The Superluminal Phenomenon of Light For The Kerr-Newman Black Hole

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Abstract We use the Kerr-Newman metric based on the general relativity to discuss the superluminal phenomenon of light at the black hole whether it is observable astronomically at infinity or the weakly gravitational place like on Earth. The black hole have the rotation term a and the charge term R_Q as well as the Schwarzschild radius R_S. The geodesic of light in the spacetime structure is $ds^2=0$ and the equation for three velocity components (dr/dt, $rd\theta/dt$, $r\sin\theta d\phi/dt$) is obtained in the spherical coordinate (r, θ, ϕ) with the coordinate time t. Then three cases of the velocity of light (dr/dt, 0, 0), (0, $rd\theta/dt$, 0), and (0, 0, $r\sin\theta d\phi/dt$) are discussed in this research. According to our discussions, only the case of (dr/dt, 0, 0) gives the possibility of the occurrence of the superluminal phenomenon for r between R_S and $(R_Q^2+a^2\sin^2\theta/2)/R_S$ at $\sin\theta>0$ when $R_S\sim R_Q$. The results reveal that the maximum speed of light and the range of the superluminal phenomenon are much related to the rotational term a and the charged term R_Q. It is at least reasonable at two poles and in the equatorial plane when light propagates along the radial direction. Generally speaking, the superluminal phenomena for light can possibly occur in these cases that the radial velocity dr/dt is dominant and the other two velocity components are comparably small. When the relative velocity between the observer coordinate frame and the black hole is not large, the superluminal phenomenon is possibly observable at infinity or in a weakly gravitational frame like on Earth.

Keywords: Superluminal phenomenon, black hole, Kerr-Newman metric, phase diagram of Quantum Chromodynamics

I. Introduction

The so called superluminal phenomenon [1] is an observation from a reference frame that the speed of particle exceeds this maximum c. It is also called the Faster-than-light (FTL) phenomenon and some laboratory experiment [2] has been reported and some astronomical observations [1,3-6] about this phenomenon have been revealed from the relativistically massive sources near supermassive bodies such as the black hole. Traditionally, the speed of light is limited in the special relativity with a maximal value of c in free space. As we know, the free space is the flat spacetime structure and this maximal speed of light is a well certified phenomenon in the special relativity. In this theory, such as an electron in the synchrotron accelerator always needs a lot of energy to reach its speed very close to c but not exceeding c. It is the relativistic effect that exists the mass-energy equivalence principle and the equivalent mass of the electron depends on its speed. Exceeding the speed of light seems not to be able to observe macroscopically on Earth. Nowadays, it continuously attracts some scientists to investigate this FTL phenomenon. When some report reveals this phenomenon, one always wants to explain it by the present theorem or try to break some concept such as the limitation of the speed of light to fit the phenomenon.

Gravitational time delay is another attracted astronomically phenomenon that the speed of light would slow down when light passes through a giant star [7-11]. This reveals that the observation of the light speed is affected by the gravity and the measured speed of light is not constant for an observer in a reference frame. Because the special relativity is based on the Minkowski metric describing a flat spacetime structure, it is not suitable to explain some astronomical phenomena. Gravitational time delay is a well-known fact predicted by general relativity, and the place nearby the supermassive star with strong gravity is good for observation. This phenomenon motivates us to think about a question whether it is possible on earth to observe the speed of light exceeding c near the supermassive bodies such as the black hole. It is the astronomical phenomenon and some astronomical observations show possibilities to investigate this kind of superluminal phenomenon for massive particles [1,3-6].

In this research, we study this observable phenomenon for light based on the general relativity with the Kerr-Newman metric [12-14] where the constant speed of light exists in a local frame with the proper time. Our discussions focus on the black hole and gives some special results for the possible occurrence of this superluminal phenomenon of light.

II. The Kerr-Newman metric and the speed of light

When we discuss the geodesic of light at the black hole, an appropriate choice is using the Kerr-Newman metric [12-14] because it considers the angular momentum J and charges Q of a black hole simultaneously. The rotation of a black hole inherits from the previous star and it may be charged because the black hole absorbs charged plasma from the high-temperature accretion clouds or neighboring stars. The expression of the Kerr-Newman metric in the spherical coordinate $(r,\ \theta,\ \phi)$ is

$$\begin{aligned} \mathrm{d}s^2 &= -c^2\mathrm{d}\tau^2 \\ &= \left(\frac{\mathrm{d}r^2}{\Delta}+\mathrm{d}\theta^2\right)\rho^2-(c\mathrm{d}t-a\sin^2\theta\mathrm{d}\phi)^2\frac{\Delta}{\rho^2} \\ &\quad +\left((r^2+a^2)\mathrm{d}\phi-ac\mathrm{d}t\right)^2\frac{\sin^2\theta}{\rho^2}, \end{aligned} \tag{1}$$

where ds is the invariant interval, τ is the proper time, t is the coordinate time, $a=J/Mc$ with mass M of the black hole is the rotational term, and

$$\rho^2=r^2+a^2\cos^2\theta. \tag{2}$$

$$\Delta = r^2 - rR_S + a^2 + R_Q^2. \tag{3}$$

The Schwarzschild radius is $R_S = 2GM/c^2$ and G is the gravitational constant. $R_Q^2 = KQ^2G/c^4$ is the term related to the charge Q and K is the Coulomb's constant. In addition, the coordinate time is the time read by the clock stationed at infinity where the proper time and coordinate time becomes identical [15]. The geodesic of light is $\mathrm{d}s^2=0$, then through deduction we have the velocity of light obeying the following equation

