The Golden Ratio in the Modified Cosmological Model

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Occupy Something, Something, Somewhere
(Dated: 7/6/2018)

The golden ratio Φ is very important in the modified cosmological model (MCM). In previous work, we have inserted it artificially rather than showing where it comes from. Where the real numbers $\mathbb{R}$ are extended to the complex numbers $\mathbb{C}$ for routine physical applications, we extend the complex numbers to the hypercomplex numbers ${}^{\star}\mathbb{C}$ and show that Φ is inherent to the transfinite structure. We formalize the transfinite concept of continuation beyond infinity. We improve upon previous motivations for deriving general relativity and the fine structure constant in the MCM, and we propose an origin for the Yang–Mills mass gap.

Consider $\mathbb{C}$-numbers $z = x + iy$ defined as in reference [1]. Define

$$y_+ = \infty - y \quad , \qquad \text{and} \qquad y_- = y + \infty \quad , \tag{1}$$

and let

$$z = \begin{cases} x + iy_+ & \text{for} \quad \mathrm{Im}(z) > 0 \\ x - iy_- & \text{for} \quad \mathrm{Im}(z) < 0 \end{cases} \quad , \tag{2}$$

where

$$x \in [-\infty, \infty] \quad , \qquad \text{and} \qquad y_\pm \in [0, \infty) \quad , \tag{3}$$

as in figure 1. The conformal coordinate on the real axis will be

$$\widetilde{x} = \tan^{-1}(x) \qquad \Longrightarrow \qquad \widetilde{x} \in [-\pi/2, \pi/2] \quad , \tag{4}$$

and we will develop some things before writing the conformal coordinate on the imaginary axis. We include infinity with the hard brackets so, evidently, we will consider the extension of the extended complex numbers $\hat{\mathbb{C}}$ into the hypercomplex numbers ${}^{\star}\mathbb{C}$. ${}^{\star}\mathbb{C}$ is derived through the fusion of $\mathbb{C}$ with hyperreal numbers $\mathbb{R}^*$ and, as $\mathbb{R}^*$ is transfinite, ${}^{\star}\mathbb{C}$ is also a transfinite number system. It is required to include infinity in anything transfinite so we use $\hat{\mathbb{C}}$ as the foundation. The most relevant details of ${}^{\star}\mathbb{C}$ appear in [1–4] and we will restate the key features here.

For the case of $\mathrm{Im}(z) = 0$, included in neither equation (2) nor definitions (3), we could take the average of $y_\pm \to \infty$ but the idea in hypercomplex analysis is to require that there is always some infinitesimal imaginary component. An $\mathbb{R}$-valued number has $\mathrm{Im}(x) = 0$, and a $\mathbb{C}$-valued number can have $\mathrm{Im}(z) = 0$, but a ${}^{\star}\mathbb{C}$-valued number will always have $\mathrm{Im}(z) \geq \alpha\hat{\Phi}^{-\infty}$ where $\alpha \in \mathbb{R}$, $\alpha \neq 0$, and $\hat{\Phi}^{-\infty}$ is the smallest possible infinitesimal element. In ${}^{\star}\mathbb{C}$, we label each tier of infinitude with $\hat{\Phi}^j$. $\hat{\Phi}^0$ is the finite tier, $\hat{\Phi}^1$ is the first infinite tier (corresponding to $\aleph_0$), $\hat{\Phi}^{-1}$ is infinitesimal, $\hat{\Phi}^{-2}$ is infinitesimal even with respect to $\hat{\Phi}^{-1}$, *etc.* We may obtain $\mathrm{Im}(z) = 0$ from a ${}^{\star}\mathbb{C}$ number by considering the sign of α to choose from $y_\pm$, and then neglect terms of order $\hat{\Phi}^{-j}$ with $j \geq 1$. All of those $\hat{\Phi}^{-j}$ terms may have vanishing coefficients but $\hat{\Phi}^{-\infty}$ may not. We refer to the tier of infinitude, a concept inherent to the hyperreals $\mathbb{R}^*$, as a "level of $\aleph$" [2, 3, 5].

Let ${}^{\star}\mathbb{C}$'s real axis x be the x^0 axis of the MCM unit cell, as in figure 2. Associate $y_\pm$ with $\chi^5_\pm$. $\chi^5_\pm$ do not

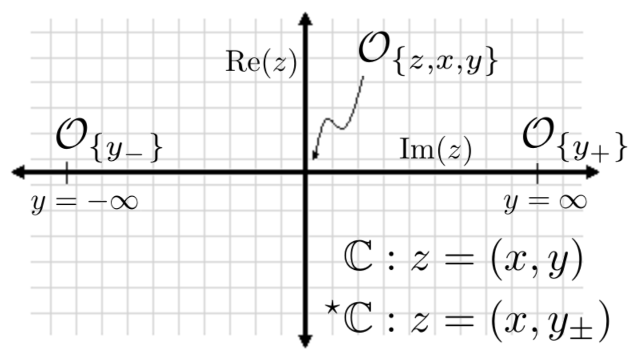

FIG. 1. To show the similarity with figure 2, the imaginary axis is in the horizontal direction. The "upper" complex half-plane is on the right, and the "lower" complex half-plane is on the left. The origin of z, x, and y is labeled $\mathcal{O}_{\{z,x,y\}}$. It is usually the origin of the $\mathrm{Im}(z)$ dimension but the new piecewise definition of $y^\pm$ puts their origins at $\mathcal{O}_{\{y^\pm\}}$. $\mathcal{O}_{\{y^\pm\}}$ lie at the points $(x, y) = (0, \pm\infty)$.

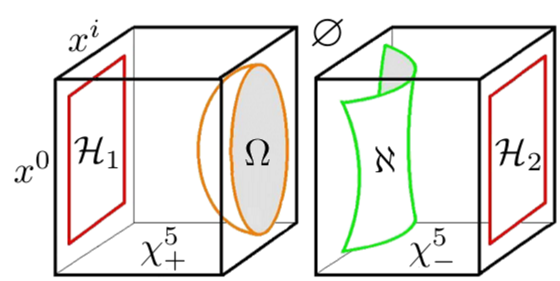

FIG. 2. This figure shows the MCM unit cell. Each cube, Σ_1^+ and Σ_2^-, is spanned by $\{x^0, x^i, \chi^5\} \sim \{\chi^0, \chi^i, \chi^5\}$. $\mathcal{H}$ is observable (real) spacetime but the bulk space is unobservable. There is, therefore, an intuitive picture in which $\chi^5_\pm$ are imaginary dimensions pointing outside of the universe spanned by x^μ.

include their boundaries at $\chi^5_\pm\!=\!0$ [6] and neither do $y_\pm$ include their boundaries at $y_\pm\!=\!\infty$ $(y\!=\!0)$. While it will be marginally beyond the scope of this paper, we could make a 5D space ($\Sigma^\pm$ are both 5-spaces) by adding the quaternions $\{\mathbf{i},\mathbf{j},\mathbf{k}\}$ to represent the x^i spatial degrees of freedom. The 5D basis of unit vectors spanning $\Sigma^\pm$ would be $\{\hat{1},\hat{\imath},\hat{\mathbf{i}},\hat{\mathbf{j}},\hat{\mathbf{k}}\}$ with the bases anchored at $\mathcal{O}_{\{y_\pm\}}$ respectively. In this way, we could begin to build a connection between the odd spatial properties of half-integer spin and the Pauli matrices which are isomorphic to the quaternions.

The initial intention in the MCM was to have a unit cell of finite width but, in recent work, it has become infinite in width along the χ^5 direction. The infinite width was introduced to join Σ^+_1 to Σ^-_2, as in figure 2. To see why this was required, we must consider the metrics of $\Sigma^\pm$ which are two 5-spaces that can, together, support a 10D string theoretical boundary condition. When we set the electromagnetic potential 4-vector $A_\mu\!=\!0$, those metrics are

$$\Sigma^\pm_{AB}=\begin{pmatrix}\eta_{ab} & 0\\ 0 & \chi^5_\pm\end{pmatrix}\quad , \tag{5}$$

with $\eta_{\mu\nu}$ being the Minkowski metric and capital Latin indices running from 1 to 5. At any given constant value of $\chi^5_\pm$, we can use that value as the the parameter of uniform curvature on a 4D brane (worldsheet) embedded in the 5D space [3]. When $\Sigma^\pm_{55}$ is taken as the hyperboloid parameter of an embedded hypersurface at each constant value of $\chi^5_\pm$, we obtain the relationship between the $\chi^a_\pm$ coordinates ($a\in\{1,2,3,4\}$) and the $x^\mu_\pm$ coordinates of the embedded hyperboloids which define the geometry of the 4-brane at that value of $\chi^5_\pm$.

$\chi^5_\pm$ were originally defined for the unit cell centered on $\mathcal{H}$, figure 3. Recently, the finite scale of $\chi^5_\pm$ was given by

$$\chi^5_-\in[-1,0)\quad ,\qquad\text{and}\qquad \chi^5_+\in(0,\Phi]\quad , \tag{6}$$

[3] with Minkowski space $\mathcal{H}$ being at the point $\chi^5_\pm=0$. The slices of constant χ^5_+ are de Sitter space because $\chi^5_+>0$ and the slices of constant χ^5_- are anti-de Sitter space because $\chi^5_-<0$. The curvature goes to zero as $\chi^5_\pm\!\to\!0^\pm$ so we can smoothly connect the slices of $\Sigma^\pm$ to $\mathcal{H}$ at $\chi^5_\pm\!=\!0$ which is the location of $\mathcal{H}$ by definition [6]. At large values of $|\chi^5_\pm|$, near $\varnothing$ in figure 2, the curvature is very large and it would be difficult to make a smooth connection between a topological space with large negative curvature and one with large positive curvature. For this reason, we extended the domain of $\chi^5_\pm$ as

$$\chi^5_-\in[-\infty,0)\quad ,\qquad\text{and}\qquad \chi^5_+\in(0,\infty]\quad , \tag{7}$$

so that we might join them on a topological singularity of infinite curvature at $\varnothing$. The purpose of the conformal coordinates $\widetilde{y}_\pm$ will be to restore finiteness to these intervals so that we may once again use $\chi^5_+\in(0,\Phi]$ to associate Φ with the increasing level of $\aleph$ defined by powers of the golden ratio $\hat{\Phi}^j$.

