1}{c^2\left(-\frac{k_4}{D}\frac{1}{r} + \frac{1}{v_0}\right)^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \quad (52)$$

In this case the apparent spacetime appears as spatial for $\left|c\left(-\frac{k_4}{D}\frac{1}{r} + \frac{1}{v_0}\right)\right| < 1$ and temporal for $\left|c\left(-\frac{k_4}{D}\frac{1}{r} + \frac{1}{v_0}\right)\right| > 1$.

For the decomposition of 3-cells from a manifold, it has been considered that these cells can manifest either as a cube force $\mathbf{F} = k_5 r^2\mathbf{r}$ or a force of inverse cube law $\mathbf{F} = k_6\mathbf{r}/r^4$ or a combination of the two. Applying the spatial Newton's second law to the cube force for radial motion we obtain

$$m\frac{d^2r}{dt^2} = k_5 r^3 \quad (53)$$

On the other hand, by applying the temporal Newton's second law the cube force for radial motion we obtain

$$D\frac{d^2t}{dr^2} = k_5 r^3 \quad (54)$$

Solutions to Equation (54) are

$$\frac{dt}{dr} = \frac{k_5}{4D}r^4 + \frac{1}{v_0} \quad (55)$$

$$t = \frac{k_5}{20D}r^5 + \frac{1}{v_0}r + t_0 \quad (56)$$

If we assume a linear approximation between space and time for the values of $v{\sim}c$, i.e., $dr/dt \sim r/t = v$, then the line elements given in Equations (11) and (12) become

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2\left(1 - c^2\left(\frac{k_5}{4D}r^4 + \frac{1}{v_0}\right)^2\right)(d\theta^2 + \sin^2\theta d\phi^2) \quad (57)$$

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 + c^2t^2\left(1 - \frac{1}{c^2\left(\frac{k_5}{4D}r^4 + \frac{1}{v_0}\right)^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \quad (58)$$

In this case the apparent spacetime appears as spatial for $\left|c\left(\frac{k_5}{4D}r^4 + \frac{1}{v_0}\right)\right| < 1$ and temporal for $\left|c\left(\frac{k_5}{4D}r^4 + \frac{1}{v_0}\right)\right| > 1$. Applying the spatial Newton's second law to the inverse cube law for radial motion we obtain

$$m\frac{d^2r}{dt^2}=\frac{k_6}{r^3} \tag{59}$$

By applying the temporal Newton's second law to the inverse cube law for radial motion we obtain

$$D\frac{d^2t}{dr^2}=\frac{k_6}{r^3} \tag{60}$$

Solutions are given as

$$\frac{dt}{dr}=-\frac{k_6}{2D}\frac{1}{r^2}+\frac{1}{v_0} \tag{61}$$

$$t=\frac{k_6}{2Dr}+\frac{1}{v_0}r+t_0 \tag{62}$$

If we assume a linear approximation between space and time for the values of $v\sim c$, i.e., $dr/dt \sim r/t = v$, then the line elements given in Equations (11) and (12) become

$$ds^2=e^{\psi}c^2dt^2-e^{\chi}dr^2-r^2\left(1-c^2\left(-\frac{k_6}{2D}\frac{1}{r^2}+\frac{1}{v_0}\right)^2\right)(d\theta^2+\sin^2\theta d\phi^2) \tag{63}$$

$$ds^2=e^{\psi}c^2dt^2-e^{\chi}dr^2+c^2t^2\left(1-\frac{1}{c^2\left(-\frac{k_6}{2D}\frac{1}{r^2}+\frac{1}{v_0}\right)^2}\right)(d\theta^2+\sin^2\theta d\phi^2) \tag{64}$$

In this case the apparent spacetime appears as spatial for $\left|c\left(-\frac{k_6}{2D}\frac{1}{r^2}+\frac{1}{v_0}\right)\right|<1$ and temporal for $\left|c\left(-\frac{k_6}{2D}\frac{1}{r^2}+\frac{1}{v_0}\right)\right|>1$.

