Lorentz Force in Rindler spacetime

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ABSTRACT

In the general relativity theory, we define the force in Rindler space-time We find Lorentz force (electromagnetic force) by electro-magnetic field transformations in Rindler space-time. In the inertial frame, Lorentz force is defined as 4-dimensional force. Hence, we had to obtain 4-dimensional force in Rindler space-time.

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1. Introduction

In the general relativity theory, our article's aim is that we find Lorentz force by electro-magnetic field transformations in Rindler space-time.

In inertial frame, Lorentz 4-force is

$$F^0 = m_0 \frac{d}{dt}(\frac{cdt}{d\tau}) = q\frac{\vec{u}}{c}\cdot\vec{E} \tag{1}$$

$$\vec{F} = m_0 \frac{d}{dt}(\frac{d\vec{x}}{d\tau}) = q[\vec{E} + \frac{\vec{u}}{c}\times\vec{B}], \quad \vec{u} = \frac{d\vec{x}}{dt} \tag{2}$$

We want to obtain the Lorentz 4-force in Rindler space-time. Hence, we define the force in Rindler space-time.

$$F_\xi^{\ 0} = m_0 \frac{d}{d\xi^0}(\frac{cd\xi^0}{d\tau})$$

$$\vec{F}_\xi = m_0 \frac{d}{d\xi^0}(\frac{d\vec{\xi}}{d\tau}) \tag{3}$$

Or,

$$F_\xi^{\ \mu} = m_0 \frac{d}{d\xi^0}(\frac{d\xi^\mu}{d\tau}) \tag{4}$$

Hence, Lorentz 4-force is in inertial frame

$$F^\alpha = m_0 \frac{d}{dt}(\frac{dx^\alpha}{d\tau}) \tag{5}$$

In this time, Minkowski force is in inertial frame or in Rindler space-time[13].

$$f^\alpha = m_0 \frac{d}{d\tau}(\frac{dx^\alpha}{d\tau}) = m_0 \frac{d^2x^\alpha}{d\tau^2} \tag{6}$$

$$f_\xi^{\ \mu} = m_0 \frac{d}{d\tau}(\frac{d\xi^\mu}{d\tau}) = m_0 \frac{d^2\xi^\mu}{d\tau^2} \tag{7}$$

2. Electro-magnetic Force in Rindler space-time

Minkowski force is

$$f^\alpha = m_0 \frac{d^2x^\alpha}{d\tau^2} = m_0 \frac{d}{d\xi^0}(\frac{\partial x^\alpha}{\partial\xi^\mu}\frac{d\xi^\mu}{d\tau})\frac{d\xi^0}{d\tau}$$

$$= m_0 \frac{d}{d\xi^0}(\frac{\partial x^\alpha}{\partial\xi^\mu})\frac{d\xi^\mu}{d\tau}\frac{d\xi^0}{d\tau} + m_0\frac{\partial x^\alpha}{\partial\xi^\mu}\frac{d}{d\xi^0}(\frac{d\xi^\mu}{d\tau})\frac{d\xi^0}{d\tau}$$

$$= m_0 \frac{d}{d\xi^0}(\frac{\partial x^\alpha}{\partial \xi^\mu})\frac{d\xi^\mu}{d\tau}\frac{d\xi^0}{d\tau} + F_\xi{}^\mu \frac{\partial x^\alpha}{\partial \xi^\mu}\frac{d\xi^0}{d\tau}$$

$$F_\xi{}^\mu = m_0 \frac{d}{d\xi^0}(\frac{d\xi^\mu}{d\tau}) \tag{8}$$

Hence,

$$f^0 = m_0 \frac{cd^2 t}{d\tau^2} = m_0 \frac{d}{d\xi^0}(\frac{\partial t}{\partial \xi^0})\frac{cd\xi^0}{d\tau}\frac{d\xi^0}{d\tau} + F_\xi{}^0 \frac{\partial t}{\partial \xi^0}\frac{d\xi^0}{d\tau}$$
$$+ m_0 \frac{d}{d\xi^0}(\frac{c\partial t}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau} + F_\xi{}^1 \frac{c\partial t}{\partial \xi^1}\frac{d\xi^0}{d\tau} \tag{9}$$

$$f^1 = m_0 \frac{d^2 x}{d\tau^2} = m_0 \frac{d}{d\xi^0}(\frac{\partial x}{c\partial \xi^0})\frac{cd\xi^0}{d\tau}\frac{d\xi^0}{d\tau} + F_\xi{}^0 \frac{\partial x}{c\partial \xi^0}\frac{d\xi^0}{d\tau}$$
$$+ m_0 \frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau} + F_\xi{}^1 \frac{\partial x}{\partial \xi^1}\frac{d\xi^0}{d\tau} \tag{10}$$

$$F_\xi{}^0 = m_0 \frac{d}{d\xi^0}(\frac{cd\xi^0}{d\tau}),\ F_\xi{}^1 = m_0 \frac{d}{d\xi^0}(\frac{d\xi^1}{d\tau})$$