GOING BEYOND INFINITY

An *ab initio* derivation of the free spin-0 particle propagator in QFT yields the expression

$$D(x-y)=\left(\frac{1}{2\pi}\right)^4\int d^4k\,\frac{e^{ik(x-y)}}{k^2-m^2}\quad . \tag{8}$$

This expression isn't great for physics because it has poles where $k^2\!=\!m^2$ meaning that we can't integrate over all values of k. The common idea is to augment the mass $m^2\to m^2+i\varepsilon$ so that the pole moves off the real axis where we can efficiently use the residue formula to solve for $D(x-y)$. This is allowed in physics because ε is an infinitesimal and $m^2+i\varepsilon$ is very nearly equal to m^2 (and we can let $\varepsilon\!\to\!0$ later.) The resulting expression is

$$D(x-y)=\left(\frac{1}{2\pi}\right)^4\int d^4k\,\frac{e^{ik(x-y)}}{k^2-m^2+i\epsilon}\quad . \tag{9}$$

To "solve for $D(x-y)$" means to integrate out the k^0 part, which is like the time part of d^4x in momentum space. Defining

$$\mathcal{A}=\left(\frac{1}{2\pi}\right)^3\int d^3k\,e^{-i\vec{k}\cdot(\vec{x}-\vec{y})}\quad , \tag{10}$$

in the signature $\{+---\}$, we can let k^0 be a complex number z and introduce the relativistic mass-energy

$$\omega_k=+\sqrt{\vec{k}^2+m^2}\quad , \tag{11}$$

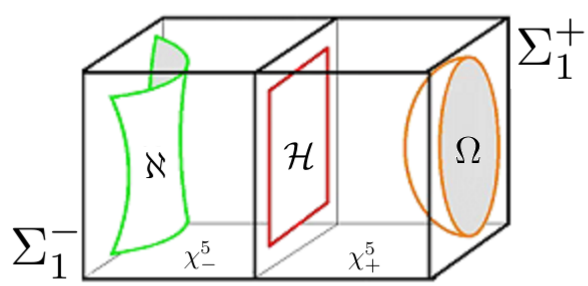

FIG. 3. This figure shows the MCM unit cell centered on $\mathcal{H}$ at $\chi^5_\pm=0$. $\mathcal{H}$ is Minkowski space with coordinates x^μ. $\aleph$ is anti-de Sitter space where the parameter of curvature that defines the embedded metric of the 4D x^μ_- coordinates is directly proportional to χ^5_-. Likewise, Ω is de Sitter space where the parameter of curvature that defines the embedded metric of the 4D x^μ_+ coordinates is directly proportional to χ^5_+.

to rewrite $D(x-y)$ as

$$D(x-y)=\frac{\mathcal{A}}{2\pi}\int dz\,\frac{e^{izt}}{z^2-\omega_k^2+i\varepsilon}\quad . \tag{12}$$

That ω_k has to be specified with a plus sign is indicative of the idea that the MCM conserves momentum in all cosmological cases [7, 8] by introducing another universe $\bar{U}$ with a sign convention such that ω_k in $\bar{U}$ uses the negative square root of $\vec{k}^2+m^2$ if our universe U uses the positive one.

The residue theorem says

$$\oint_{C_0} dz\,f(z)=2\pi i\sum_j R_j \quad , \tag{13}$$

where R_j are the residues at the poles of $f(z)$ which are enclosed by the closed contour C_0. Regarding $D(x-y)$, let us define the curve C_0 as in figure 4. The contour encloses the pole in the upper complex half-plane so that

$$\oint_{C_0} dz\,f(z)=\int_C dz\,f(z)+\int_{C'} dz\,f(z) \tag{14}$$

The path over C is the line integral in equation (12) and the integral over C' is equal to zero because the $z^2=\infty^2$ in the denominator dominates along C'. We have referred to this path integral many times in this research program, notably we used the symbol $\delta(t)$ to describe that which moves the pole off the k^0 axis in [9].

A general idea in the MCM is to "go beyond infinity" [2–4] and now we will formalize that idea using the contour integral for reference. The MCM operator $\hat{M}^3$ [9, 10] is defined such that it takes an initial condition in $\mathcal{H}_1$ and returns a final condition on $\mathcal{H}_2$:

$$\hat{M}^3 \quad : \quad \mathcal{H}_1\to\Omega\to\aleph\to\mathcal{H}_2 \quad . \tag{15}$$

Each of these spaces $\{\mathcal{H}_1,\Omega,\aleph,\mathcal{H}_2\}$ have their own coordinates [9]: $x^\mu_{\{1\}}\in\mathcal{H}_1$, $x^\mu_+\in\Omega$, $x^\mu_-\in\aleph$ and $x^\mu_{\{2\}}\in\mathcal{H}_2$. The idea in going beyond infinity is to somehow smoothly evolve a curve parameterized in the coordinates of one space into a curve parameterized in the coordinates of another where each space has its own origin of coordinates that is infinitely far away from the others. (If they were not infinitely far away, then they would have non-trivial interactions exceeding the "trivial" MCM dark energy interaction [7, 8, 11].) By implementing the imaginary axis $y_\pm$ of ${}^\star\mathbb{C}$ as

$$y_+=\infty-y \quad , \qquad \text{with} \qquad y_-=y+\infty \quad , \tag{16}$$

we have put together everything required to implement a rigorous transfinite analysis well-suited to these purposes.

Consider the path in figure 5. The sum of the integrals over C_1 and C_3 should be exactly equal to the the integral over C in figure 4. The curve at infinity $C'\leftrightarrow C_2$ should contribute nothing. C_2 is defined with

$$y_+(\widetilde{x})=\tan(\widetilde{x}) \quad , \tag{17}$$

and it easy to confirm that C_1 connects to C_2. We may extract from equations (16)

$$y=\infty-y_+ \ , \tag{18}$$

which concisely demonstrates the connection. When $\tan(\widetilde{x}=\pi/2)=\infty$, $y=0$ and so C_2 is connected to C_1. We

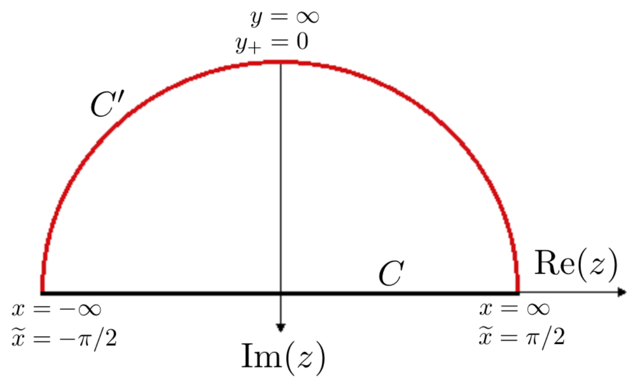

FIG. 4. This figure shows the closed contour typically used to solve equation (12). C_0 in equation (13) is a path C along the entire real axis taken together with another path C' around infinity. Since the integral of $D(x-y)$ over C' is equal to zero, the closed contour integral in equation (13) is exactly equal to the line integral over C which is the path in equation (12).

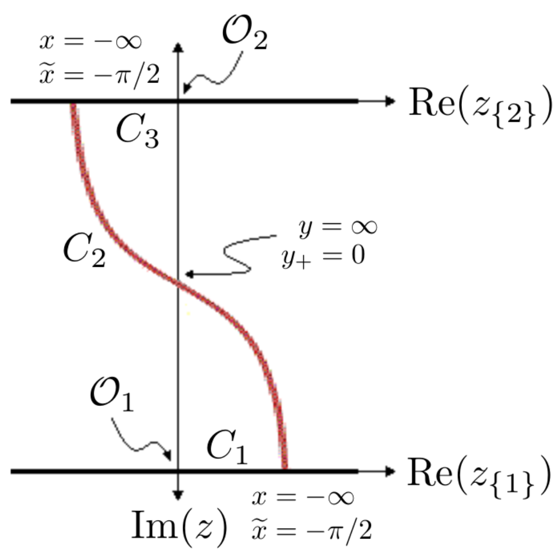

FIG. 5. The path integral over $C_1\cup C_2\cup C_3$ should be exactly equal to the closed contour integral in figure 4.

have defined $y_+ \in [0,\infty)$ so it only approaches the connection point at $(\widetilde{x},y)=(\pi/2,0)$ but that point contributes nothing to the integral and everything works out in the limit. When $\tan(\widetilde{x}=0)=0$, $y=\infty$ and $y_+=0$. Then, continuing along the tangent curve, we go beyond infinity by continuing the domain of $y_+\in[0,\infty)$ to the range of the tangent function sans the endpoints: $y_+\in(-\infty,\infty)$. Since $\tan(\widetilde{x}=-\pi/2)=-\infty$, equation (18) gives an interesting, transfinite point at $(x,y)=(-\pi/2,2\infty)$. Since y_+ is constrained to be $y_+\in[0,\infty)$, and $2\infty\notin\hat{\mathbb{C}}$, we will define the conformal coordinate on y_+ as

$$\widetilde{y}_+ = \tan^{-1}(y_+) \quad , \qquad \text{with} \qquad \widetilde{y}_+ \in (-\pi/2,\pi/2) \quad . \tag{19}$$

Therefore, we can retain the definition $y_+\in[0,\infty)$ and let the conformal coordinate be the one that goes beyond infinity. With equation (19) and

$$\widetilde{x} = \tan^{-1}(x) \qquad \Longrightarrow \qquad \widetilde{x} \in [-\pi/2,\pi/2] \quad , \tag{20}$$

we have defined the conformal coordinates in ${}^\star\mathbb{C}$. Equation (19) restores finite width to the MCM unit cell, albeit proportional to π instead of the highly desirable Φ. However, we still have room to make things work out later with more conformalism.

The semicircle C' passes through the point $y_+=0$ with zero slope so the tangent curve C_2 goes inside the semicircle before reaching that point. The slope of tangent at zero is $\sec^2(0)=1$ so it approaches the line $\widetilde{x}=0$ from inside the circle. Indeed, making the change of notation $y_+(\widetilde{x})\to y_+(x)$ or $y_+(\widetilde{x})\to\widetilde{y}_+(\widetilde{x})$ shows that C_2 is a straight line. If the semicircle is the curve that defines $z=\infty$ through $z=re^{i\theta}$ then $f(z)$ might not identically vanish at all points of C_2. That could cause an undesirable non-vanishing contribution from the path around infinity. However, since we have gone all the way to infinity along the real axis, and then we will only go back inside relative to that point, we can say that the integral vanishes at every point on C_2. Considering the z^{-2} term that should dominate the value of $D(x-y)$ along C_2, let us examine the ordinary coordinates $z=x+iy$. Along C_2, we have

$$x = \infty - \Delta x \tag{21}$$

and

$$\begin{aligned} y &= \infty - y_+ &(22)\\ &= \infty - \tan(\widetilde{x}) &(23)\\ &= \infty - \tan\left[\tan^{-1}(\infty-\Delta x)\right] &(24)\\ &= \Delta x \quad . &(25)\end{aligned}$$

When we plug that into z^{-2}, we see

$$\begin{aligned} z^{-2} &= \big(x+iy\big)^{-2} &(26)\\ &= \big[(\infty-\Delta x)+i\Delta x\big]^{-2} &(27)\\ &\approx \infty^{-2} \quad . &(28)\end{aligned}$$

Therefore, the integral along C_2 should contribute nothing because it gets all the way to $x=\infty$ before working its way back. This relies on some hokeyness regarding a radius smaller than infinity being equal to infinity but this is what we will consider. Since we have to reach infinity before going onto the hokey curve, which is the line $y_+(x)=\tan\left[\tan^{-1}(x)\right]=x$, everything should be ok.