3.2 Spacetime structures due to temporal geometric interactions

In this section we will extend our previous discussions into temporal geometric interactions in which time considered as a three-dimensional continuum which may possess the geometric and topological structures of a CW complex. We showed in our work on temporal geometric interactions that these interactions could also be classified according to *n*-cells that are decomposed from elementary particles which are assumed to possess the geometric and topological structures of CW complexes in which the evolution of the geometric processes that involve with the intrinsic geometric structure of a manifold can be described by the Ricci flow and Einstein field equations of the gravitational field. We showed that interesting results could be obtained from temporal dynamics in which negative mass considered as a form of inertial reaction to the change of time, similar to the inertial reaction by inertial mass in spatial dynamics in Newtonian mechanics. In particular, we showed that magnetic monopole can be considered as a topological structure of the temporal manifold and Dirac relationship between the electric charge and the magnetic charge $\hbar c/q_e q_m = 2$ can be derived purely in terms of topology as $k/q_e q_m = n_S n_T$. From these considerations and if we assume a complete symmetry between space and time then a temporal differentiable manifold that is associated with an elementary particle should also be a CW complex. As in the case of spatial CW complexes, in order to describe the evolution of a temporal geometric process as a physical interaction we assume that an assembly of cells of a specified dimension will give rise to a certain form of physical interactions and the intermediate particles, which are the force carriers of physical fields decomposed during a geometric evolution, may possess the geometric structures of the n-spheres and the n-tori and there should also be a classification of the geometric interactions of the temporal CW complexes. In particular, if an elementary particle is considered to be composed of not only a three-dimensional spatial manifold but also a three-dimensional temporal manifold then there are four different types of temporal geometric interactions that are resulted from the decomposition of 0-cells, 1-cells, 2-cells and 3-cells. Spacetime structures that are formed from temporal geometric interactions can be formulated similar to the case of spatial geometric interactions. For example, for the case of 1-cells, it can be suggested that forces associated with 1-cells will manifest as either a linear force $\mathbf{F} = h_1 \mathbf{t}$ or a force of inverse law $\mathbf{F} = h_2 \mathbf{t}/t^2$ or a combination of the two. Now, if we apply the spatial Newton's second law for radial motion to the linear force then we obtain

$$m\frac{d^2r}{dt^2} = h_1 t \qquad (65)$$

Solutions to Equation (65) are found as

$$\frac{dr}{dt} = \frac{h_1}{2m}t^2 + v_0 \qquad (66)$$

$$r = \frac{h_1}{6m}t^3 + v_0 t + r_0 \qquad (67)$$

If we assume a linear approximation between space and time for the values of $v \sim c$, i.e., $dr/dt \sim r/t = v$, then the line elements given in Equations (11) and (12) become

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2\left(1 - \frac{1}{c^2\left(\frac{h_1}{2m}t^2 + v_0\right)^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \qquad (68)$$

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 + c^2t^2\left(1 - c^2\left(\frac{h_1}{2m}t^2 + v_0\right)^2\right)(d\theta^2 + \sin^2\theta d\phi^2) \qquad (69)$$

In this case the apparent spacetime appears as spatial for $\left|c\left(\frac{h_1}{2m}t^2 + v_0\right)\right| > 1$ and temporal for $\left|c\left(\frac{h_1}{2m}t^2 + v_0\right)\right| < 1$. On the other hand, by applying the temporal Newton's second law for radial motion to the linear force we obtain

$$D\frac{d^2t}{dr^2} = h_1 t \qquad (70)$$

$$t = c_1 e^{\sqrt{h_1/D}r} + c_2 e^{-\sqrt{h_1/D}r} \tag{71}$$

If $D < 0$ and $h_1 > 0$, where m is the inertial mass, then we have

$$t = A\sin(\omega r) \tag{72}$$

where $\omega = \sqrt{-h_1/D}$. By differentiation, we obtain

$$\frac{dt}{dr} = A\omega\cos(\omega r) \tag{73}$$

If we assume a linear approximation between space and time for the values of $v \sim c$, i.e., $dr/dt \sim r/t = v$, then Equations (11) and (12) become

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2\left(1 - \frac{1}{c^2\big(A\omega\cos(\omega r)\big)^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{74}$$

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 + c^2t^2\left(1 - c^2\big(A\omega\cos(\omega r)\big)^2\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{75}$$