$$\begin{aligned}
&\frac{\rho^4}{\Delta(\Delta - a^2\sin^2\theta)}\left(\frac{\mathrm{d}r}{dt}\right)^2 + \frac{\rho^4}{r^2(\Delta - a^2\sin^2\theta)}\left(r\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 \\
&\quad - \frac{(\Delta a^2\sin^2\theta - (r^2 + a^2)^2)}{r^2(\Delta - a^2\sin\theta)}\left(r\sin\theta\frac{\mathrm{d}\phi}{dt}\right)^2 \\
&\quad - \frac{2ac(-\Delta + (r^2 + a^2))\sin\theta}{r(\Delta - a^2\sin^2\theta)}\left(r\sin\theta\frac{\mathrm{d}\phi}{dt}\right) = c^2.
\end{aligned} \tag{4}$$

In Eq. (4), $\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)$, $\left(r\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)$, and $\left(r\sin\theta\frac{\mathrm{d}\phi}{dt}\right)$ are the three velocity components of light in the spherical coordinate. It is also the geodesic of light in space. This way to obtain the velocity of light from $\mathrm{d}s^2=0$ has been used to get the velocity of light in the Schwarzschild metric [16-19]. It reveals that the velocity of light at the black hole is much different from the Minkowski spacetime structure, and the form in Eq. (4) is much complicated and dependent on the spherical coordinate, the mass, the angular momentum as well as the charge of a black hole. In the following, we discuss the possibility of the superluminal phenomenon for each velocity component individually.

Because we discuss the rotating black hole, we check the gravitational dragging [18] or the frame-dragging effect [16] to make sure the reasonableness of our results. The black hole has the angular velocity ω. We consider the instantaneously local initial system rotating with angular velocity ω and tangential velocity v respect to the black hole. Then a light beam in the equatorial plane ($\theta=\pi/2$) propagates in the radial direction toward the center of the Kerr-Newman black hole. Two persons at different initial frame observe the trajectories of light. One observer A is in this instantaneously dragging frame and the other observer B is in the non-rotating or very weak dragging frame far away the black hole. It is alike a situation that a man, the observer A, stays on a transparent train with a velocity v moving toward the right, and another man, the observer B, stand at rest on the platform. From the viewpoint of the observer B, a light beam perpendicular to the movement of the train passes through it directly as the light trajectory L_1 shown in Fig. 1. However, due to the dragging-frame effect, the observer A will see the same light beam propagating along the light trajectory L_2. It means that the observer on earth has the same viewpoint as that of the observer B when light propagates in the equatorial plane from the place far away the Kerr-Newman black hole along the radial direction to its center. The dragging-frame effect can be ignored in this situation. It is also true when light propagates toward two poles.

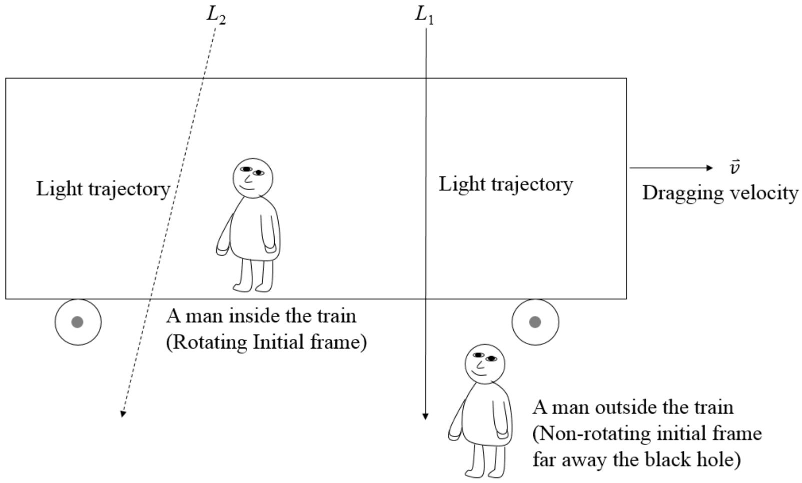


Figure 1. The observations of the light trajectory from different initial frames. The rotating initial frame has the tangential velocity v with respect to the Kerr-Newman black hole as the transparent train. An observer stays with the train and the other one is at rest on the platform. A light beam observed by the observer on the platform is perpendicular to the movement of the train as the light trajectory L_1. The same light beam observed by the observer moving with the train is the light trajectory L_2.

Before discussing, there is a basic requirement that the time is real at any reference frame. When we consider the geodesic along the radial direction without including the $\mathrm{d}\phi$ term, then it requires the $\mathrm{d}t^2$ term in Eq. (1) having

$$\rho^2 > 0, \tag{5}$$

$$(\Delta - a^2\sin^2\theta) > 0. \tag{6}$$

From Eq. (6), it can be expanded as

$$r^2 - rR_S + R_Q^2 + a^2\cos^2\theta > 0. \tag{7}$$

For any real r, Eq. (7) further requires the condition at $r=R_S/2$

$$R_S^2 \leq 4(a^2\cos^2\theta + R_Q^2). \tag{8}$$

It is the condition for the black hole at $r=R_S/2$ but at other place $r>0$ the condition is different. Such as at $r=R_S$, it only requires