REMARKS REGARDING GLOBAL CONSISTENCY

We have defined the C_2 curve in the conformal coordinate $\widetilde{y}_+$ as

$$\widetilde{y}_+(x) = \tan^{-1}\left\{\tan\left[\tan^{-1}(x)\right]\right\} \quad , \tag{29}$$

This is somewhat similar to the idea in [10, 12, 13] to go across the MCM unit cell through the dual tangent space. Considering figure 6, we see that the quantum clockwork of $\hat{M}^3$ [9] is such that the Riemann sphere is created on $\mathcal{H}_1$, we apply the inversion map to put the origin at $\varnothing$ (which is like applying the inversion map directly to $\mathcal{H}_1$), then we move to the second sphere through the point where the spheres are *tangent*, and then we invert again to get to $\mathcal{H}_2$. Comparing to equation (29), we invert $\mathcal{H}_1$ with the inverse tangent function, move to the other sphere via their point of tangency with the tangent function, and then invert the second sphere to get $\mathcal{H}_2$ as

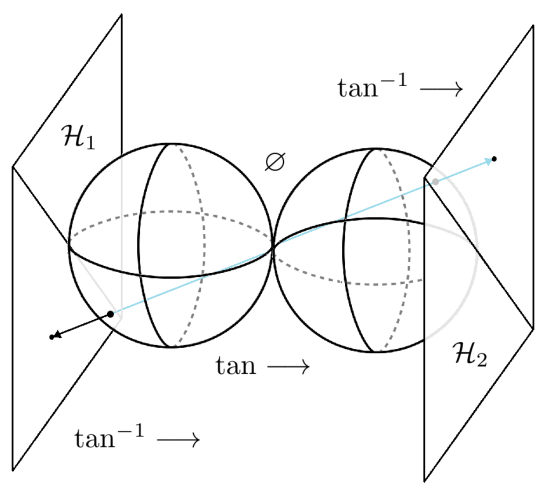

FIG. 6. To cross the unit cell, we begin in $\mathcal{H}_1$, create the Riemann sphere, permute its poles with the inversion map, go into the tangent sphere, and then use the inversion map again to move the origin into $\mathcal{H}_2$.

a plane tangent to the second sphere. Furthermore, the Riemann sphere is projected onto $\mathbb{C}$ at its polar points giving the surfaces $\{\mathcal{H}_1, \varnothing, \mathcal{H}_2\}$ and it is also projected onto $\mathbb{C}$ as plane passing through its equator. These equatorial planes would be Ω on the first sphere and $\aleph$ on the second.

Nota bene, each period of tangent has two asymptotes, figure 7, so the range of the pseudo-affine transfinite curve $y_+(\widetilde{x})$ is well-suited to take the pseudo-affine parameter [4, 12, 14–17]

$$\chi^5 \quad \equiv \quad \chi^5_+ \otimes \chi^5_\varnothing \otimes \chi^5_- \quad , \tag{30}$$

used to build smooth curves, or strings, across the MCM unit cell. To better understand the pseudo-affine curve $y_+(\widetilde{x})$, we should put together a tangent and a tangent inverse to simplify equation (29). The conformal imaginary coordinate $\widetilde{y}_+$ along C_2 as function of the conformal real coordinate $\widetilde{x} = \tan^{-1}(x)$ is

$$\widetilde{y}_+(\widetilde{x}) = \tan^{-1}\left[\tan(\widetilde{x})\right] = \widetilde{x} \quad . \tag{31}$$

Nota molto bene, this is the equation of a straight line and describes the identically flat shortcut through the cotangent space exposed so well in [3, 10, 12].

Whereas the tangent is a periodic function, and depending on the nuance, we will need to define the branch cut of each of the inverse tangent functions which define the hypercomplex curve around infinity $\widetilde{y}_+(\widetilde{x})$. Since inverse tangent has two branch cuts, and two inverse tangents appear in equation (29), we should probably use one branch for each to define an information channel. If chosen judiciously, each path around infinity can translate between the periodic domains of the tangent function which are separated by its asymptotes, as in figure 7. For instance, where $\tan^{-1}(x) = \theta$ we could make some convention to move to the next domain when computing $\widetilde{y}_+$ such that $\tan^{-1}\big[\tan(\theta)\big] = \theta + \pi$. This would be useful in defining a mechanism for chugging along across the unit cell from the $\hat{\pi}_0$-site in $\mathcal{H}_1$ to the $\hat{\pi}_1$-site in $\mathcal{H}_2$ [18]. Indeed, the inherent sub-domain of periodicity of the tangent function has width π so moving to the next sub-domain is at least qualitatively like the system of two co-$\hat{\pi}$'s which is a fundamental object in the MCM [7, 8, 11, 18]. (In principle, we want to connect pairs of co-$\hat{\pi}$s to form a circle (with U(1) symmetry), let them share information, then disconnect them (break the U(1) symmetry), and then reconnect them to other co-$\hat{\pi}$'s such that information propagates. The twisting mechanism in [7] gives a good example when co-$\hat{\pi}$s are halves of a circle of circumference 2π.) An application of the tangent inverse mechanism for $\theta \to \theta + \pi$ might be to define a new origin of coordinates, such as that in figure 5, which is shifted to the left or right instead of directly above the first origin. Furthermore, where the most important dimensionless coupling constant of classical physics $1/4\pi$ is attached to $\hat{\pi}$ [19], and the conformal complex numbers $\widetilde{z} = \widetilde{x} \pm i\widetilde{y}_\pm$ are such that the real and imaginary axes have length π, we might speculate that the conformal complex plane is itself the union of two co-$\hat{\pi}$s.

When the complex plane is mapped onto the Riemann sphere, the desire to go beyond infinity requires us to go into the tangent space to the Riemann sphere at the polar point opposite the origin of coordinates: conformal infinity. For each level of $\aleph$ to be self-similar, the object of tangency at one sphere's north pole should be the south pole of another sphere [2]. The "tangent" function seems intuitively well-suited to these purposes. When the complex plane is mapped onto the Riemann sphere, the upper half of figure 5 (the part that is beyond infinity) lies inside the "hypercomplexly infinitesimal neighborhood" around conformal infinity [2]. Therefore, now that we know how to arrange the coordinates to go past infinity, the methods developed here can be used to further develop the methods presented in [2]. That paper was almost completely dedicated to the idea of going beyond infinity when we argued against the Riemann hypothesis.

The final point of interest that we will raise before moving on with Φ is that the conformal coordinate $\widetilde{y}_+$ pushes beyond infinity in *exactly* the way required to accommodate the structure of $\hat{\Phi}^1$ and $\hat{\Phi}^2$ used to argue against the Riemann hypothesis in [2]. In the projection of the complex plane onto the Riemann sphere, using language from [2], the $\hat{\Phi}^1$ region begins at $\widetilde{y}_+ = 0$ and the $\hat{\Phi}^2$ region begins at $\widetilde{y}_+ \to -\pi/2$. This is the point $y = 2\infty$ where C_2 meets C_3 in figure 5. We will not use the concept of 2∞ here; the domain of y is $[-\infty, \infty]$. It is only the conformal coordinate that goes beyond infinity. Once we have gone beyond infinity, we need to redefine the magnitude of infinity relative to the new level of $\aleph$. Increasing magnitudes of infinity bring us back to the development of the golden ratio in ${}^\star\mathbb{C}$.

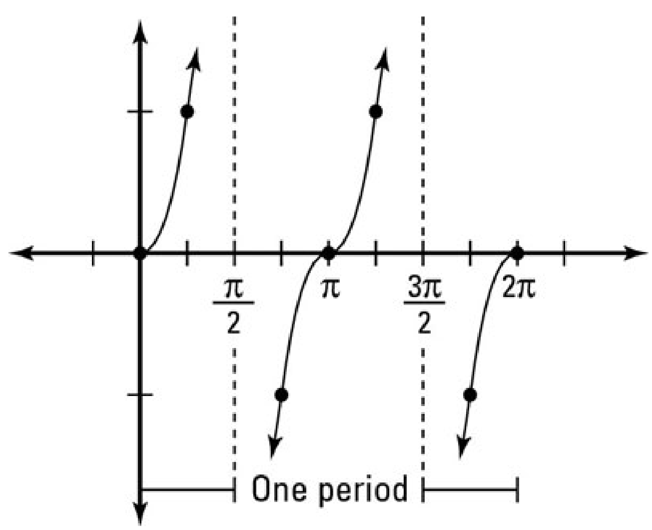

FIG. 7. The tangent function is has a period of width 2π constructed from a periodic domain of width π and two lengths of $\pi/2$ separated by asymptotes.

INCREASING INFINITUDE

How should we define $y_-^{\{2\}}$ relative to the new origin $\mathcal{O}_2$ (figure 5) which is on the level of $\aleph\,\hat{\Phi}^{j+1}$ relative to $\hat{\Phi}^j$ on $\mathcal{O}_1$? We know that complex conjugation is an operation that swaps the upper and lower complex half-planes, and we want $\widetilde{y}_+^{\{1\}} \to y_-^{\{2\}}$ which, according to

$$z = \begin{cases} x + iy_+ & \text{for} \quad \text{Im}(z) > 0 \\ x - iy_- & \text{for} \quad \text{Im}(z) < 0 \end{cases} \quad , \tag{32}$$

will have the effect of sending the upper complex half-plane to the lower complex half-plane on the higher level of $\aleph$. Since $y_\pm$ has i factored out of it in $z = x \pm iy_\pm$ a simple sign change will suffice to change the orientation of $y_\pm$. Physically, we should implement the change of sign with the phase change $e^{i\theta} \to e^{i(\theta+\pi)}$ attendant to a specular reflection off of the topological singularity at $\varnothing$. We also need to include the notion that infinity relative to $\mathcal{O}_2$ is bigger than infinity relative $\mathcal{O}_1$ [3, 5]. Furthermore $y_+^{\{1\}}$ is not the coordinate that goes beyond infinity; it is defined on $[0,\infty)$. Therefore, we can define bigger infinity by using the entire interval $\widetilde{y}_+^{\{1\}} \in (-\pi/2, \pi/2)$ to define the domain of non-conformal $y_-^{\{2\}}$, as in figure 8. If all of $\widetilde{y}_+^{\{1\}} \in (-\pi/2,\pi/2)$ becomes $y_-^{\{2\}}$ in the hypercomplex conjugation of $z_{\{1\}} = \widetilde{x} + i\widetilde{y}_+^{\{1\}}$ then the point $y_-^{\{2\}} = 0$ will be at $\mathcal{O}_1$ and the scale of $y_-^{\{2\}}$ is twice that of $y_-^{\{1\}}$. We use the term "hypercomplex conjugation" to refer to the sign change together with a rescaling of the magnitude of infinity. When $y_\pm^{\{2\}}$ is defined as $y_\pm^{\{2\}} \in [0,\infty)$, in accordance with the rules of $^\star\mathbb{C}$, the ∞ symbol will refer to the new, bigger infinity. Therefore, for clarity, we should write

$$y_\pm^{\{2\}} \in [0, \infty_2) \quad . \tag{33}$$

If we started at $\mathcal{O}_0$ instead of $\mathcal{O}_1$ then we could use notation like $\infty_0 \equiv \aleph_0$, $\infty_1 \equiv \aleph_1$, *etc.* When we start at $\mathcal{O}_1$, we have something like $\infty_j \equiv \aleph_{j-1}$.

