The generalized Seiberg-Witten equations

A.Balan

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Abstract

We show a set of equations which generalizes the Seiberg-Witten equations

1 The Seiberg-Witten equations

The Seiberg-Witten equations are the following ones [F] [M]:
1)

$$\mathcal{D}_A(\psi) = 0$$

2)

$$F_+(A) = -(1/4)\omega(\psi)$$

2 The generalization of the SW equations

We consider two spinors ϕ, ψ and we define [F] the coupled Seiberg-Witten equations (A, A', f, ϕ, ψ):
1)

$$\mathcal{D}_A(f\phi) = 0$$

2)

$$\mathcal{D}_{A'}((1/f)\psi) = 0$$

3)

$$F_+(A) = -(1/4)\omega(\phi)$$

4)

$$F_+(A') = -(1/4)\omega(\psi)$$

5)

$$(f^2)^*A = (1/f^2)^*A'$$

A, A' are connections $f : M \to S^1$. If $f = 1$, then we have the Seiberg-Witten equations.

The gauge group acts:

$$g.(A, A', f, \phi, \psi) = ((1/g^2)^*A, (g^2)^*A', fg, g\phi, (1/g)\psi)$$

Morover, the situation can be generalized to n solutions of the Seiberg-Witten equations:
1)

$$\mathcal{D}_{A_i}(f_i\psi_i) = 0$$

2)

$$F_+(A_i) = -(1/4)\omega(\psi_i)$$

3)

$$(f_i^2)^* A_i = B$$

4)

$$\prod_i f_i = 1$$

3 The invariants of Seiberg-Witten generalized

We have to prove compacity of the moduli spaces and to define the invariants of Seiberg-Witten over them.

References

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