Therefore,

$$F_\xi{}^0 \frac{\partial t}{\partial \xi^0}\frac{d\xi^0}{d\tau} + F_\xi{}^1 \frac{c\partial t}{\partial \xi^1}\frac{d\xi^0}{d\tau}$$
$$= f^0 - m_0 \frac{d}{d\xi^0}(\frac{\partial t}{\partial \xi^0})\frac{cd\xi^0}{d\tau}\frac{d\xi^0}{d\tau} - m_0 \frac{d}{d\xi^0}(\frac{c\partial t}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau} = A \tag{11}$$

$$F_\xi{}^0 \frac{\partial x}{c\partial \xi^0}\frac{d\xi^0}{d\tau} + F_\xi{}^1 \frac{\partial x}{\partial \xi^1}\frac{d\xi^0}{d\tau}$$
$$= f^1 - m_0 \frac{d}{d\xi^0}(\frac{\partial x}{c\partial \xi^0})\frac{cd\xi^0}{d\tau}\frac{d\xi^0}{d\tau} - m_0 \frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau} = B \tag{12}$$

If we represent by the matrix,

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \frac{\partial t}{\partial \xi^0}\frac{d\xi^0}{d\tau} & \frac{c\partial t}{\partial \xi^1}\frac{d\xi^0}{d\tau} \\ \frac{\partial x}{c\partial \xi^0}\frac{d\xi^0}{d\tau} & \frac{\partial x}{\partial \xi^1}\frac{d\xi^0}{d\tau} \end{pmatrix} \begin{pmatrix} F_\xi{}^0 \\ F_\xi{}^1 \end{pmatrix} \tag{13}$$

In this time, Rindler coordinate is

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0 \xi^0}{c}), x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0 \xi^0}{c}) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3$$

(14)

So,

$$\frac{\partial t}{\partial \xi^0} = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}, \quad \frac{\partial x}{c\partial \xi^0} = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}$$

$$\frac{c\partial t}{\partial \xi^1} = \sinh(\frac{a_0\xi^0}{c}), \quad \frac{\partial x}{\partial \xi^1} = \cosh(\frac{a_0\xi^0}{c}) \tag{15}$$

Hence,

$$Det = (\frac{\partial t}{\partial \xi^0}\frac{\partial x}{\partial \xi^1} - \frac{c\partial t}{\partial \xi^1}\frac{\partial x}{c\partial \xi^0})(\frac{d\xi^0}{d\tau})^2 = (1 + \frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})^2 \tag{16}$$

Therefore,

$$\begin{pmatrix} F_\xi^{\ 0} \\ F_\xi^{\ 1} \end{pmatrix} = \frac{1}{(1 + \frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})^2}\begin{pmatrix} \frac{\partial x}{\partial \xi^1}\frac{d\xi^0}{d\tau} & -\frac{c\partial t}{\partial \xi^1}\frac{d\xi^0}{d\tau} \\ -\frac{\partial x}{c\partial \xi^0}\frac{d\xi^0}{d\tau} & \frac{\partial t}{\partial \xi^0}\frac{d\xi^0}{d\tau} \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} \tag{17}$$

Hence,

$$F_\xi^{\ 0} = \frac{1}{(1 + \frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})^2}[A\frac{\partial x}{\partial \xi^1}\frac{d\xi^0}{d\tau} - B\frac{c\partial t}{\partial \xi^1}\frac{d\xi^0}{d\tau}]$$

$$= \frac{1}{(1 + \frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\frac{\partial x}{\partial \xi^1}[f^0 - m_0\frac{d}{d\xi^0}(\frac{\partial t}{\partial \xi^0})c(\frac{d\xi^0}{d\tau})^2 - m_0\frac{d}{d\xi^0}(\frac{c\partial t}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}]$$

$$- \frac{1}{(1 + \frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\frac{c\partial t}{\partial \xi^1}[f^1 - m_0\frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^0})c(\frac{d\xi^0}{d\tau})^2 - m_0\frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}] \tag{18}$$