$$R_Q^2 + a^2\cos^2\theta > 0, \tag{9}$$

and $r>R_S$ Eq. (7) automatically establishes till to the place far away from the black hole. Although the event horizon depends on θ, it is convenient to discuss the phenomenon using R_S as a reference position and the event horizon approximates to a spherical surface while $a<<R_S$ and $R_Q<<R_S$. Because the conditions of Eq. (8) holds for all θ, then it gives the lowest requirement

$$R_S^2 \leq 4R_Q^2 \tag{10}$$

at $r=R_S/2$ and $\theta=\pi/2$, and Eq. (9) gives

$$R_Q^2 > 0, \tag{11}$$

at $r=R_S$. This is just the condition for the charged black hole. The other requirement for the $\mathrm{d}r^2$ term in Eq. (1) is

$$\Delta > 0. \tag{12}$$

It also gives the other condition at $r=R_S/2$ and $\theta=\pi/2$

$$R_S^2 \leq 4(a^2 + R_Q^2). \tag{13}$$

From Eqs. (10) and (13), the minimum rotated condition can be obtained

$$0 \leq |a|. \tag{14}$$

However, at $r=R_S$ similar to Eq. (10), the requirement is

$$R_Q^2 + a^2 > 0, \tag{15}$$

which automatically establishes. The other factor worth mentioning is ρ^2 when it is at the denominator. It arises a mathematical singularity at $r=0$ and $\theta=\pi/2$. If the black hole has finite-size nucleus, this singularity will automatically remove because $J=0$, $Q=0$ as well as zero gravity at $r=0$. According to Eqs. (10) and (14), it means that even the massive star is very heavy, the formation of a black hole exists some basic conditions.

III. The Judgement of The Superluminal Requirements From The Velocity Component d*r*/d*t* of Light

According to Eq. (4), when we discuss the speed of light in the radial direction, t the other velocity components are zero. This choice is the convenient way to discuss the superluminal conditions. The rule used here is also applied to discuss other velocity components individually. We first focus on the $\mathrm{d}r/\mathrm{d}t$ velocity component to check whether the superluminal phenomenon of light exists or not. When an observer rests in a reference frame such as on Earth or the place with very weak gravitation, Eq. (1) gives the time relationship between the proper time and the coordinate time

$$\mathrm{d}\tau^2 = \frac{(\Delta - a^2 \sin^2\theta)}{\rho^2} \mathrm{d}t^2. \tag{16}$$

According to the equivalence principle in general relativity, the time dilation requires the coefficient of the $\mathrm{d}t^2$ less than one which gives the condition

$$r > R_Q^2 / R_S. \tag{17}$$

The range for this requirement also exists between 0 and R_S, and considering Eq. (11) at $r=R_S$ it requires

$$R_S^2 > R_Q^2 > 0. \tag{18}$$

When $r > R_S$, the time dilation automatically establishes because Eq. (18) gives the maximum of R_Q less than R_S. However, it seems that Eq. (17) is not well-defined for the region $R_Q^2/R_S > r \geq 0$. It is the reason that we adopt a singularity at the center of the black hole where all mass and charges gather there. When we use the model of a finite-size nucleus in the black hole, the Coulomb's repulsive force as well as the strong interaction make all particles not shrink to a singularity and the problem can be solved by establishing the charge distribution between 0 and R_S. Then R_Q is a function of r and θ related to the totally enclosed charges at (r,θ), that is,

$$R_Q = R_Q(r,\theta). \tag{19}$$

Eqs. (8) and (9) support this assumption. It also means that a is a function of $(r,\ \theta)$ between 0 and R_S which might be due to the distribution of its mass M. From the viewpoint of the rotational movement, Eq. (19) is reasonable for a rotationally charged black hole. It means that the charge distributions in Eq. (19) have to ensure Eq. (17) between 0 and R_S correctly and the time dilation is still correct from $r \geq 0$. Using $r = \alpha R_S$ with $0 < \alpha < 1$, then Eq. (7) becomes

$$(R_Q^2 + a^2\cos^2\theta)/(\alpha - \alpha^2) > R_S^2. \tag{20}$$

This inequality holds for all θ. For very small a, combing Eq. (17) with Eq. (20) gives

$$\alpha > R_Q^2/R_S^2 > (\alpha - \alpha^2). \tag{21}$$

By Eq. (21), it reveals the minimum and maximum of the charge distribution varying with the radial distance r form $r=0$ to $r=Rs$ as shown in Fig. 2 (a) and (b).

If the superluminal phenomenon occurs, it means $\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right) > c$. Then according to the dr/dt term in Eq. (4), it gives the requirement

$$\frac{\Delta(\Delta - a^2\sin^2\theta)}{\rho^4} > 1. \tag{22}$$

Because $\rho^4 > 0$, it becomes

$$(\Delta^2 - \Delta a^2\sin^2\theta - \rho^4) > 0. \tag{23}$$

Substituting Eqs (2) and (3) into Eq. (23) gives the following relation

$$\begin{aligned} 0 < 2r^2(-rR_S + R_Q^2) + r^2a^2\sin^2\theta + r^2R_S^2 - 2rR_SR_Q^2 + R_Q^4 \\ +(a^2 + a^2\cos^2\theta)(-rR_S + R_Q^2) + a^4\cos^2\theta\sin^2\theta. \end{aligned} \tag{24}$$

