The Gravity Primer

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Abstract. It was shown in [1] that gravitational interaction can be expressed as an algebraic quadratic invariant form $V_1^2 - E^2 \equiv const \geq 0$, where $V_1(r) := V(r \equiv 1)$ is the potential of a subparticle in the gravitational system whose center of mass is taken to be in the origin of location coordinates, r is the (Euclidean) distance of the subparticle from the origin (center of mass), and E is the subparticle's energy.
This allows the decomposition of the entire gravitational system into the sum of squares of energies of its composing subparticles. Still then, we ran into serious problems, when it came to figure out the Hamiltonian and calculate the total energy of the system from that. (Equivalently put, the algebraic invariant above is not a Hamiltonian one.)
Simply, the problem is: What goes wrong? This is what this article is about, and the answer is very simple, too.

1. Decomposition and Free Theory

Let there be n particles in the gravitational system, sticking stably together in time, with its center of mass staying at rest in the origin, as in [1]. Then $T^2 := \sum_{1 \leq k \leq n} \left(|V_{1,k}|^2 - |E_k|^2 \right)$ can be defined as the square of temperature, where $\sum_{1 \leq k \leq n} |E_k|^2$ is the sum of squares of energy of the particle energies E_k, which is the square of an Euclidean n-vector $\vec{E} := (E_1, \ldots, E_n)$.
Now, let's substitute (formally): $E_k \to m_k c^2$, $V_{1,k} \to E_k$, $V_{1,k} - E_k \to p_k c$, where c is the speed of light and where the m_k are (rest) masses. We get

$$\sum_k |E_k|^2 = \sum_k (m_k^2 c^4 + |p_k|^2 c^2).$$

That is the Euclidean square norm of an n-vector of n free particles, each obeying the basic relativistic formula $E_k^2 = m_k^2 c^4 + |p_k|^2 c^2$, $(1 \leq k \leq n)$. It is here, where mathematics bites physics: both equations are mathematically equivalent; but a mathematical equivalence never is an accident.

2. What's a Free Theory?

There is much useless debate as to whether n particles in space-time can be free, or whether they need to be interacting, whereas that is just a matter of a proper mathematical definition: When $n = 1$, things are trivial: it's a particle moving in space-time at constant momentum. Then for $n > 1$, this is almost trivial: it's an n-vector of n particles, such that the projections to each (particle) component are free 1-particle systems. That's all to it.

3. Adding up the Energy

For a free n-particle system, that means in particular, that each of it lives in its 3-dimensional location coordinate system, or, equivalently in the conjugated space of 3-dimensional momentum coordinates; as such, they don't interfere. But what about time and its conjugate, the energy coordinate?
That is where the error happens: As to the definition above, we have to add the energies of the n particles as components of a vector, and not as a scalar! In other words, the (total) energy of a system of n free particles is not the sum of square roots $\sum_k \sqrt{m_k^2 c^4 + |p_k|^2 c^2}$, but the Euclidean norm of the vector $\vec{E} := (m_1 c^2 + |p_1| c, \ldots, m_n c^2 + |p_n| c)$, instead.

4. Then some classical laws need correction and/or further experimental proofs

The very first law affected by this, is certainly the universal gas law which in turn affects thermodynamics: The square root of $\vec{E}^2 = \sum m_k^2 c^4 + T^2$ will not be T, so, in particular, no matter can turn into a black-body radiator for $T \to 0$, and Planck radiation will not be explainable through thermodynamics, but by Bohr theory alone.

The 2^{nd} law affected is gravity itself: Gravity relies on the the principle of equivalence of inert and gravitational mass, and by the principle of relativity, inert mass is to be equivalent to its energy. So, given that $\left|\vec{E}\right| = (\sum m_k^2 c^4 + T^2)^{1/2}$ is the energy, inert mass and therefore gravitational mass must increase (even slightly non-linearly) with temperature.
First off, be it theoretically accepted or not: what counts only are experiments that show, whether these relations hold or do not hold! Sadly, there is no mention of these in the physics literature. The good thing is that these experiments are straight forward and relatively cheap to carry out.

Because of the high degree of experimental precision in special relativity, it is more than unlikely that the equivalence of inert mass and energy ever breaks. So, it will be expected that the inert mass increases with temperature according to the above formula.

Similarly, experimental reality has it, that a raised temperature expands a solid body, so heat does work against gravity, and if the temperature is high enough, the body will evaporate. That can only mean that gravitity will decrease with temperature, rather than increase. Therefore, we should hence expect the classical theory of gravitation and the theory of gasses (thermodynamics) to be at opposite ends of idealized limits of $\left|\vec{E}\right|^2 = \sum_k m_k^2 c^4 + T^2$: For $T \to 0$ we'll get Newtonian gravity, whilst thermodynamics will result from $\sum_k m_k^2 c^4 \to 0$.