It follows from

$$\hat{\Phi}^j \to \hat{\Phi}^{j+1} \quad : \quad \begin{cases} \infty \to 1 \\ 1 \to \varepsilon \\ \varepsilon \to \varepsilon^2 \end{cases} \quad , \tag{34}$$

that "2∞" needs to be like "∞^2" if the pattern of the cases is to hold across every level of $\aleph$. Even when we use ∞^2, we can write that as $\infty \times \infty \hat{\Phi}^j$ which will become $1 \times 1\hat{\Phi}^{j+1}$ under the rules. Therefore, we will make use of the ∞_j notation to describe transfinite infinities which exceed $[-\infty,\infty]$. If we *did* want to implement a scheme like ∞^2 then we could do so as follows. Where the radius of curvature is ∞ along a flat path like $\widetilde{y}_+(\widetilde{x}) = \widetilde{x}$, which is like the portion of a path inside the identically flat hypercomplexly infinitesimal neighborhood around a sphere's polar point, we might let the curvature kick in at $y = \infty$ ($\widetilde{y}_+ = 0$) when the path exits $\hat{\mathbb{C}}$ along a transfinite curve whose radius of curvature becomes finite at infinity. This is tantamount to letting the region $\widetilde{y}_+ > 0$ be the hypercomplex neighborhood around the south pole of a sphere on a higher level of $\aleph$.

In [2], we discussed how the hypercomplexly infinitesimal neighborhood around a pole can be a flat region on an otherwise curved sphere and figure 9 shows the

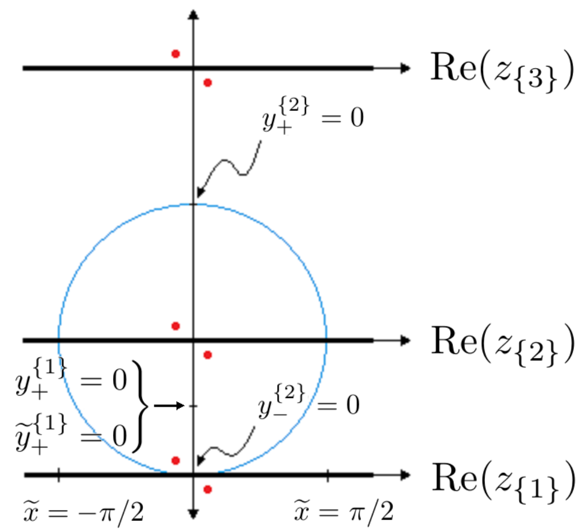

FIG. 8. In this figure, the poles of $D(x-y)$ are red dots. The contour around the entire complex plane defined around the $z_{\{2\}}$ origin encloses three poles (red dots) rather than the two that are usually considered. This figure shows that the $\hat{\Phi}^1$ and $\hat{\Phi}^2$ levels of $\aleph$ are needed to construct the transfinite framework [2]; the integral around the whole complex plane relative to $z_{\{1\}}$ would not include an extra pole.

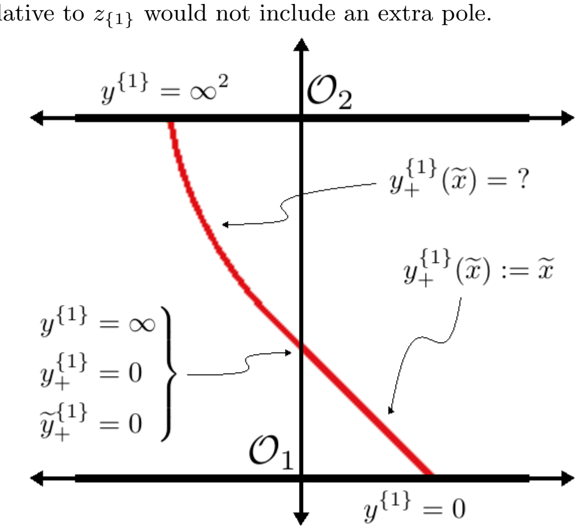

FIG. 9. This figure shows an alternative path beyond infinity which lets infinity grow as its square instead of merely doubling. The path from $\mathcal{H}$ to Ω is linear, but the path from Ω to $\varnothing$ accelerates.

smooth continuation. Aiming for ∞^2 makes the path beyond infinity more complicated than a straight line but if the only thing that will happen beyond infinity (before converting to the next $\hat{\mathbb{C}}$ coordinates) is a specular reflection from a topological singularity then there is probably a useful symmetry reduction related to $\varnothing$ having no width between Ω and $\aleph$ [3].

MASS

Now we will consider an application. If we draw a closed contour around the entire complex plane defined at $\mathcal{O}_2$, a circle with radius ∞_2, as in figure 8, it will enclose the two poles of $D(x-y)$ at

$$z = \pm\sqrt{\omega_k^2 + i\varepsilon} \quad , \tag{35}$$

which sum to zero, and it will also include the pole that was in the upper complex half-plane relative to $\mathcal{O}_1$. Figure 8 shows why the imbalance below is not offset by another term above. To get the pole in the lower complex half-plane relative to $\mathcal{O}_3$, we would have to use the conformal coordinate $\widetilde{y}_+^{\{2\}}$ to go beyond infinity. Figure 8 also shows that the result for the contour integral in the upper complex half-plane is preserved in translation from $\mathcal{O}_1$ to $\mathcal{O}_2$, and will be at $\mathcal{O}_3$, *etc.* Since the pole residue is usually associated with a particle's properties, such as mass through the relativistic energy ω_k, we might extend this principle into a formal proof of the existence of the Yang–Mills mass gap. When the sum of the poles over an entire level of $\aleph$ in the hypercomplex plane is not equal to zero, there should always be some non-vanishing mass term. However, detractors of this research program *and others* are surely aware that this writer has not yet read Yang's and Mills' 1954 paper so we will not progress in that direction presently. Instead, we will emphasize the physical aspects of the theory of infinite complexity which should manifest in the hypercomplex analysis of $z \in {}^\star\mathbb{C}$. Before doing so, it must be noted that the propagator and the residue theorem are very important in QFT so there are probably a lot of other direct applications for the MCM construction of ${}^\star\mathbb{C}$ demonstrated in figure 8.

One such application improves the motivation for the $i\varepsilon$ term added to the denominator of $D(x-y)$ through unsightly, though not invalid reasoning. When $z \in \mathbb{C}$, we can add and subtract real numbers like m^2 because an $\mathbb{R}$-number is just a $\mathbb{C}$-number with $\mathrm{Im}(z)=0$ and we are allowed to do analysis with $m^2 \in \mathbb{C}$. However, there is no such thing as a ${}^\star\mathbb{C}$-number with $\mathrm{Im}(z)=0$. Therefore, it makes no sense to subtract m^2 from a ${}^\star\mathbb{C}$-number. When we make the substitution $k^0 \to z \in {}^\star\mathbb{C}$, we are compelled to make the complimentary substitution $m^2 \to m^2 + i\varepsilon$ because otherwise the operation in the denominator is ill-defined. This should alleviate the need to make an artificial substitution which moves the poles of $f(z)$ off the real axis. This complements the result of [20] wherein we demonstrated how to alleviate the artificial substitution of imaginary time during analytic continuation via Wick rotation: another very important, though likewise *ad hoc*, method in QFT.

THE MCM UNIT CELL

Consider the infinite width of the MCM unit cell needed to derive a topological singularity at $\chi_+^5 = \infty$ and that we we want to restore the original definition

$$\chi_+^5 \in (0, \Phi] \quad , \tag{36}$$

which supports the $\hat{\Phi}^j$ notation for levels of $\aleph$. We will say that the conformal coordinate on $\chi_\pm^5$ is $\widetilde{\chi}_\pm^5 \equiv x_\pm^5$ where $x_\pm^5$ is the de Sitter parameter of curvature in the de Sitter or anti-de Sitter metric of each respective slice of constant $\chi_\pm^5$. This means that the location of the topological singularity $\varnothing$ is defined by $x_\pm^5 = \infty$ rather than $\chi_\pm^5 = \infty$ (which allows us to restore $\chi_+^5 \in (0, \Phi]$.) We have reversed the convention for the conformal tilde on $\widetilde{\chi}_\pm^5$ because the conformal coordinates are the ones used to create the singularity at $x_\pm^5 = \pm\infty$. This is only a matter of semantics between which one is labeled "conformal."

Let the relationship between χ_+^5 and $\widetilde{y}_+$ be

$$\chi_+^5 = \frac{\pi}{2} - \widetilde{y}_+ \quad . \tag{37}$$

This excludes $\chi_+^5 = 0$ as required. The conformal complex plane coordinate $\widetilde{y}_+ = 0$ is at $\chi_+^5 = \pi/2$, as in figure 10. If we let the conformal relationship between $\chi_\pm^5$ and $x_\pm^5$ be

$$x_\pm^5 = \tan(\chi_\pm^5) \quad , \tag{38}$$

then $\varnothing$ will lie at $\chi_+^5 = \pi/2$ and that is not the desired behavior. We want to put the singularity at $\chi_+^5 = \Phi$. We get the singularity at $\chi_+^5 = \Phi$ when the conformal

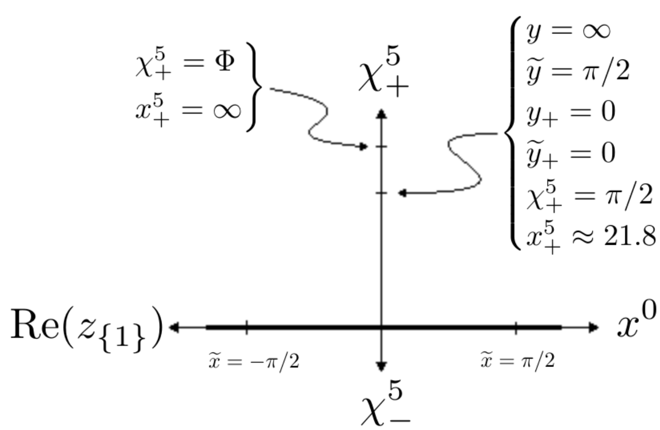

FIG. 10. The conformal coordinate to the chirological coordinate $\chi_\pm^5$ is the de Sitter parameter $x_\pm^5$.

relationship between the chirological coordinate χ^5_+ and the de Sitter parameter x^5_+ is

$$x^5_+ = \tan\left(\frac{\pi\chi^5_+}{2\Phi}\right) \quad . \tag{39}$$

Therefore, the complex plane centered on $\mathcal{O}_1$ reaches infinity at $\chi^5_+ = \pi/2$. The topological singularity at $\chi^5_+ = \Phi$ lies beyond the boundary of the extended complex plane $\hat{\mathbb{C}}$ on the level of $\aleph$ corresponding to $\mathcal{O}_1$.

When we place a singularity at $\chi^5_+ = \Phi$ and send a plane wave past infinity along the conformal coordinate that goes beyond infinity, and neglecting that entire books have been written about the nuance of wave and heat equations in curved space (and neglecting that only the $x^\mu_\pm$ spaces are curved, not the identically flat $\chi^a_\pm$ spaces [3]), then we expect that the wave will reflect off the topological singularity. Furthermore, where general relativity says that the singularity will be a sink, not a reflector, we may consider the loss into a black hole on one level of $\aleph$ as emission from a white hole on the next, as per usual in conformal spacetime (with the new level of $\aleph$ concept added.) As stated above, complex conjugation is the operation that does reflection along the imaginary axis. Therefore, using a reflection operator to construct the black/white hole throughput will be a good operation to make $\widetilde{y}^{\{1\}}_+$ become $y^{\{2\}}_-$. If the sign changes by reflection, then we should use $z = x + iy_\pm$ instead of the $z = x \pm iy_\pm$ used in

$$z = \begin{cases} x + iy_+ & \text{for} \quad \mathrm{Im}(z) > 0 \\ x - iy_- & \text{for} \quad \mathrm{Im}(z) < 0 \end{cases} \quad , \tag{40}$$

because we will have $y^{\{2\}}_- \in (-\infty, 0]$.