Further rearranging Eq. (24), then we have

$$\begin{aligned} (-rR_S + R_Q^2 + a^2\sin^2\theta/2)(2r^2 - rR_S + R_Q^2 + a^2/2 + 3a^2\cos^2\theta/2) \\ > a^4\sin^4\theta/4, \end{aligned} \tag{25}$$

or

$$(rR_S - R_Q^2 - a^2\sin^2\theta/2)(2r^2 - rR_S + R_Q^2 + a^2/2 + 3a^2\cos^2\theta/2) < -a^4\sin^4\theta/4. \tag{25$'$}$$

This inequality allows us to discuss the range for occurring superluminal phenomenon. First, the case at θ=0 or π is discussed, then Eq. (23) becomes

$$(-rR_S + R_Q^2)(2r^2 - rR_S + 2a^2 + R_Q^2) > 0. \tag{26}$$

The solutions of Eq. (26) are

$$-rR_S + R_Q^2 > 0 \text{ and} \tag{27a}$$
$$2r^2 - rR_S + 2a^2 + R_Q^2 > 0, \tag{27b}$$

or

$$-rR_S + R_Q^2 < 0 \text{ and} \tag{28a}$$
$$2r^2 - rR_S + 2a^2 + R_Q^2 < 0. \tag{28b}$$

From Eqs. (27a) and (27b), it gives the ranges of r that

$$R_Q^2/R_S > r, \tag{29a}$$

$$r < \frac{R_S - [R_S^2 - 8(2a^2 + R_Q^2)]^{1/2}}{4}, \tag{29b}$$

$$r > \frac{R_S + [R_S^2 - 8(2a^2 + R_Q^2)]^{1/2}}{4}, \tag{29c}$$

accompanying with the condition due to the real r

$$R_S^2 \geq 8(R_Q^2 + 2a^2). \tag{30}$$

However, Eq. (29a) doesn't satisfy the requirement in Eq. (17), and Eq. (30) obviously violates Eq. (13) at r=R_S/2 so we have to look for the other solution. Then Eqs. (28a) and (28b) give the other ranges for the superluminal phenomenon

$$R_Q^2/R_S < r, \tag{31a}$$

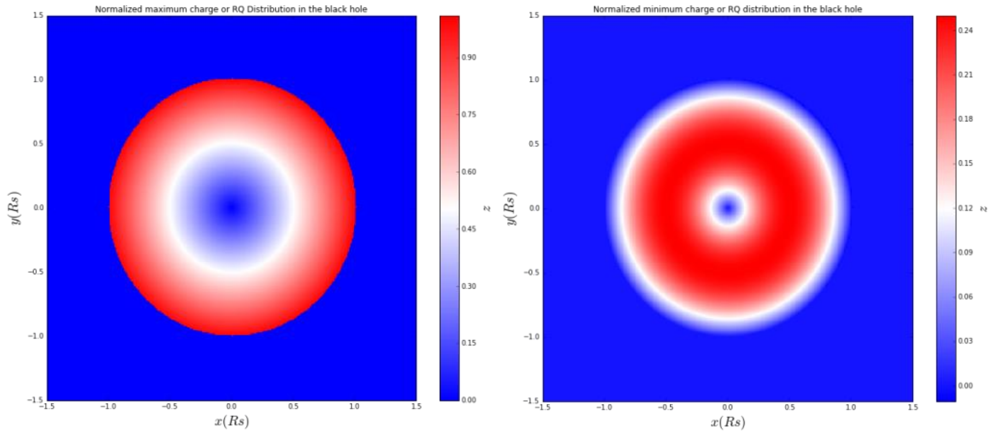
$$\frac{R_S - [R_S^2 - 8(2a^2 + R_Q^2)]^{1/2}}{4} < r < \frac{R_S + \left[R_S^2 - 8(2a^2 + R_Q^2)\right]^{1/2}}{4}, \tag{31b}$$

with the same condition as Eq.(30). Both solutions for r cannot give satisfied ranges. To sum up, the discussions from Eqs. (22) to (31) are for the requirements and solutions of v_r^2, not $v_{r,}$.

Then we discuss this phenomenon directly from the expression of the only velocity component (dr/dt) term obtaining from Eq. (4). This term is

$$v_{r,pole} = \left.\frac{\mathrm{d}r}{\mathrm{d}t}\right|_{\theta=0,\pi} = \pm c\frac{r^2 - rR_S + a^2 + R_Q^2}{r^2 + a^2}. \tag{32}$$

There are two expressions for (dr/dt), '+' means light leaving away from the center of the black hole, and '-' means light propagating toward the center of the black hole. So the superluminal solution leaving away the center satisfies the condition $R_Q^2/R_S > r$. However, it still violates the requirement in Eq. (17) and Eq. (18) gives $r<R_S$ in Eq (32). It means that the superluminal phenomenon doesn't happen when light leaves away from the center of the black hole at θ=0 or π. The other superluminal solution toward the center has the same r condition that the superluminal phenomenon also doesn't happen when light propagates towards the center of the black hole at θ=0 or π.



(a) (b)

Figure 2. (a) The minimal distribution of R_Q and (b) the maximal distribution of R_Q varying with the radial distance r for very small a. The color bar is in unit of R_S.