In this case the apparent spacetime appears as spatial for $\left|c\big(A\omega\cos(\omega r)\big)\right| > 1$ and temporal for $\left|c\big(A\omega\cos(\omega r)\big)\right| < 1$. Applying the spatial Newton's second law to the inverse law for radial motion we obtain

$$m\frac{d^2r}{dt^2} = \frac{h_2}{t} \tag{76}$$

Solutions are

$$\frac{dr}{dt} = \frac{h_2}{m}\ln(t) + v_0 \tag{77}$$

$$r = \frac{h_2}{m}t\ln(t) + v_0t + r_0 \tag{78}$$

If we assume a linear approximation between space and time for the values of $v \sim c$, i.e., $dr/dt \sim r/t = v$, then the line elements given in Equations (11) and (12) become

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2\left(1 - \frac{1}{c^2\left(\frac{h_2}{m}\ln(t) + v_0\right)^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{79}$$

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 + c^2t^2\left(1 - c^2\left(\frac{h_2}{m}\ln(t) + v_0\right)^2\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{80}$$

In this case the apparent spacetime appears as spatial for $\left|c\left(\frac{h_2}{m}\ln(t)+v_0\right)\right|>1$ and temporal for $\left|c\left(\frac{h_2}{m}\ln(t)+v_0\right)\right|<1$. By applying the temporal Newton's second law to the inverse law for radial motion we obtain

$$D\frac{d^2t}{dr^2}=\frac{h_2}{t} \tag{81}$$

Solutions to Equation (81) can be found in terms of the inverse of the Gauss error function $\mathrm{erf}(z)=\frac{2}{\sqrt{\pi}}\int_0^z e^{-x^2}\,dx$ as follows [5]

$$t=\exp\left(\frac{-c_1-2a\,\mathrm{erf}^{-1}\left(\pm i\sqrt{\frac{2}{\pi}}\sqrt{ae^{\frac{c_1}{a}}(c_2+r)^2}\right)^2}{2a}\right) \tag{82}$$

4. Euclidean relativistic multiverse

As a further discussion, in this section we will extend our presentation on multiverse to the spatiotemporal manifold that is endowed with a Euclidean relativistic metric [22]. In Euclidean relativity, a general six-dimensional centrally symmetric line element can be written as

$$ds^2=e^{\psi}c^2dt^2+c^2t^2\left(d\theta_T{}^2+\sin^2\theta_T d\phi_T{}^2\right)+e^{\chi}dr^2+r^2\left(d\theta_S{}^2+\sin^2\theta_S d\phi_S{}^2\right) \tag{83}$$

If we also rearrange the (θ,ϕ) directions of both the spatial and the temporal cells so that they coincide, $\theta_S=\theta_T=\theta$ and $\phi_S=\phi_T=\phi$, then we have

$$ds^2=e^{\psi}c^2dt^2+e^{\chi}dr^2+(r^2+c^2t^2)(d\theta^2+\sin^2\theta d\phi^2) \tag{84}$$

The line element given in Equation (84) can be re-written in the forms

$$ds^2=e^{\psi}c^2dt^2+e^{\chi}dr^2+r^2\left(1+\frac{c^2}{v^2}\right)(d\theta^2+\sin^2\theta d\phi^2) \tag{85}$$

$$ds^2=e^{\psi}c^2dt^2+e^{\chi}dr^2+c^2t^2\left(1+\frac{v^2}{c^2}\right)(d\theta^2+\sin^2\theta d\phi^2) \tag{86}$$

As in the case of pseudo-Euclidean relativity, it is seen from Equation (85) that when $v\to\infty$ then the spatiotemporal cells appear as a Euclidean relativistic spatial manifold. On the other hand, from Equation (86) we see that if $v\to 0$ then the spatiotemporal cells appear as a Euclidean relativistic temporal manifold. However, for the special case when $v=c$ the line element given in Equation (85) reduces to the Euclidean relativistic line element of a spatial manifold and the line element given in Equation (86) reduces to the Euclidean relativistic line element of a temporal manifold. Therefore, in Euclidean relativity both spatial and temporal manifolds equally appear at the universal speed and the spatiotemporal manifold does not appear as a wave as in pseudo-Euclidean relativity. For example, consider the decomposition of 1-cells with the linear force $\mathbf{F} = k_1\mathbf{r}$ and the force of inverse law $\mathbf{F} = k_2\mathbf{r}/r^2$. Applying the spatial Newton's second law for radial motion for a linear force we obtain