To consider reflection off the singularity, consider specular reflection in classical optics. The reflected wave gains π radians of phase which is exactly what is required to reverse the sign of $\widetilde{y}^{\{1\}}_+$ so that it may become $y^{\{2\}}_-$. The same can be said for the reflection of a string wave where $\varnothing$ is like a string's fixed endpoint.

THE ROLE OF Φ IN THE MCM

In equation (39), we've just stuck Φ in there, and that is fine, but it would be better if Φ came out by itself. To that end, consider our motivations for using Φ. The first strong evidence for the appropriateness of the golden ratio in the MCM came in the form of the fine structure constant [21]

$$\alpha^{-1}_{\text{MCM}} = 2\pi + \left(\Phi\pi\right)^3 \quad , \tag{41}$$

which differs from $\alpha^{-1}_{\text{QED}} \approx 137$ by about 0.4%. Einstein's equation is derived in the MCM [9, 18, 22, 23] from some algebraic operation

$$\hat{M}^3\psi = \hat{M}^3\phi \qquad \longrightarrow \qquad \partial^3_t\psi = \pi\Phi^2\phi \quad , \tag{42}$$

where ψ is a wavefunction and ϕ is something. Φ is also needed to get the dimensionless coupling constant $1/4\pi$ of the Poisson equations for both Newtonian gravity

$$\rho = \frac{1}{4\pi}\,\nabla^2\phi \quad , \tag{43}$$

and classical electromagnetism

$$J^\mu = \frac{1}{4\pi}\,\eta^{\mu\nu}\partial_\nu\partial_\rho A^\rho \quad , \tag{44}$$

out of the ontological resolution of the identity

$$\hat{1} = \frac{1}{4\pi}\,\hat{\pi} - \frac{\varphi}{4}\,\hat{\Phi} + \frac{1}{8}\,\hat{2} - \frac{i}{4}\,\hat{i} \quad . \tag{45}$$

Without Φ, the coefficient on the $\hat{\pi}$ term would be $1/3\pi$ and no physicist ever heard of 3π. So, there is a lot of evidence that the correct physical theory should have Φ in it and we would like to get it out without putting it in.

How should the golden ratio appear? If we can use the increasing scale of infinity to begin to construct a Fibonacci sequence, such as, for example, that which causes $y^{\{2\}}_-$ to be twice as infinite as $y^{\{1\}}_+$, as in figure 8, then we could take the limit of the Fibonacci sequence to infinity and obtain Φ as the ratio of $\infty\hat{\Phi}^{j+1}$ to $\infty\hat{\Phi}^j$. This makes perfect sense because

$$\frac{\left|\infty\hat{\Phi}^{j+1}\right|}{\left|\infty\hat{\Phi}^{j}\right|} = \Phi \quad . \tag{46}$$

Taking the limit of an infinitely late place in the Fibonacci sequence is exactly what we have suggested in [5] when claiming that there is no way for the observer to determine his absolute level of $\aleph$. By choosing an arbitrary level of $\aleph$ as finite and measuring infinitude relative to that level, we would choose a point on the golden spiral, figure 11, where the ratio of the length of the sides of each successive box to the previous one's has already reached its asymptotic value Φ. How shall we construct such a sequence?

What is special about the golden ratio? Whatever it is, it shows up in figure 11. Let the $\hat{\Phi}^0$ box be the ordinary complex plane. $\mathcal{O}_1$ is exactly in center of that box. In the conformal complex coordinates $\widetilde{z} = \widetilde{x} + i\widetilde{y}_\pm$, the area of the complex plane is π^2. Therefore, the area of next complex plane on the higher level of $\aleph$ is $\Phi^2\pi^2$. We can get the critical value $\pi\Phi^2$ [22] from equation (42) by operating on that area with the operator that projects into the $\hat{\pi}$-site [18]. That operator is

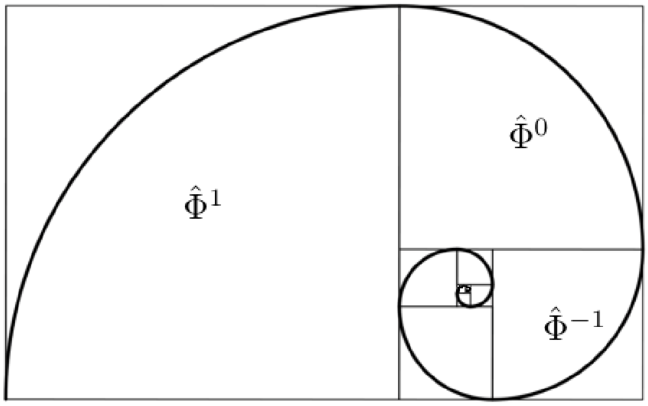

FIG. 11. This figure captures the "golden" quality of the golden ratio Φ. The area of each larger box is Φ^2 as large as the area of the previous box.

$$\hat{\mathcal{P}}_\pi \quad \equiv \quad \hat{1} = \frac{1}{\pi}\,\hat{\pi} \quad . \tag{47}$$

If we label the area of the $\hat{\Phi}^j$ box with A_j then $A_0 = \pi^2$ and we obtain

$$\hat{\mathcal{P}}_\pi A_1 = \pi\Phi^2\hat{\pi} \quad . \tag{48}$$

This exactly the critical value [22] needed to derive Einstein's equation but it remains to show why we should use $\hat{\mathcal{P}}_\pi$ or put it in an equality with $-i\omega^3\psi$. Both issues are beyond the scope of this paper but we will discuss them briefly.

We should assume that projecting into the $\hat{\pi}$-site means to change the $\hat{\Phi}^j$ of a given box to $\hat{\Phi}^0 = \hat{1}$ because we will have to use finite numbers on the $\hat{\Phi}^0$ level of $\aleph$ to do physics there. When we do this to the $\hat{\Phi}^1$ box with $\hat{\mathcal{P}}_\pi A_1$, it has the effect of discrete translation up the golden spiral. In a more formal treatment, we would project the wavefunction of A_1 into the (finite) basis associated with A_0, and then relabel everything, and that wouldn't change anything except the renormalized area because the golden spiral is perfectly self-similar. This self-similarity imposes the stability of the dynamics needed to be able to apply $\hat{M}^3$ over and over, and over, where we keep projecting the next higher box into $\hat{\pi}$-site. The next higher box is either the $\hat{2}$-site or the $\hat{\Phi}$-site. Generally, we could consider two perpendicular spirals [5] where $\hat{\Phi}$ points into the spiral which has the box labeled $\hat{\Phi}^1$ in its asymptotic limit while $\hat{2}$ would point into a new spiral growing as the Fibonacci sequence $\{1, 1, 2, ...\}$.

Where we have proposed that torsion should act everywhere in the MCM unit cell except for $\mathcal{H}$ [13, 19], that can be a mechanism that twists a curve belonging to a spiral in the plane of the page onto another spiral in the perpendicular direction. Since $\mathcal{H}_2$ is two levels of $\aleph$ above $\mathcal{H}_1$, there is a lot of intermediate space in which to do tricky things. Indeed, the level of $\aleph$ only increases at twice during $\mathcal{H} \to \Omega \to \varnothing \to \aleph \to \mathcal{H}$ so we have some reason to consider adjacent boxes of the same size where spirals intersect, as in $\{1, 1, 2, ...\}$ and $\{1, \Phi, \Phi^2, ...\}$.

MCM GENERAL RELATIVITY

To demonstrate the utility of Φ, we will rederive Einstein's equation from

$$\partial_t^3\psi = \pi\Phi^2\phi \quad . \tag{49}$$

The main hypothesis in the MCM [24] around which our application of the scientific method is structured is that equation (49) is true. Using the notation that $\psi\hat{\pi} \sim |\psi; \hat{\pi}\rangle$ and assuming that ψ is plane wave with $\partial_t\psi = i\omega\psi$, we may derive from equation (49)

$$\partial_t^3|\psi; \hat{\pi}\rangle = \pi\Phi^2|\phi; \hat{\pi}\rangle \tag{50}$$
$$\omega^3|\psi; \hat{\pi}\rangle = i\pi\Phi|\phi; \hat{\pi}\rangle + i\pi|\phi; \hat{\pi}\rangle \tag{51}$$
$$8\pi^3 f^3|\psi; \hat{\pi}\rangle = i\pi^2|\phi; \hat{\Phi}\rangle + \pi^2|\phi; \hat{i}\rangle \tag{52}$$
$$8\pi f^3|\psi; \hat{\pi}\rangle = i|\phi; \hat{\Phi}\rangle + |\phi; \hat{i}\rangle \quad . \tag{53}$$

This generates the formulation of gravity where the present is the sum of contributions from the past and future [8, 22]: $\hat{\pi}$ vectors live in $\mathcal{H}'$ which is the Hilbert space of states in the present, $\hat{\Phi}$ vectors live in Ω' which contains position eigenstates the in future (geometric final conditions), and $\hat{i}$ vectors live in $\aleph$ which is the space of quantum mechanical initial conditions in the past with respect to $\mathcal{H}'$. $\{\aleph', \mathcal{H}', \Omega'\}$ form a Gelfand triple, or a rigged Hilbert space [3].

After adding $\hat{2}$ to the ontological basis [19, 22] in work that followed the initial reporting of equations (50-53) [9], we supposed that we should insert $|\psi; \hat{2}\rangle$ into equation (53) but we will not do so here. Instead, consider that there exist two golden spirals [5] and that their boxes are labeled with even and odd powers of infinity respectively. In that case, the general relativity of the $\hat{\pi}$-site whose level of $\aleph$ is $\hat{\Phi}^0$ can take contributions from the past $\aleph$ and future Ω defined in two adjacent boxes of the other spiral: those labeled $\hat{\Phi}^{\pm 1}$. Indeed, where the intersection of two spirals might define the $\{1, 1, 2, ...\}$ portion of a new golden spiral, the domains

$$\chi_-^5 \in [-1, 0) \quad , \qquad \text{and} \qquad \chi_+^5 \in (0, \Phi] \quad , \tag{54}$$

suggest one box of equal size and one box scaled by Φ. Regarding $\hat{\Phi}^{-1}$, we don't want to consider negative levels of $\aleph$ [13] so perhaps we should put $\mathcal{H}$ (the $\hat{\pi}$-site) in the $\hat{\Phi}^1$ box with the odd powers of $\hat{\Phi}$. To make this change, we could say that taking the asymptotic limit of large boxes requires $\hat{\Phi}^1$ as the finite level and that $\hat{\Phi}^0$ is like $\hat{\Phi}^{-\infty}$ where there is no lower box in which to define $\aleph$

at the $\hat{\imath}$-site. However, that will be irrelevant presently and we will say $\mathcal{H}$ has the even level of $\aleph\ \hat{\Phi}^0$ (and $\hat{\Phi}^{-1}$ is replaced with $\hat{\varphi}$ [13].) Regarding $\hat{2}$, if $\hat{\Phi}$ and $\hat{\imath}$ are the past and future, then what should $\hat{2}$ be? We might use a representation in which $\{\hat{2}, \hat{\Phi}, \hat{\imath}\}$ represent contributions to the geometry of the hypersurface of the present associated with spacelike, timelike, and null infinity or the conformal regions beyond those infinities. Whatever it is, the application is beyond the scope of this paper.