Next, Eq. (24) is discussed for any θ situations. A tricky way to solve Eq. (24) is to define

$$a^4\sin^4\theta/4 = (\alpha a^2)(\beta a^2). \tag{33}$$

Then Eq. (24) can be directly divided into two terms

$$(-rR_S + R_Q^2 + a^2\sin^2\theta/2) \geq \alpha a^2, \tag{34}$$

$$(2r^2 - rR_S + R_Q^2 + a^2/2 + 3a^2\cos^2\theta/2) \geq \beta a^2. \tag{35}$$

Eq. (34) gives the range for the superluminal phenomenon

$$r < [R_Q^2 + a^2(\sin^2\theta/2 - \alpha)]/R_S. \tag{36}$$

When Eq. (36) combines with Eq. (17), the range of r for the superluminal phenomenon is given

$$R_Q^2/R_S \leq r < [R_Q^2 + a^2(\sin^2\theta/2 - \alpha)]/R_S. \tag{37}$$

It means that the superluminal phenomenon possibly occurs when this condition in Eq. (37) satisfies. Then Eq. (37) further gives

$$\sin^2\theta/2 - \alpha > 0, \tag{38}$$

or

$$\sin^2\theta/2 > \alpha > 0. \tag{38'}$$

In Eq. (38'), the first condition of α is defined. In the meanwhile, the first condition of β is given as

$$\beta > \sin^2\theta/2. \tag{39}$$

In Eq (35), it gives the second condition of β between R_Q, a, and R_S for the superluminal phenomenon

$$[8(a^2/2 + 3a^2\cos^2\theta/2 + R_Q^2) - R_S^2]/8 > \beta a^2. \tag{40}$$

In the meanwhile, it also gives the second condition of α using Eqs. (33) and (39), that is,

$$2a^4\sin^4\theta/[8(a^2/2 + 3a^2\cos^2\theta/2 + R_Q^2) - R_S^2] < \alpha a^2. \tag{41}$$

Combining Eqs. (39) with (40), and (38') with (41), they give the limited conditions for α and β respectively

$$[8(a^2/2 + 3a^2\cos^2\theta/2 + R_Q^2) - R_S^2]/8a^2 > \beta > \sin^2\theta/2, \tag{42}$$

and

$$\sin^2\theta/2 > \alpha > 2a^2\sin^4\theta/[8(a^2/2 + 3a^2\cos^2\theta/2 + R_Q^2) - R_S^2]. \tag{43}$$

Furthermore, comparing the upper limitation with the lower limitation in Eq. (42) gives another condition for the other requirement of R_Q^2

$$8(2a^2\cos^2\theta + R_Q^2) > R_S^2. \tag{44}$$

This requirement is to consider the superluminal phenomenon. After discussing above conditions, the upper limitation of r can be obtained. Considering $R_S \sim R_Q$, Eq. (37) reveals that the superluminal phenomena can be observed in the range

$$R_S < r < R_S + a^2\sin^2\theta/2R_S, \tag{45}$$

which is function of θ. An example of the region occurring the superluminal phenomenon for a black hole with a=2R_S and R_Q=0.999R_S is given in Fig. 3(a), where the deep blue region is a spherical region with a radius of R_S and the yellow region means the region for the occurrence of the superluminal phenomena. It means that the region of $r \geq R_S$ is discussed, and R_S is the boundary because it exists the case of which the event horizon is close to a spherical surface when both $a \ll R_S$ and $R_Q \ll R_S$. The furthest distance from the center of the black hole in Fig. 3(a) is about 3R_S at the equator of $\theta=\pi/2$. All the rotating axes in Figs. 3(a) to 3(d) are parallel to the y-axis. According to Eq. (4) in the case of (dr/dt, 0, 0), the speed distribution of light is shown in Fig. 3(b) where the unit of the color bar is c. The velocity distribution matches the region of the superluminal occurrence in Fig. 3(a) and the maximum is about 2.20c at $r=R_S$ and $\theta=\pi/2$. When a is increased to $8R_S$ and R_Q is held at $0.999R_S$, the speed distribution of light is shown in Fig. 3(c). The maximum velocity of light is about 8c at $r=R_S$ and $\theta=\pi/2$, and the furthest distance of the superluminal phenomenon is $33R_S$ from the center of the black hole in Fig. 3(c). For the case of $a=20R_S$ and $R_Q=0.999R_S$, the speed distribution of light is shown in Fig. 3(d). The maximum speed of light is about 20c at $r=R_S$ and $\theta=\pi/2$, and the furthest distance of the superluminal phenomenon is $201R_S$ from the center of the black hole in Fig. 3(d). From Figs. 3(c) to 3(d), the occurrences of the high speed of light is centered more and more to the region near $\theta=\pi/2$. Our discussion is using the Kerr-Newman metric that is a spacetime solution in general relativity, so considering light bending near the high-speed rotational supermassive black holes, it possibly explains some astronomical observations about the superluminal phenomena from the relativistically massive jet [1,3-6]. This result can be also extended to some supermassive stars with very high density, large a, and R_Q.