As,

$$F_\xi^{\ 1} = \frac{1}{(1 + \frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})^2}[-A\frac{\partial x}{c\partial \xi^0}\frac{d\xi^0}{d\tau} + B\frac{c\partial t}{\partial \xi^0}\frac{d\xi^0}{d\tau}]$$

$$= -\frac{1}{(1 + \frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\frac{\partial x}{c\partial \xi^0}[f^0 - m_0\frac{d}{d\xi^0}(\frac{\partial t}{\partial \xi^0})c(\frac{d\xi^0}{d\tau})^2 - m_0\frac{d}{d\xi^0}(\frac{c\partial t}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}]$$

$$+ \frac{1}{(1 + \frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\frac{\partial t}{\partial \xi^0}[f^1 - m_0\frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^0})c(\frac{d\xi^0}{d\tau})^2 - m_0\frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^1})\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}] \tag{19}$$

In this time,

$$\frac{d}{d\xi^0}(\frac{\partial t}{\partial \xi^0}) = \frac{d\xi^1}{d\xi^0}\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2} + (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{{a_0}^2}{c^3} \quad ,$$

$$\frac{d}{d\xi^0}(\frac{\partial x}{c\partial \xi^0}) = \frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2} + (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{{a_0}^2}{c^3}$$

$$\frac{d}{d\xi^0}(\frac{c\partial t}{\partial \xi^1}) = \cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c}, \quad \frac{d}{d\xi^0}(\frac{\partial x}{\partial \xi^1}) = \sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c} \tag{20}$$

Therefore, Eq(18) is by Eq(15),Eq(20). Lorentz force ${F_\xi}^0$ is in Rindler space-time.

$${F_\xi}^0$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\cosh(\frac{a_0\xi^0}{c})$$

$$\times[f^0 - m_0\{\frac{d\xi^1}{d\xi^0}\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2} + (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{{a_0}^2}{c^3}\}c(\frac{d\xi^0}{d\tau})^2$$

$$- m_0\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c}\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}]$$

$$- \frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}\sinh(\frac{a_0\xi^0}{c})$$

$$\times[f^1 - m_0\{\frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2} + (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{{a_0}^2}{c^3}\}c(\frac{d\xi^0}{d\tau})^2$$

$$- m_0\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}] \tag{21}$$

Therefore, Eq(19) is by Eq(15),Eq(20).Lorentz force ${F_\xi}^1$ is in Rindler space-time.

$${F_\xi}^1$$

$$=-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}(\frac{c^2}{a_0}+\xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}$$

$$\times[f^0-m_0\{\frac{d\xi^1}{d\xi^0}\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}+(\frac{c^2}{a_0}+\xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0{}^2}{c^3}\}c(\frac{d\xi^0}{d\tau})^2$$

$$-m_0\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c}\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}]$$

$$-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)(\frac{d\xi^0}{d\tau})}(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})$$

$$\times[f^1-m_0\{\frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}+(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0{}^2}{c^3}\}c(\frac{d\xi^0}{d\tau})^2$$

$$-m_0\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}\frac{d\xi^1}{d\tau}\frac{d\xi^0}{d\tau}] \qquad (22)$$

In this time, the transformation of electromagnetic field is[12]

$$E_x=E_{\xi^1}\ ,$$

$$E_y=E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z=E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$B_x=B_{\xi^1},$$

$$B_y=B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z=B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}) \qquad (23)$$

Hence,

$$f^0=F^0\frac{dt}{d\tau}=q\frac{\vec{u}}{c}\cdot\vec{E}\frac{dt}{d\tau}=q\frac{\vec{u}'}{c}\cdot\vec{E},\vec{u}=\frac{d\vec{x}}{dt},\vec{u}'=\frac{d\vec{x}}{d\tau}$$

$$=q\frac{1}{c}[\frac{dx}{d\tau}E_x+\frac{dy}{d\tau}E_y+\frac{dz}{d\tau}E_z],\ \frac{dx}{d\tau}=\frac{\partial x}{\partial\xi^0}\frac{d\xi^0}{d\tau}+\frac{\partial x}{\partial\xi^1}\frac{d\xi^1}{d\tau}$$

$$= \frac{q}{c}[\{(\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}\frac{d\xi^0}{d\tau} + \cosh(\frac{a_0\xi^0}{c})\frac{d\xi^1}{d\tau}\}E_{\xi^1}$$

$$+ \frac{d\xi^2}{d\tau}\{E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})\}$$

$$+ \frac{d\xi^3}{d\tau}\{E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})\}] \qquad (24)$$

$$f^1 = F^1\frac{dt}{d\tau},$$

$$F^1 = q[E_x + \frac{1}{c}(u_yB_z - u_zB_y)] \qquad ,\vec{u} = \frac{d\vec{x}}{dt}, \vec{u}' = \frac{d\vec{x}}{d\tau}$$