$$m\frac{d^2r}{dt^2} = k_1 r \tag{87}$$

$$r = c_1 e^{\sqrt{k_1/m}t} + c_2 e^{-\sqrt{k_1/m}t} \tag{88}$$

If we consider only the case $m > 0$ and $k_1 < 0$ then we can obtain a solution as

$$r = A\sin(\omega t) \tag{89}$$

where $\omega = \sqrt{-k_1/m}$. By differentiation, we obtain

$$\frac{dr}{dt} = A\omega\cos(\omega t) \tag{90}$$

If we assume a linear approximation between space and time for the values of $v{\sim}c$, i.e., $dr/dt \sim r/t = v$, then the line elements given in Equations (85) and (86) become

$$ds^2 = e^{\psi}c^2dt^2 + e^{\chi}dr^2 + r^2\left(1 + \frac{c^2}{A^2\omega^2\cos^2(\omega t)}\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{91}$$

$$ds^2 = e^{\psi}c^2dt^2 + e^{\chi}dr^2 + c^2t^2\left(1 + \frac{A^2\omega^2\cos^2(\omega t)}{c^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{92}$$

It is observed from the above line elements that the Euclidean relativistic temporal continuum is defined for all values of t but the Euclidean relativistic spatial continuum is only defined for values of t for which $A\omega\cos(\omega t) \neq 0$.

References

[1] Wikipedia, *Multiverse.*

[2] M. Tegmark, *Parallel Universes*, Scientific American, Vol. **288**, 2003.

[3] Wikipedia, *Multiverse* (Religion).

[4] Vu B Ho, *Spacetime Structures of Quantum Particles* (Preprint, ResearchGate, 2017), viXra 1708.0192v2, Int. J. Phys. vol 6, no 4 (2018): 105-115. doi: 10.12691/ijp-6-4-2.

[5] Vu B Ho, *A Classification of Geometric Interactions* (Preprint, ResearchGate, 2018), viXra 1805.0329v1.

[6] Vu B Ho, *Temporal Geometric Interactions* (Preprint, ResearchGate, 2018), viXra 1807.0134v1.

[7] Vu B Ho, *On Dirac Negative Mass and Magnetic Monopole* (Preprint, ResearchGate, 2018), viXra 1806.0319v1.

[8] Vu B Ho, *On the Principle of Least Action*, ResearchGate (2016), viXra 1708.0226v2, Int. J. Phys, vol. 6, no. 2 (2018): 47-52. doi: 10.12691/ijp-6-4-2.

[9] Isaac Newton, *Opticks*, 1730.

[10] Hugh Everett, *Theory of the Universal Wavefunction*, PhD thesis, Princeton University, 1956.

[11] Ray D'Inverno, *Introducing Einstein's Relativity* (Clarendon Press, Oxford, 1992).

[12] A. Einstein, *The Principle of Relativity* (Dover Publications, New York, 1952).

[13] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation*, (W. H. Freeman and Company, New York, 1971).

[14] Lewis Ryder, *Introduction to General Relativity* (Cambridge University Press, Melbourne, 2009).

[15] J. A. Romeu, *Derivation of Friedman Equations* (Preprint, Universitat de Barcelona, 2014).

[16] Vu B Ho, *A Derivation of the Ricci Flow* (Preprint, ResearchGate, 2017), viXra 1708.0191v1.

[17] Vu B Ho, *A Covariant Ricci Flow* (Preprint, ResearchGate, 2017), viXra 1708.0424v1.

[18] Richard S. Hamilton, *Three-Manifolds with Positive Ricci Curvature*, J. Diff. Geo., **17** (1982) 255-306.

[19] Vu B Ho, *On the Cosmological Evolution* (Preprint, ResearchGate, 2017), viXra 1709.0051v1.

[20] Vu B Ho, *Spacetime Structures of Electromagnetic and Matter Waves* (Preprint, ResearchGate, 2018), viXra 1807.0419v1.

[21] WolframAlpha, *Differential Equations*.

[22] Vu B Ho, *Euclidean Relativity* (Preprint, ResearchGate, 2017), viXra 1710.0302v1.