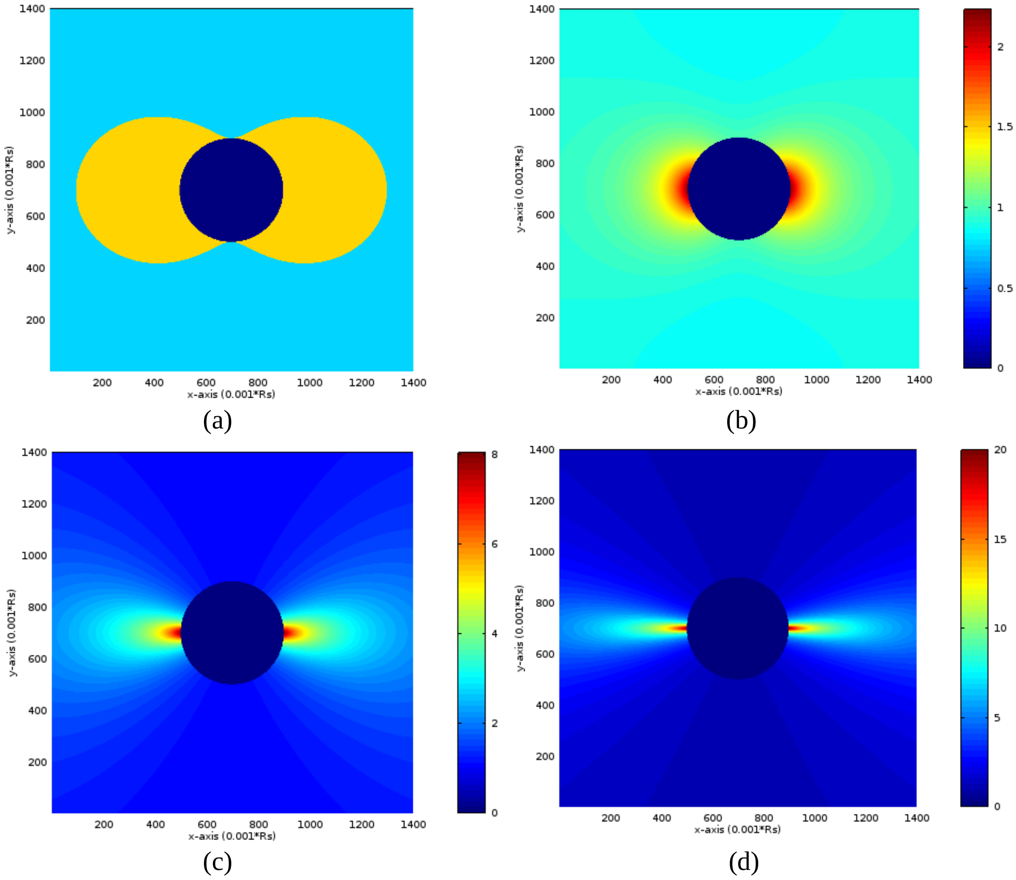


Figure 3. (a) The superluminal region is denoted by the yellow color. The center of the picture is a spherical region with a radius of R_S (deep blue color). In this case, $a = 2R_S$ and $R_Q=0.999R_S$. The maximum distance for the superluminal phenomenon from the center of the black hole is $3R_S$ at $\theta=\pi/2$. (b) The speed distribution of light at $a=2R_S$ and $R_Q = 0.999R_S$. (c) The speed distribution of light at $a=8R_S$ and $R_Q=0.999R_S$. The maximum distance of the superluminal phenomenon is $33R_S$ from the center of the black hole in this case. (d) The speed distribution at $a=20R_S$ and $R_Q=0.999R_S$. The maximum distance of the superluminal phenomenon is $201R_S$ from the center of the black hole in this case. All these cases the rotational axes are parallel to the y-axis and the color bars show in unit of c.

IV. The Judgement of The Superluminal Requirements For The Velocity Component *r*(d*θ*/d*t*) of Light

The second study case is the velocity component $r\mathrm{d}\theta/\mathrm{d}t$ term in Eq. (4). All the other velocity components are zero. This term is easy to check whether the superluminal phenomenon exists or not. Assuming that it happens, then

$$\frac{r^2(\Delta - a^2\sin^2\theta)}{\rho^4} > 1. \tag{46}$$

Expanding above equation, we have

$$r^2(-rR_S + R_Q^2) - r^2a^2\cos^2\theta - a^4\cos^4\theta > 0. \tag{47}$$

It can be further rearranged as

$$(-rR_S + R_Q^2 - a^2\cos^2\theta)r^2 > a^4\cos^4\theta. \tag{48}$$

Similar to the discussions of the velocity component $\mathrm{d}r/\mathrm{d}t$, a tricky way is to assume

$$\alpha^2\beta = \cos^4\theta. \tag{49}$$

Then Eq. (48) gives the requirements of r

$$r^2 > \alpha^2a^2, \tag{50}$$

$$-rR_S + R_Q^2 - a^2\cos^2\theta > \beta a^2. \tag{51}$$

Combining Eqs. (51) with (17), and considering the condition of Eq. (8), the range of r for the occurrence of the superluminal phenomenon is given by

$$R_Q^2/R_S < r < (R_Q^2 - a^2\cos^2\theta - \beta a^2)/R_S. \tag{52}$$

Because $\beta \geq 0$, this requirement doesn't exsti. Eq. (52) means that in this case of the velocity component $r\mathrm{d}\theta/\mathrm{d}t$ the superluminal phenomenon doesn't exist and isn't observable astronomically on Earth.