Einstein's equation is derived from equation (53) with three rigorously-defined, mathematically perfect maps

$$f^3\big|\psi;\hat{\pi}\big\rangle \quad \mapsto \quad T_{\mu\nu} \tag{55}$$

$$i\big|\phi;\hat{\Phi}\big\rangle \quad \mapsto \quad Rg_{\mu\nu} \tag{56}$$

$$\big|\phi;\hat{\imath}\big\rangle \quad \mapsto \quad g_{\mu\nu}\Lambda \quad . \tag{57}$$

Substituting into equation (53), we get

$$8\pi T_{\mu\nu} = R_{\mu\nu} + g_{\mu\nu}\Lambda \quad . \tag{58}$$

This is Einstein's equation for general relativity as it has been known for more than one hundred years. Where $T_{\mu\nu}$ is the stress-energy tensor, we have previously pointed out that a classical energy density is proportional to the cube of the frequency [6], as in map (55). Since $T_{\mu\nu}$ is the stress-energy tensor that says what the energy density is, it is very good that there is already an energy density law proportional to the cube of the frequency: Planck's law. In units where $\hbar = c = 1$, Planck's law is

$$B_f(f,T) = 4\pi f^3\,\frac{1}{e^{2\pi f/k_B T}-1} \quad . \tag{59}$$

It is chock-full of ontological numbers. Note that the coefficient 4π is like the coefficient $1/4\pi$ of $\hat{\pi}$ in the ontological resolution of the identity: equation (45). 4π, therefore, is also like the $\hat{\pi}$ map, map (55), associated with Planck's law through the cube of the frequency. Planck's law tells us how much electromagnetic energy a perfect blackbody at temperature T will shed at frequency f so the stress-energy tensor $T_{\mu\nu}$ on the right side of map (55) should be like a slice of the black body curve at the f whose state is ψ.

Surely we would have cited Planck's law by name in every previous MCM derivation of Einstein's equation [9, 11, 15, 16, 18, 22–25] but this writer had become fixated on the fifth power of the wavelength in

$$B_\lambda(\lambda,T) = \frac{4\pi}{\lambda^5}\,\frac{1}{e^{2\pi/\lambda k_B T}-1} \quad , \tag{60}$$

being somehow related to $\dim(\Sigma^\pm)$. The fifth power of a quantity is very unusual and not typically observed in physics. Since frequency and wavelength are related by the inversion map

$$f = \frac{1}{\lambda} \quad , \tag{61}$$

which swaps the poles of the Riemann sphere, one would expect λ^{-5} to show up as $f^5\ B_f$ but it does not. It shows up as f^3. The discrepancy arises because increasing f or λ go in different directions with respect to increasing energy and the tails of the energy distribution are uneven.

The point $\lambda = 0$ is at $f = \infty$, and vice versa, so there are a lot of connections to be made with our origins separated by infinity. Indeed, Planck's law is the "origin" in physics of the concept of quantized energy packets. Therefore, in a devoted physical treatment to appear elsewhere, we should consider quantization arising from the arrangement of various powers of $\hat{\Phi}$ occupying some configuration on two separate golden spirals connected by coordinates whose origins are infinitely separated, as are the origins of f and λ in equations (59) and (60). In fact, where we have proposed to use torsion in the MCM, torsion together with reflection and rescaling can construct from the curve of the golden spiral a wave with a given f or λ, or vice versa.

When gravity is an expected interaction between a box on one spiral and two boxes on the other, as in equation (53), and it has the same coefficient f^3 as Planck's law, and the coefficient 4π of Planck's law is inversely associated with the $\hat{\pi}$ in $\big|\psi;\hat{\pi}\big\rangle$, as in map (55) and equation (59), then it is likely that we can use these relationships to learn something about quantum gravity. For example, we have proposed to solve the divergent energy of the QFT vacuum [8, 11] when the 4D gravitational branes in the unit cell have no 5D hypervolume. The solution relies on the principle that infinite energy divided by finite volume in the canonical system is infinite, but infinite energy divided by zero volume in the MCM system can be replaced with a derivative dE/dV. We get an energy from Planck's law by integrating over some range of f or λ and in the MCM we will need to integrate dE/dV over some width of χ^5 to obtain a finite answer for the energy density in hyperspacetime. Since $\mathcal{H}$ has no width in χ^5, and the Kaluza–Klein metric $\Sigma^\pm_{AB}$ only works when there is no 5D matter-energy, we might integrate over x^0 instead of χ^5 to get a finite energy density in 4D spacetime. Moreover, Planck introduced his eponymous constant and the concept of quantization specifically to stop the divergent explosion of the energy density described by his law.

We have a lot of hints that the maps between the MCM algebra, equation (49), and Einstein's equations are well-motivated. We have demonstrated in equations (50) and (51) that the magic all comes from the special property of the golden ratio $\Phi^2 = \Phi + 1$. Another nice motivation for gravity can be derived from the Cauchy–Riemann equations. Since those are inherently true, if we can derive $\pi\Phi^2$ from them then we won't need to resort to any interpretive twisting to say why general relativity is natural to the MCM structure. The hypothesis, equation (49), will be confirmed.

When

$$f(z) \quad \equiv \quad f(x,y) = u(x,y) + iv(x,y) \quad , \tag{62}$$

with u and v being real-valued functions, the Cauchy–Riemann equations are

$$\partial_x u = \partial_y v \quad , \qquad \text{and} \qquad \partial_y u = -\partial_x v \quad . \tag{63}$$

The wavefunction is a complex-valued function like $f(z)$ so we may suppose that Ψ is a wavefunction and that

$$\Psi = \psi + i\phi \quad . \tag{64}$$

This is not far-removed from the origin of ϕ proposed in [26]. (Neither is it far-removed from equation (53).) Since we have chosen $x \equiv x^0$, we can apply ∂_x^2 to a Cauchy–Riemann equation and use $\partial_x \psi = i\omega\psi$ to very nearly obtain the MCM equation as

$$\partial_x^3 \psi = \partial_x^2 \partial_y \phi \qquad \Longrightarrow \qquad \omega^3 \psi = i\partial_x^2 \partial_y \phi \quad , \tag{65}$$

The operator on the LHS has the form required to return the critical value $\pi\Phi^2$ [22] (up to a phase) if

$$\partial_x \phi := \Phi\phi \quad , \qquad \text{and} \qquad \partial_y \phi := \pi\phi \quad . \tag{66}$$

These conditions are like those we have treated previously in [15]. Usually ψ is like e^{ikx} so it does not fit the form of equation (62). However, since quaternions have the property

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1 \quad , \tag{67}$$

we may use any one of them to make a different sort of plane wave. It is *plain* to see that we may define plane waves as

$$\psi = e^{\mathbf{i}(\omega t - \vec{k}\cdot\vec{x})} \quad . \tag{68}$$

There is some subtle feature which is different than the imaginary number i when the quaternion triple product is

$$\mathbf{ijk} = -1 \quad , \tag{69}$$

so the implications of equation (69) for a set of complex planes using the imaginary number and/or the quaternions will need to be cataloged should we move forward with $\psi := \mathbf{i}$.

If $\mathrm{Im}(\psi)$ returns the part of ψ multiplied by i then we will have

$$\mathrm{Im}(\psi) = 0 \quad , \tag{70}$$

and Ψ in the form of equation (62). The quaternion plane wave preserves equation (65) with an $\mathbf{i}$ instead of an i. If we tried to ignore all of this and write $\Psi = \phi + i\psi$ with ψ being a regular plane wave, then it would not work because plane waves have real and imaginary parts, and u and v are strictly real in the Cauchy–Riemann equations. When the plane wave part of Ψ in encoded on ψ, that leaves us to define ϕ as a geometric piece which will have the correct derivatives. In [15], we demonstrated an operator

$$\partial_\chi^3 \quad \equiv \quad \partial_+ \partial_\varnothing \partial_- \quad , \tag{71}$$

to get the value $\pi\Phi^2$ but, in equation (65), we have two x derivatives when it is the y derivative that would seem amenable to representation as $\partial_\pm \phi := \Phi\phi$. Likewise, it is the periodic domain structure on x would lend itself more naturally to $\partial_\varnothing \phi := \pi\phi$. If we could rotate ϕ, or rotate $\hat{\mathcal{R}}z = z' = \chi^5 \pm ix^0$, then this would work out. We will not completely solve all of hypercomplex gravity in this paper but we will consider the rotation in a later section after we show the origin of the golden ratio in the structure of ${}^\star\mathbb{C}$ so that it could contribute naturally, in some way, to $\partial_y \phi = \Phi\phi$.

THE MCM FINE STRUCTURE CONSTANT

Not only does MCM general relativity depend on the golden ratio through its square, MCM quantum theory depends on its cube through the fine structure constant

$$\alpha_{\mathrm{MCM}}^{-1} = \big(\Phi\pi\big)^3 + 2\pi \quad . \tag{72}$$

We have considered gravity regarding the temporal $\{x^0, \chi^5_\pm\}$ part of $\Sigma^\pm$ and, here, we will consider the fine structure constant regarding the spatial $x^i \sim \chi^j$ part of $\Sigma^\pm$. If α_{MCM} comes from the volume of x^i while gravity comes from the Cauchy–Riemann equations of x^0 and χ^5 then we would have two nicely distinct sectors. Since quantum theory mostly deals with states in a timeless Hilbert space, it is good to consider the timeless spatial part of $\Sigma^\pm$ for the origin of $\alpha_{\mathrm{MCM}}^{-1}$. To that end, assign conformal coordinates

$$\widetilde{x}^i = \tan^{-1}(x^i) \quad , \tag{73}$$

so that the volume of all of space is π^3. When space increases by Φ along the golden spiral, the volume of the space on the higher level of $\aleph$ will be $\big(\Phi\pi\big)^3$. This is very nearly $\alpha_{\mathrm{MCM}}^{-1}$. Our goal will be to add to this volume the volume of the connector between all of space in one box and all of space in the next. We have previously proposed a few ways by which we might obtain the value 2π required for α^{-1}_{MCM} [15, 21] and now we will develop a better one. We have worked with A_j pertaining to Einstein's equation above and now we will work with

$$V_0 = \pi^3 \quad , \qquad \text{and} \qquad V_1 = \Phi^3\pi^3 \quad , \tag{74}$$

pertaining to the fine structure constant.

