Proof of the Legendre's Conjecture

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***Abstract*—This article solves the problem for the second time using the Formula of Disjoint Sets of Odd Numbers numbers that I proposed**

***Index Terms*—algorithm**

I. Definition of the Legendre's Conjecture

Is it true that between n^2 and $(n+1)^2$ there is always a prime number (y_o), where $n \in \mathbb{N}^*$ and $\mathbb{N}^*$ be natural numbers without zero?

II. Algorithm for Proof of the Legendre's Conjecture

After number 2 the sequence of primes $\{y_o\}$ enters an infinite sequence of odd numbers $\{y\}$, the formulation of the Legendre's Conjecture must be changed to consider this sequence.
For this, if n^2 or $(n+1)^2$ are even numbers, they will be replaced by odd numbers (n^2-1) or $((n+1)^2-1)$, respectively, which does not change the very essence of the question, since these numbers are composite.

Let the odd number $n^2 = y^2$, then the even number $(n+1)^2 = (y+1)^2 = y^2+2y+1$. Let's replace this even number in a sequence of odd numbers $\{y\}$:

$$y^2+2y+1-1 = y^2+2y = y(y+2). \qquad (1)$$

Thus, let's must consider each set:

$$\{y_n \mid y_k^2 < y_n < y_k(y_k+2),\ y_k \geq 3\}. \qquad (2)$$

The number of terms in each set (2):

$$N_{y_n} = y_k - 1. \qquad (3)$$

But in the sets:

$$\{y_n \mid y_k(y_k-2) < y_n < y_k^2,\ y_k \geq 3\} \qquad (4)$$

the number of terms is also equal to (3).
It is logical to consider (2) and (4) with respect to sets with equal N_{y_n}. But the segments between $y_k(y_k-2)$ and y_k^2, y_k^2 and $y_k(y_k+2)$ are segments in a sequence of odd numbers for which the Formula of Disjoint Sets of Odd Numbers is valid. For the entire sequence of odd numbers $\{y\}$, it has the form of the following expression:

$$\begin{aligned} Z_y = \Bigg(& 0{,}0\ldots01\%(1) + 33{,}3\ldots3\%(\{3y\}) + \\ &+ \sum_{n=3}^{n\to\infty} Z_{y_{on}}\Big(\{y_{on}y_n \mid \frac{y_n}{3} \notin \mathbb{N}^*, \ldots \\ &\ldots, \frac{y_n}{y_{o(n-1)}} \notin \mathbb{N}^*\}\Big)\Bigg) \to 100\%, \end{aligned} \qquad (5)$$

where:
Z_y is of appearance of all odd numbers y;
the number of digits represented by $(\ldots)$ in the first two terms $\to \infty$;
$Z_{y_{on}}$ is the frequency of appearance of the given set (in %) in the sequence $\{y\}$;
n is the number of a member of a sequence of odd primes;
y_n is a sequence of odd numbers with the conditions given in the formula;
$y_{o(n-1)}$ is the prime number in sequence of primes just before y_{on}.

For the segments (3) of (2) and (4) Formula of Disjoint Sets of Odd Numbers (5) takes the following form, where the percentage of the sets remains unchanged:

$$\begin{aligned} & Z_{y_{comp}}(\{y_{comp}\}) = \\ & = 33{,}3\ldots3\%(\{3y \mid y \geq 3,\ 3y = y_n\}) + \\ & + \sum_{m=3} Z_{y_{om}}\Big(\{y_{om}y_m \mid y_m \geq y_{om},\ y_{om}y_m = y_n, \\ & y_{om} < N_{y_n}, \frac{y_m}{3} \notin \mathbb{N}^*, \ldots \frac{y_m}{y_{o(m-1)}} \notin \mathbb{N}^*\}\Big), \end{aligned} \qquad (6)$$

where:
$Z_{y_{comp}}$ is the frequency of the appearance of composite numbers (in %) in a given segment of the sequence {y};
y_{comp} is a composite odd number in a given segment of a sequence of odd numbers y;
the number of digits represented by $(\ldots)$ in the first term, $\to \infty$;
m is the number of a member of a sequence of odd primes;
$Z_{y_{om}}$ is the frequency of appearance of the given set (in %) in the sequence $\{y\}$;
y_m is a sequence of odd numbers with the conditions given in the formula;
N_{y_n} is the number of terms in (2) or (4) (see (3)).

But since in the whole sequence of odd numbers y the frequency of appearance of known sets according to (5) only tends to 100%, then in the case of (6):

$$Z_{y_{comp}}(\{y_{comp}\}) < 100\%. \tag{7}$$

That is, between n^2 and $(n+1)^2$ there is always a prime number.

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REFERENCES