V. The Judgement of The Superluminal Requirements For The Velocity Component *r*sin*θ*(d*ϕ*/d*t*) of Light

The velocity component $r\sin\theta(\mathrm{d}\phi/\mathrm{d}t)$ term is the third case for discussing the possibility of the superluminal phenomenon. All the other velocity components are zero. From Eq. (4), the velocity equation for this case is

$$-\frac{(\Delta a^2\sin^2\theta - (r^2+a^2)^2)}{r^2(\Delta - a^2\sin\theta)}\left(r\sin\theta\frac{\mathrm{d}\phi}{dt}\right)^2$$

$$-\frac{2ac\big(-\Delta + (r^2+a^2)\big)\sin\theta}{r(\Delta - a^2\sin^2\theta)}\left(r\sin\theta\frac{\mathrm{d}\phi}{dt}\right) = c^2. \tag{53}$$

Next, we replace $r\sin\theta(\mathrm{d}\phi/\mathrm{d}t)$ with hc, where h is a real value. Then the equation becomes

$$-\frac{(\Delta a^2\sin^2\theta-(r^2+a^2)^2)}{r^2(\Delta - a^2\sin^2\theta)}h^2 - \frac{2a\left(-\Delta+(r^2+a^2)\right)\sin\theta}{r(\Delta - a^2\sin^2\theta)}h = 1. \tag{54}$$

If the superluminal phenomenon happens, it means h>1. Eq. (54) is the second-order equation in the general form $Ah^2+Bh+C=0$. It requires $0\leq B^2-4AC$ to make sure the real solutions existing. According to this, we have

$$0 \leq \frac{\{2a[-\Delta+(r^2+a^2)]\sin\theta\}^2-4[\Delta a^2\sin^2\theta-(r^2+a^2)^2](\Delta-a^2\sin^2\theta)}{r^2(\Delta - a^2\sin^2\theta)^2}. \tag{55}$$

After rearrangement, it gives

$$0 \leq \frac{4\left(\mathrm{r}^2+a^2-rR_S+R_Q^2\right)(r^2+a^2\cos^2\theta)^2}{r^2(\Delta - a^2\sin^2\theta)^2}, \tag{56}$$

or

$$0 \leq \frac{4\Delta\rho^4}{r^2(\Delta - a^2\sin^2\theta)^2}. \tag{56$'$}$$

Because ρ^4 ≥0 as well as the denominator $r^2(\Delta - a^2\sin^2\theta)^2 \geq 0$, it requires Δ ≥0 and r>0. The former condition has been shown in Eq. (8). Eq. (56') makes sure that Eq. (54) has real solutions and then we can further discuss whether the superluminal phenomenon exists or not in this case.

In the following, we solve Eq. (54) directly to obtain two solutions of h, that is,

$$h_\pm = \frac{-\frac{2a(-\Delta+(r^2+a^2))\sin\theta}{r(\Delta - a^2\sin^2\theta)} \pm \frac{2(r^2+a^2-rR_S+R_Q^2)^{1/2}(r^2+a^2\cos^2\theta)}{r(\Delta-a^2\sin^2\theta)}}{2\frac{(\Delta a^2\sin^2\theta-(r^2+a^2)^2)}{r^2(\Delta - a^2\sin^2\theta)}}$$

$$= \frac{-ra(-\Delta+(r^2+a^2))\sin\theta \pm r\,(r^2+a^2-rR_S+R_Q^2)^{1/2}(r^2+a^2\cos^2\theta)}{(\Delta a^2\sin^2\theta-(r^2+a^2)^2)}$$

$$= \frac{ra(rR_S-R_Q^2)\sin\theta \pm r\,(r^2+a^2-rR_S+R_Q^2)^{1/2}(r^2+a^2cos^2\theta)}{(r^2+a^2)(r^2+a^2\cos^2\theta)+(rR_S-R_Q^2)a^2sin^2\theta}. \tag{57}$$

It can be further expressed as

$$h_\pm = \pm\frac{r\left(r^2+a^2-rR_S+R_Q^2\right)^{1/2}}{r^2+a^2}$$

$$+r\left(rR_S-R_Q^2\right)a\sin\theta\frac{1\mp\left(r^2+a^2-rR_S+R_Q^2\right)^{1/2}a\sin\theta/(r^2+a^2)}{(r^2+a^2)(r^2+a^2\cos^2\theta)+\left(rR_S-R_Q^2\right)a^2\sin^2\theta}. \tag{58}$$

The other two expressions of $h_{\pm}$ are

$$h_{\pm} = \frac{r}{a\sin\theta} + \frac{r(r^2+a^2)(r^2+a^2\cos^2\theta)}{a\sin\theta}\frac{\pm\left(r^2+a^2-rR_S+R_Q^2\right)^{1/2}a\sin\theta/(r^2+a^2)-1}{(r^2+a^2)(r^2+a^2\cos^2\theta)+\left(rR_S-R_Q^2\right)a^2\sin^2\theta}. \tag{59}$$

Then the condition is whether h can be greater than one or not. Eq. (59) reveals a possible situation for

$$\frac{r}{a\sin\theta} > 1. \tag{60}$$

However, we have to discuss it with the second long term in the right-hand side and this velocity component at sinθ=0 in Eq. (59) is the same as the velocity component r(dθ/dt). It is not easy to deal with so we use the expression in Eq. (58) to judge the occurrence of the superluminal phenomenon. When considering the solution for $h_+>1$, the requirement from Eq. (58) is

$$r^2(r^2+a^2\cos^2\theta)^2\left(r^2+a^2-rR_S+R_Q^2\right) -[(r^2+a^2)(r^2+a^2\cos^2\theta)-(-rR_S+R_Q^2)a\sin\theta(r-a\sin\theta)]^2>0. \tag{61}$$

