Proof of the Legendre's Conjecture (Third Landau's Problem)

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1 Definition of the Legendre's Conjecture

Definition: Is it true that between n^2 and $(n+1)^2$ there is always a prime number (y_o)?

2 Algorithm for Proof of the Legendre's Conjecture

Since after 2 the sequence of primes $\{y_o\}$ enters an infinite sequence of odd numbers $\{y\}$, the formulation of the **Legendre's Conjecture** must be changed to consider this sequence, which we know from the **Proof of the Goldbach's Conjecture**.
For this, if n^2 or $(n+1)^2$ are even numbers, they will be replaced by odd numbers (n^2-1) or $((n+1)^2-1)$, respectively, which does not change the very essence of the question, since these numbers are composite.

Let the odd number $n^2 = y^2$, then the even number $(n+1)^2 = (y+1)^2 = y^2+2y+1$. Let's replace this even number in a sequence of odd numbers $\{y\}$:

$$y^2 + 2y + 1 - 1 = y^2 + 2y = y(y+2). \tag{1}$$

Thus, let's must consider each set:

$$\{y_n \mid y_k^2 < y_n < y_k(y_k+2),\ y_k \geq 3\}. \tag{2}$$

The number of terms in each set (2):

$$N_{y_n} = y_k - 1. \tag{3}$$

But in the sets:

$$\{y_n \mid y_k(y_k-2) < y_n < y_k^2,\ y_k \geq 3\} \tag{4}$$

the number of terms is also equal to (3).
It is logical to consider the sets (2) and (4) with respect to sets with equal N_{y_n}. But the segments between $y_k(y_k-2)$ and y_k^2, y_k^2 and $y_k(y_k+2)$ are segments in a sequence of odd numbers for which the Formula

of Disjoint Sets of Odd Numbers of **Proof of the Goldbach's Conjecture** is valid. For the entire sequence of odd numbers $\{y\}$, it has the form of the following expression:

$$\left(0{,}0\ldots01\%(1)+33{,}3\ldots3\%(\{3y\})+\sum_{n=3}^{n\to\infty}Z_{y_{on}}\left(\{y_{on}y_n \mid \frac{y_n}{3}\notin\mathbb{N},\ldots,\frac{y_n}{y_{o(n-1)}}\notin\mathbb{N}\}\right)\right)\to 100\%, \tag{5}$$

where: the number of digits represented by $(\ldots)$ in the first two terms $\to\infty$;

n is the number of a member of a sequence of odd primes;

y_n is a sequence of odd numbers with the conditions given in the formula;

$y_{o(n-1)}$ is the prime number in sequence of primes just before y_{on};

$Z_{y_{on}}$ is the frequency of appearance of the given set (in %) in the sequence $\{y\}$.

For the segments (3) of the sets (2) and (4) Formula of Disjoint Sets of Odd Numbers (5) takes the following form, where the percentage of the sets remains unchanged:

$$\begin{aligned}Z_{y_{comp}}(\{y_{comp}\}) &= 33{,}3\ldots3\%(\{3y \mid y\geq 3,\ 3y=y_n\})+\\ &+\sum_{m=3}Z_{y_{om}}\left(\{y_{om}y_m \mid y_m\geq y_{om},\ y_{om}y_m=y_n,\ y_{om}<N_{y_n},\frac{y_m}{3}\notin\mathbb{N},\ldots,\frac{y_m}{y_{o(m-1)}}\notin\mathbb{N}\}\right),\end{aligned} \tag{6}$$

where: the number of digits represented by $(\ldots)$ in the first term, $\to\infty$;

m is the number of a member of a sequence of odd primes;

y_{comp} is a composite odd number in a given segment of a sequence of odd numbers $\{y\}$;

$Z_{y_{comp}}$ is the frequency of the appearance of composite numbers (in %) in a given segment of the sequence $\{y\}$;

N_{y_n} is the number of terms in the sets (2) or (4);

$N_{y_n}=y_k-1$;

y_m is a sequence of odd numbers with the conditions given in the formula.

But since in the whole sequence of odd numbers $\{y\}$ the frequency of appearance of known sets according to (5) only tends to 100%, then in the case of (6):

$$Z_{y_{comp}}(\{y_{comp}\})<100\%. \tag{7}$$

That is, between n^2 and $(n+1)^2$ there is always a prime number.

Publications: http://samlib.ru/editors/b/bezymjannyj_a/w4.shtml