$$= q[E_x\frac{dt}{d\tau} + \frac{1}{c}(\frac{dy}{d\tau}B_z - \frac{dz}{d\tau}B_y)],\ \frac{dt}{d\tau} = \frac{\partial t}{\partial\xi^0}\frac{d\xi^0}{d\tau} + \frac{\partial t}{\partial\xi^1}\frac{d\xi^1}{d\tau}$$

$$= q[\{(\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}\frac{d\xi^0}{d\tau} + \sinh(\frac{a_0\xi^0}{c})\frac{1}{c}\frac{d\xi^1}{d\tau}\}E_{\xi^1}$$

$$+ \frac{1}{c}\{\frac{d\xi^2}{d\tau}(B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}))$$

$$- \frac{d\xi^3}{d\tau}(B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})\}] \qquad (25)$$

In Eq(8), f^2 is

$$f^2 = m_0\frac{d^2y}{d\tau^2} = F_\xi^{\ 2}\frac{d\xi^0}{d\tau}, \quad F_\xi^{\ 2} = m_0\frac{d}{dt}(\frac{dy}{d\tau})\frac{dt}{d\xi^0} = F^2\frac{dt}{d\xi^0} \qquad (26)$$

Therefore, Lorentz force $F_\xi^{\ 2}$ is in Rindler space-time.

$$F_\xi^{\ 2} = m_0\frac{d}{d\xi^0}(\frac{d\xi^2}{d\tau}) = F^2\frac{dt}{d\xi^0}$$

$$= q[E_y + \frac{1}{c}(u_zB_x - u_xB_z)]\frac{dt}{d\xi^0}$$

$$= q[\{E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})\}$$

$$\times\{(\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2} + \frac{1}{c}\frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\} + \frac{1}{c}\{\frac{d\xi^3}{d\tau}B_{\xi^1}$$

$$-\{(\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c} + \cosh(\frac{a_0\xi^0}{c})\frac{d\xi^1}{d\xi^0}\}$$

$$\times\{B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})\}\}] \qquad (27)$$

In Eq(8), f^3 is

$$f^3=m_0\frac{d^2z}{d\tau^2}=F_\xi{}^3\frac{d\xi^0}{d\tau},\quad F_\xi{}^3=m_0\frac{d}{dt}(\frac{dz}{d\tau})\frac{dt}{d\xi^0}=F^3\frac{dt}{d\xi^0} \qquad (28)$$

Therefore, Lorentz force $F_\xi{}^3$ is in Rindler space-time.

$$F_\xi{}^3=m_0\frac{d}{d\xi^0}(\frac{d\xi^3}{d\tau})=F^3\frac{dt}{d\xi^0}$$

$$=q[E_z+\frac{1}{c}(u_xB_y-u_yB_x)]\frac{dt}{d\xi^0}$$

$$=q[\{E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})\}$$

$$\times\{(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0\xi^0}{c})\frac{a_0}{c^2}+\frac{1}{c}\frac{d\xi^1}{d\xi^0}\sinh(\frac{a_0\xi^0}{c})\}$$

$$+\frac{1}{c}\{\{(\frac{c^2}{a_0}+\xi^1)\sinh(\frac{a_0\xi^0}{c})\frac{a_0}{c}+\cosh(\frac{a_0\xi^0}{c})\frac{d\xi^1}{d\xi^0}\}$$

$$\times\{B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})\}-\frac{d\xi^2}{d\xi^0}B_{\xi^1}\}] \qquad (29)$$

3. Conclusion

We find the electro-magnetic force in uniformly accelerated frame.

Reference

[1]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[2]W.Rindler, Am.J.Phys.**34**.1174(1966)

[3]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[4]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[5]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[6]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)

[7]A.Miller, Albert Einstein's Special Theory of Relativity(Addison-Wesley Publishing Co., Inc., 1981)

[8]W.Rindler, Special Relativity(2nd ed., Oliver and Boyd, Edinburg,1966)

[9]Massimo Pauri, Michele Vallisner, "Marzke-Wheeler coordinates for accelerated observers in special relativity":Arxiv:gr-qc/0006095(2000)

[10]A. Einstein, " Zur Elektrodynamik bewegter K¨orper", Annalen der Physik. 17:891(1905)

[11]J.W.Maluf and F.F.Faria,”The electromagnetic field in accelerated frames”:Arxiv:gr-qc/1110.5367v1(2011)
[12]S.Yi, ”Electromagnetic Field Equation and Lorentz Gauge in Rindler Spacetime”, The African Review of Physics,**11**;33(2016)
[13]D.J. Griffith,” INTRODUCTION TO ELECTRODYNAMICS”,(2nd ed.,Prentice Hall,Inc.1981)