When sinθ~0, this requirement becomes

$$-a^2(r^2+a^2)^3+\left(-rR_S+R_Q^2\right)r^2(r^2+a^2)^2>0. \tag{62}$$

However, both terms in the left-hand side are negative when $r>R_S$, so the superluminal phenomenon doesn't occur at $r>R_S$ when sinθ~0. Next, we discuss all other cases of $\sin\theta$. Through expanding and rearranging Eq. (61), it gives the requirement

$$\sin\theta > \frac{1}{a(r-a\sin\theta)}\left[\frac{a^2(r^2+a^2\cos^2\theta)}{2(rR_S-R_Q^2)}+\frac{r^2(r^2+a^2\cos^2\theta)}{2(r^2+a^2)} + \frac{(rR_S-R_Q^2)a^2\sin^2\theta(r-a\sin\theta)^2}{2(r^2+a^2)(r^2+a^2\cos^2\theta)}\right]. \tag{63}$$

The three terms in the right-hand side of Eq. (63) are all positive. According to the geometric inequality in which the first term is equal to the third term, Eq. (63) can further simplify to the strict condition

$$\sin\theta > \frac{1}{a(r-a\sin\theta)}\left[\frac{a^2(r^2+a^2\cos^2\theta)}{2(rR_S-R_Q^2)}+\frac{r^2(r^2+a^2\cos^2\theta)}{2(r^2+a^2)} + \frac{(rR_S-R_Q^2)a^2\sin^2\theta(r-a\sin\theta)^2}{2(r^2+a^2)(r^2+a^2\cos^2\theta)}\right]$$

$$\geq \frac{1}{a(r-a\sin\theta)}\left[\frac{a^2\sin\theta(r-a\sin\theta)}{(r^2+a^2)^{\frac{1}{2}}}+\frac{r^2(r^2+a^2\cos^2\theta)}{2(r^2+a^2)}\right]. \tag{64}$$

It means the mostly possible place for the superluminal phenomenon in this case at $\sin\theta$ =1. It also requires $r>a\sin\theta$. This requirement needs three terms in right-hand side to be small enough. When we look at the pre-factor in the right-hand side of Eq. (64), it gives the minimum value when

$$a=\frac{r}{2\sin\theta}. \tag{65}$$

Using $\sin\theta$ =1 and combing the pre-factor, it gives the minimum

$$\frac{a\sin\theta}{(r^2+a^2)^{1/2}}+\frac{1}{a(r-a\sin\theta)}\frac{r^2(r^2+a^2\cos^2\theta)}{2(r^2+a^2)} \geq \frac{1}{\sqrt{5}}+\frac{8}{5}>1. \tag{66}$$

It means that Eq. (66) doesn't satisfy Eq. (63) because $\sin\theta\leq 1$ and the superluminal phenomenon doesn't occur in this case of the velocity component $r\sin\theta(\mathrm{d}\phi/\mathrm{d}t)$ when $r>0$.

VI. Discussion

Above discussions show that only the case of the velocity of ($\mathrm{d}r/\mathrm{d}t$, 0, 0) for light can possibly occur the superluminal phenomenon at $\theta>0$. The maximum speed of light is much related to the rotational term a and the charged term R_Q of a black hole. The results are at least reasonable at two poles and in the equatorial plane. The other two cases of the velocities of (0, $r\mathrm{d}\theta/\mathrm{d}t$, 0) and (0, 0, $r\sin\theta\mathrm{d}\phi/\mathrm{d}t$) for light don't have the possibility of the superluminal phenomenon. However, light can have at least one velocity component in the vicinity of a black hole. Generally speaking, the superluminal phenomenon also possibly occur in these cases of ($\mathrm{d}r/\mathrm{d}t$, $r\mathrm{d}\theta/\mathrm{d}t$, 0), ($\mathrm{d}r/\mathrm{d}t$, 0, $r\sin\theta\mathrm{d}\phi/\mathrm{d}t$), or ($\mathrm{d}r/\mathrm{d}t$, $r\mathrm{d}\theta/\mathrm{d}t$, $r\sin\theta\mathrm{d}\phi/\mathrm{d}t$). In those cases, the radial velocity component is dominant for the occurrences of the superluminal phenomena.

VII. Conclusion

The superluminal phenomenon is an attracted research and this phenomenon can be discussed based on the general relativity with a given spacetime structure. In this research, the Kerr-Newman metric is chosen for describing the spacetime structure at the black hole and its vicinity. The Kerr-Newman metric considers both a and R_Q terms that all kinds of the black hole at present knowledge are included. Because the black hole possesses strong gravity, it is a good astronomical example for studying the superluminal phenomenon. According to the Kerr-Newman metric, the geodesic as well as the velocity components of light can be established. In order to study this phenomenon, three velocity components are independently discussed, and they are (dr/dt, 0, 0), (0, rdθ/dt, 0), and (0, 0, $r\sin\theta$dϕ/dt). From our analysis, only the case of (dr/dt, 0, 0) has the possibility of the occurrence of the superluminal phenomenon between R_S and $[R_Q^2 + (a^2\sin^2\theta)/2]/R_S$ at $\sin\theta>0$ when $R_S \sim R_Q$. The result reveals that the superluminal phenomenon can possibly happen outer the black hole from the observer at infinity or in a reference frame with a very weak gravity. The maximum speed of light and the range of the superluminal phenomenon are much related to the rotational term a and the charged term R_Q of a black hole. The results are at least reasonable at two poles and in the equatorial plane when light propagates in the radial direction. Generally speaking, the superluminal phenomena for light can possibly occur in these cases that the radial velocity dr/dt is dominant and the other two velocity components are comparably small or zero. Furthermore, the superluminal phenomenon here just means the results of the measurements from an observer in a reference frame like on Earth. This conclusion can be also applied on some stars with very high density, large a, and big R_Q.

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