We have defined the operation that connects one box to the next as the inversion of the Riemann sphere. Therefore, space in one box is connected to space in the next box through the sphere's polar point. When we attach x^i to the plane spanned by $\{x^0, \chi^5\}$ and then use conformalism to send that plane to a sphere, the light cone structure of spacetime will only be preserved if all of space terminates at the polar point. When we say "terminates" there is obviously some nuance because we will continue space through the termination at conformal infinity as if through a big bounce [7]. Therefore, when we want to add to the volume of all of space the volume of the connective piece, that must be the volume of the bounce which, incidentally, was non-zero in the progenitive LQC formulation [7, 8, 11]. Without referring to LQC, we want to include the volume of the hypercomplex neighborhood of the polar point in the volume of all of space. If there is a big bang and a big crunch at the ends of each box then we are not integrating over everything if we don't include them. If we include both of them then we will be double counting when we exchange bangs and crunches for cyclic bouncing in the continuation beyond conformal infinity. Therefore, we should add to $\big(\Phi\pi\big)^3$ the volume of the hypercomplexly infinitesimal neighborhood around one of the sphere's poles. This volume vanishes trivially under ordinary geometric conditions but those neglect the hypercomplexity.

We will define the connection between adjacent levels of $\aleph$ such that the connective point is defined according to the lower level while residing on the upper level. This embodies what it means for two levels to be connected. In this way, the $\hat{\Phi}^1$ sphere will have its south pole defined on the $\hat{\Phi}^0$ level of $\aleph$ and its north pole, which is defined by the $\hat{\Phi}^1$ level, will reside at the south pole of the Riemann sphere on the $\hat{\Phi}^2$ level. This allows us to avoid double counting while enforcing some concept of connection between levels of $\aleph$.

The hypercomplex neighborhood around the pole lies entirely within the point of tangency between spheres [2]. Therefore, we may use an infinite radius of curvature to write the area of the hypercomplex neighborhood as a flat disc. The motivation for the infinite radius of curvature is that the radius of the $\hat{\Phi}^1$ sphere is infinite with respect to the finite $\hat{\Phi}^0$ level of $\aleph$ of its south pole. Therefore, we can use plane polar coordinates $\{r, \theta\}$ to describe the area of the disc. Since we want to do a volume integral, we will add cylindrical polar z'. The non-hypercomplex volume is

$$\int_{\text{Pole}} dV = \int_0^0 dz' \int_0^0 dr\, r \int_0^{2\pi} d\theta = 0 \quad . \tag{75}$$

We derive the limits on z because the point lies in the plane and we derive limits on r because the radius of a point is zero. To extract a non-zero value from equation (75), consider the relationship between $\hat{\Phi}^0$ and $\hat{\Phi}^1$. We have defined $^\star\mathbb{C}$ such that

$$\hat{\Phi}^j \to \hat{\Phi}^{j+1} \quad : \quad \begin{cases} \infty \to 1 \\ 1 \to \varepsilon \\ \varepsilon \to \varepsilon^2 \end{cases} \quad , \tag{76}$$

[2, 3] where the meaning is that

$$\infty\hat{\Phi}^j = \hat{\Phi}^{j+1} \quad . \tag{77}$$

Now we will write the hypercomplex volume. On the $\hat{\Phi}^1$ level, the volume of the ball is finite: $\big(\Phi\pi\big)^3$. Therefore, the scale of the pole with respect to a hypercomplex volume element dV_1 is on the order of an infinitesimal, but it is not zero. We should consider a radius ε instead of the 0 radius given in equation (75). If the point has a radius, then it has a height too. By symmetry, the height is centered on $z'=0$ and we may write

$$\int_{\text{Pole}} dV_1 = 2\pi \int_{-\varepsilon}^{\varepsilon} dz' \int_0^{\varepsilon} dr\, r \quad . \tag{78}$$

Since ε is infinitesimal, the pole still has zero volume relative to $\hat{\Phi}^1$, as expected. To consider the connection to the lower level, we want to consider the pole's volume defined according to $\hat{\Phi}^0$. From definition (76), we see that $\hat{\Phi}^1 \to \hat{\Phi}^0$ induces $\varepsilon \to 1$ so

$$\int_{\text{Pole}} dV_0 = 2\pi \int_{-1}^{1} dz \int_0^1 dr\, r = 2\pi \quad . \tag{79}$$

This has units of dimensionless volume and we have greatly improved upon previous efforts [15, 21] to obtain a contribution of 2π to the volume $\big(\Phi\pi\big)^3$. In [15], we used a π-normalized delta function at a sphere's two poles, but that had an implied double counting effect so the 2π derived here is better. Furthermore, the Riemann sphere does not include two poles so it is not obviously globally consistent that we should integrate over two of them.

Why does this piece contribute to V_1 but not A_1? The fact is, we might not need A_1 at all to get MCM general relativity when $\pi\Phi^2$ comes instead from $\partial_y^2\partial_x$. Even when we discussed the projection $\hat{\mathcal{P}}_\pi A_1$, a great reason for doing so was not manifest. The formulation based on the Cauchy–Riemann equations, however, is extremely well-motivated and has nothing to do with $\hat{\mathcal{P}}_\pi A_1$. Whatever the outstanding issues are, we have obtained a good result that the hypercomplex neighborhood around the polar point has 2π units of dimensionless volume which are well-suited to addition with a rectangular volume element $\left(\Phi\pi\right)^3$.

The volume of a periodic box contributes to a physical lattice theory as its inverse and this is how we aim to derive $\alpha_{\rm MCM}$ from $\alpha^{-1}_{\rm MCM}$. In [15], we wrote:

> "In the case of a function f on a Bravais lattice satisfying $f(\vec{r}+\vec{R})=f(\vec{r})$, the function can be expanded as follows when $\vec{K}$ are reciprocal lattice vectors and $f_{\vec{K}}$ are the Fourier coefficients.
>
> $$f(\vec{r}) = \sum_{\vec{K}} f_{\vec{K}}\, e^{i\vec{K}\cdot\vec{r}} \tag{80}$$
>
> $$f_{\vec{K}} = \frac{1}{V}\int_C d\vec{r}\, e^{-i\vec{K}\cdot\vec{r}} f(\vec{r}) \tag{81}$$
>
> "The integral $\int_C d\vec{r}$ is over one unit cell C and in the case of the cosmological lattice that should be understood as $[\Sigma^+ \; or \; \Sigma^-.]$ In equation (81), V is the volume of $[a]$ unit cell in the Bravais lattice of additive periodicity. $[W]$hen it is adapted to to the cosmological lattice of multiplicative periodicity we can expect a series of terms like V^{-N}. [*This is*] exactly like the analytical form of perturbative expansions in QED. It only remains to show how the volume could be $2\pi + \left(\Phi\pi\right)^3$ when volumes typically look like $\left(\Phi\pi\right)^3$ without the π and the other π added."

The non-Bravais condition of multiplicative periodicity is something like $f(\vec{r}\hat{\Phi}^{j+k}) = f(\vec{r}\hat{\Phi}^j)\hat{\Phi}^k$ but the details are not presently relevant. The gist is that since each lower level of $\aleph$ is contained inside the polar point of the higher one, they are related by multiplication with $\hat{\Phi}$ instead of addition with $\vec{R}$. We know that probabilities in perturbative quantum electrodynamics are defined with power series in $\alpha_{\rm QED}$ and, when we substitute $\alpha_{\rm MCM}$, we can write real-valued quantum mechanical probabilities as

$$P[\psi] := \sum_{j=1}^{\infty} \frac{c_j}{\left[2\pi + \left(\Phi\pi\right)^3\right]^j} \quad . \tag{82}$$

This will be like equation (81) when $V^{-1}\to\sum V^{-j}$. So, here we have a lot of evidence that the golden ratio fits in very nicely with physics, both in the quantum regime and the gravitational regime. Perturbative QED works because $\alpha_{\rm QED}$ is a very small number so it should also work with $\alpha_{\rm MCM}$ which is similarly small.

Where we have replaced (defined) imaginary zero with $\alpha\hat{\Phi}^{-\infty}$, we might use $\alpha^{-1}_{\rm MCM}$ for that α. It could be the volume of a hypothetical cell lower than the final cell all the way at the center of the golden spiral where the Fibonacci sequence has two ones, as in figure 12, which would be a good place to connect two spirals. Even when we invoke the principle that the observer cannot know his absolute level of $\aleph$, he does have to choose one box and call it $\hat{\Phi}^0$. That which we have described as the existence of two spirals might imply that whenever we choose a box for the $\hat{\pi}$-site as $\hat{\Phi}^0$, there is attached another box of the same size, possibly associated with the unitary $\hat{\imath}$-site. Since $\alpha_{\rm QED}$ is a non-classical constant of quantum theory, this strongly agrees with our intention to replace $\hat{\Phi}^{-1}$ with a quantum piece $\hat{\varphi}$ [13]. When hypercomplex analysis restricts to only three simultaneous level of $\aleph$ [2], that gives us reason to define imaginary zero as $\alpha\hat{\Phi}^{-2}$ instead of $\alpha\hat{\Phi}^{-\infty}$. Then $\alpha^{-1}_{\rm MCM}$ can be the volume of the next box on the second spiral which is associated with the quantum sector of negative powers of $\hat{\Phi}^j$.

The empirical fine structure constant $\alpha_{\rm QED}$ is most famously recognized in Schwinger's derivation of the correction to the electronic g-factor. Therefore, in future work, we should investigate how the correction is adjusted when $\alpha_{\rm MCM}$ is substituted for $\alpha_{\rm QED}$ in the derivation of the correction. Does it increase or decrease the contribution from the higher order corrections?

CONTINUITY AT INFINITY

To see why the level of $\aleph$ increases by two between adjacent instances of $\mathcal{H}$, consider $\sin(\infty)=0$ [1]. If the wave continued as $\hat{\Phi}^j\to\hat{\Phi}^{j+1}$ then, using $\infty\to 1$, the first point past infinity would be 1 which is not good for the wave's continuity because it truncates the domain $[0,1)$.

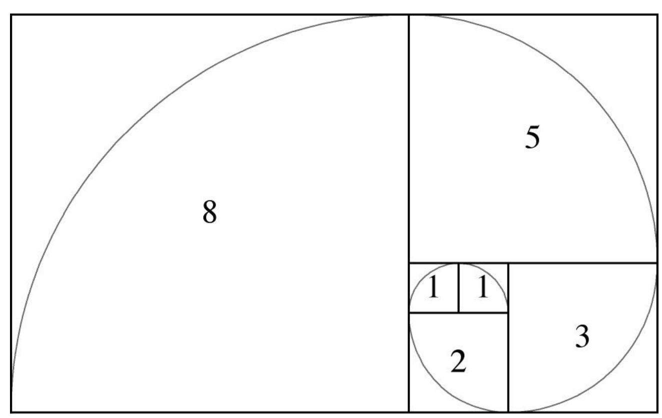

FIG. 12. This figure show a representation of the golden spiral wherein the ratio of the sides of the successive boxes are not equal to the golden ratio. In the asymptotic regime where each successive side is scaled by Φ, as in figure 11, the golden spiral is perfectly self-similar forever. When one attempts to construct the golden ratio from scratch, there is a final box where the spiral terminates.