5. What's in the rest mass?

Let's make an extreme experiment and assemble a cold substance A consisting of 4 parts of nitroglycerin, 10 parts of H_2, and 12 parts of carbon C. That is known to be transformable into 12 parts of CO_2, 10 parts of H_2O, 5 parts N_2, and 2 parts of NO. At the same cold temperature, let's assemble the same amount of that transformed mixture B, say. Then the question is whether or not inert and gravitational masses of A and B differ from eachother.
As to inertness, the answer is simple: With a little deviation of pressure, A will will exothermally transform into the constituents B is composed of, and in a short moment of time, these constituents will escape the center of mass at maximal speed, at which point they will move freely. So, A evolves into a free system C of particles after explosion. Since the energy of the substance is the same before and after the explosion, the inert masses of A and C must be equal. Likewise, the rest mass of B is equal to the rest mass of C, and it follows that the square of the inert mass of A is that of B plus the equivalent of the square kinetic energy of C.
The problem remains whether or not A and B gravitationally weigh the same. Again, if the answer is yes, then this will put an end to the universality of the principle of equivalence of inert and gravitational mass.

6. Another Experiment

Let's make another, not so extreme experiment:
Given three identical spherical shells of iron of one ton of weight, enclosing $1m^3$ of atmospheric gas at 0°C and normal atmospheric pressure, all three equipped with an identical valve. Let's call these shells A, B, and C, say.
From the interior of A we extract 200g of gas, and we spray 200g of iron to the outer outer surface of the shell. From B we take away 220g of iron, spray 20g of iron to its surface again, just to make it look the same as A and C from the outside, and then we inject 200g of gas at 0°C through the valve into its interior.

Now, let's compare the inert and gravitational masses of the three shells: A and B are under considerable pressure, yet of opposite sign. From the perspective of mechanical kintetics, the pressure is simplistically argued away

by adding all the forces up to a single 3-vector: as long as A and B do not change in time, that addition results onto a zero vector, such deducing that the pressure P was to be a big bluff, and A, B, C were to deliver identical results.
I hope, you'll be with me that this is the wrong calculation, because the opposite non-zero forces attach to different particles, oppositely sited in the iron shell: Doing the calculation correctly, would mean to calculate the 3n-vector of net forces of each and every of the n particles, and then determine the Euclidean norm of that vector to get at the scalar net force of A and of B. That gives the same norm of non-zero net force for A and B, because they have the same surface area, their absolute value pressure of non-zero pressure $|P|$ is the same, and compared with C, the same amount of energy $|P|\,V$ has been added to A and B, where V is the volume of 1 cubic meter.
And can we can check that by hitting A, B, and C hard enough, cracking the adhesive, short ranged forces within the iron lattice structure: all three will transform into free particle systems, but the kinetical energy for A and B will be the same and exceed that for the cracked shell C. Above all: before cracking, that kinetic energy must have been potential energy within A and B, but as such these are of opposite sign!

In all, one would definitely expect A and B to be of equal inert mass M which increases with $|P|$ according to $\sqrt{M_0^2c^4 + (PV)^2}$. That said, let's inspect C, which is interesting, since the iron atoms within their solid lattice firmly stick together: That means that neighboured particles exert non-zero forces between eachother, and again, if we do it correctly, we'll add up the net forces for each particle to a vector component for that particle. And the sum is a vector with an Euclidean norm of formidable positive value, which again relates to its binding energy. Then we can proceed with the interaction between nucleons and electrons, and among themselves.
And now the dooming question is: if at all, what part of binding energy is contributing to gravitation itself?

7. Outlook

My own point of view has been stated in the cited articles, below: if only we take the square of energy within a massy substance, that will include Standard model and gravity, and necessarily anti-gravity. What part of that whole is gravitationally effective or not, that has to be decided upon by experiment. And at least the ones proposed above, namely weighing a metal shell filled with gas of varying pressure and at varying temperatures, are surprisingly simple and affordable.

(It should be remarked that they will not just test against the universality of equivalence of inert and gravitational mass, but with it, will also test against the fundaments of General Relativity and cosmology.)

References

[1] H. D. Hüttenbach, *An Algebraic Invariant of Gravity*, `http://vixra.org/abs/1804.0068`,2018.

[2] H. D. Hüttenbach, *From Bernoulli to Laplace and Beyond*, `http://vixra.org/abs/1801.0025`, 2018.

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