However, if the sine wave continues beyond infinity as $\hat{\Phi}^j \to \hat{\Phi}^{j+2}$, with $\infty \to \varepsilon$, then the wave proceeds smoothly through infinity. Where does the wave go on the intermediate level of $\aleph$? We might say that it went to "the other spiral" or, more formally, we might invoke the vanishing width of $\chi^5_\varnothing$ across the unit cell. If it has vanishing width, then the path there is a like a loop, and the corrections to the electronic g-factor discussed above are referred to as loop corrections. Whatever the global phenomena are, the limits of sine and cosine at infinity [1], and their continuity there, will be a good foundation on which to lay a more advanced analysis of the transition into the region beyond infinity.

THE ORIGIN OF Φ IN THE MCM

By now, we have discussed the main applications of the golden ratio in the MCM and we need to show where it comes from. If we aim to do so by constructing a Fibonacci sequence, then we must note that the scheme in figure 13 (replicated in figure 8) is insufficient. Even if we added another 1 at the beginning and continued the pattern of figure 13, we would obtain successive radii $\{1, 1, 2, 4, 8, ...\}$. These are not the Fibonacci numbers and they will not generate the golden ratio. The problem is that this scheme neglects the origins of $\aleph$ and Ω. We need to include the growth of infinity in the $\aleph$ and Ω coordinates to generate the Fibonacci sequence. When $\mathcal{O}_1$ is in $\mathcal{H}_1$, the origin of Ω will be at the point labeled $y{=}\infty$ in figure 13, and $\varnothing$'s origin will be at $y{=}2\infty$. This is labeled $\mathcal{O}_2$ but it will be $\mathcal{O}_3$ if we include the origin of Ω between them. Figure 14 shows how these features are inherent to the system heretofore considered even when they were not labeled.

Now we will carefully, and tersely, build the Fibonacci sequence using the conventions in figure 15. This is the key principle for our construction: hypercomplexity spans three simultaneous levels of $\aleph$ [2], and we will work in the convention that the level increases on $\varnothing$ and $\mathcal{H}_2$. The rule for defining the increasing scale of infinity will be to include only one lower level of $\aleph$ within the infinite radius defined for a given level. Then, by symmetry, we can include a higher level, and then the conformal coordinate which goes beyond infinity can grab a third level.

Regarding figure 15, begin at $\mathcal{H}_1$. The radius of infinity there, in units of ∞, is self-evidently one. Therefore $\widetilde{y}^{\{1\}}_+{=}0$ is one tick mark higher than $\mathcal{O}_1$ where it defines the origin $\mathcal{O}_2$ of Ω. For Ω to grab one lower level of $\aleph$, $y^{\{2\}}_-$ has to reach back the $\mathcal{H}$. However, infinity also has to grow and it will not grow if it only reaches back one tick mark. Therefore, it must reach down to the instance of $\aleph$ below $\mathcal{H}_1$ (not pictured in figure 15.) The reader is invited to envision $\hat{\Phi}$ objects attached to the $\mathcal{H}$ and $\varnothing$ origins so that is clear what it means to "grab" a lower level of $\aleph$. When $y^{\{2\}}_-$ is so defined, the radius on Ω is two units of ∞: $\widetilde{y}^{\{2\}}_- = 0$ must be one tick mark below $\mathcal{H}_1$, as indicated in the lower left of the figure. $\widetilde{y}^{\{2\}}_+$ has to have the same scale as $\widetilde{y}^{\{2\}}_-$ so $\widetilde{y}^{\{2\}}_+{=}0$ defines the origin of $\varnothing$ two tick marks above Ω, and three higher than $\mathcal{H}$. This is somewhere between the $\widetilde{y}^{\{1\}}{=}2\infty$ and $\widetilde{y}^{\{1\}}{=}\infty^2$ values considered earlier for the location of $\varnothing$.

How big should infinity be relative to $\varnothing$? To grab one lower level of $\aleph$ it needs to reach back to $\mathcal{H}_1$; we must define $\widetilde{y}^{\{3\}}_- = 0$ at $\mathcal{H}_1$. Therefore, the radius around $\varnothing$ has three units of ∞. $\widetilde{y}^{\{3\}}_+$ must have the same scale as $\widetilde{y}^{\{3\}}_-$ so $\widetilde{y}^{\{3\}}_+{=}0$ defines the origin of $\aleph$ three tick marks higher than $\varnothing$, and five higher than Ω.

How big should infinity be relative to $\aleph$? If it only reaches back to $\varnothing$ then infinity won't grow. We must put $\widetilde{y}^{\{4\}}_- = 0$ at Ω. Then the radius around $\aleph$ has five units of infinity. $\widetilde{y}^{\{4\}}_+$ must have the same scale as $\widetilde{y}^{\{4\}}_-$

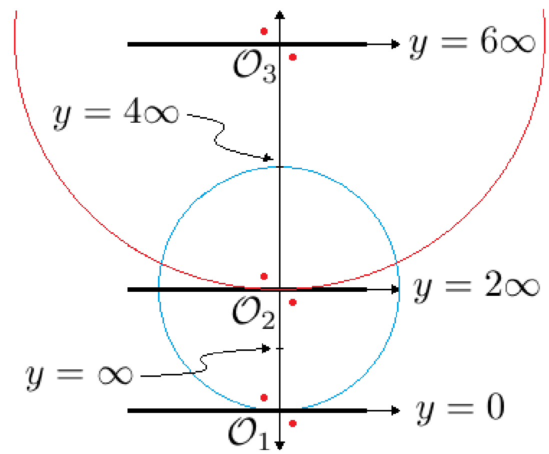

FIG. 13. If we continue the pattern of increasing infinity shown in figure 8 then we do not obtain Fibonacci numbers. The radius of the circle around $\mathcal{O}_1$ (circle not shown) is ∞. The radius around $\mathcal{O}_2$ is 2∞. Around $\mathcal{O}_3$ it is 4∞ and we clearly will not build the Fibonacci sequence.

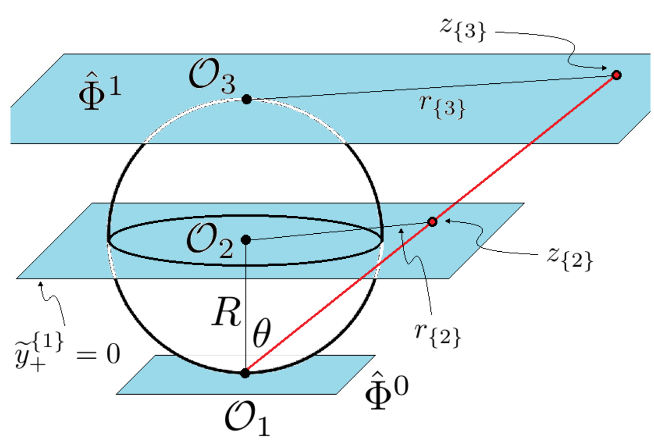

FIG. 14. This figure demonstrates that when the Riemann sphere shares an origin with one complex plane, it can define another instance of $\mathbb{C}$ at height R and a third at height $2R$ corresponding to the ∞ and 2∞ of figure 13.

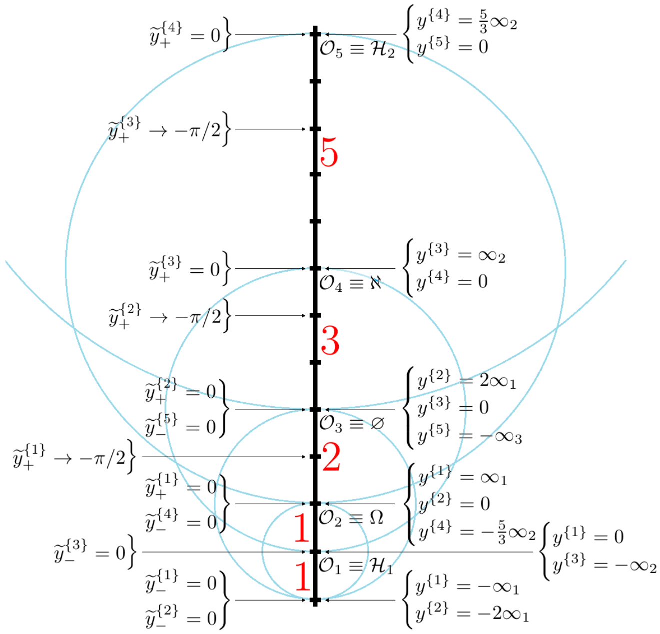

FIG. 15. This figure demonstrates how infinity grows according to the Fibonacci sequence. When we apply the principle that the observer has no way to detect his absolute level of $\aleph$, that will be an instruction to consider the asymptotic behavior when the ratio of one infinity to the next reaches its limit Φ. A first circle of radius one, not shown but corresponding to the first big red 1, should be associated with the $\hat{\varphi}$ piece [13]. The labels on ∞ do not agree with the labels on $\mathcal{O}$ because we have included $\aleph$ and Ω.

so $\widetilde{y}_+^{\{4\}}=0$ defines the origin of $\mathcal{H}_2$ five tick marks higher than $\aleph$. For infinity on $\mathcal{H}_2$ to grab one lower level of $\aleph$, we must put $\widetilde{y}_-^{\{5\}}=0$ at $\varnothing\equiv\mathcal{O}_3$. Then the radius there has eight units of infinity and it follows by inspection that infinity will continue to increase as the Fibonacci numbers. Therefore, when we impose the physical condition that the observer cannot know his absolute level $\aleph$, that will be an instruction to choose a $\hat{\Phi}^0$ in the region where the ratio of the next infinity to the current one is Φ.

The radius around $\mathcal{O}_5\equiv\mathcal{H}_2$ is ∞_3 so the level of $\aleph$ has correctly increased by two between $\mathcal{H}_1$ and $\mathcal{H}_2$. Note that the values $\{2,5/3,...\}$ on the right side of figure 15 will converge to Φ in the asymptotic limit of the sequence. In that case, we would see values like $\Phi\infty_j$ for the increasing scale infinity between the branes on which the level of $\aleph$ increases.

Figure 15 contains another Fibonacci sequence that we did not yet describe. The gap between $\widetilde{y}_+^{\{1\}}=-\pi/2$ and $\mathcal{O}_3$ is one unit of ∞. The gap between $\widetilde{y}_+^{\{2\}}=-\pi/2$ and $\mathcal{O}_4$ is one unit of ∞. The gap between $y_+^{\{3\}}=-\pi/2$ and $\mathcal{O}_5$ is two units of ∞. If we extended the figure to $\mathcal{O}_6$, we would see that the gap between $\widetilde{y}_+^{\{4\}}=-\pi/2$ and $\mathcal{O}_6$ is three units of ∞, and so on. Therefore, the growth of the unincluded $-\pi/2$ endpoint of $\widetilde{y}_\pm\in(-\pi/2,\pi,2)$ independently increases according to the golden ratio in the asymptotic region far from the absolute $\mathcal{O}_1$. This reflects the increasing size of the ε which separates $\widetilde{y}_\pm$ from $\mathrm{Im}(z)=0$ as the level of $\aleph$ increases. Note the good agreement between this growth of ε and

$$\hat{\Phi}^j\to\hat{\Phi}^{j+1}\quad:\quad\begin{cases}\infty\to1\\1\to\varepsilon\\\varepsilon\to\varepsilon^2\end{cases}\quad. \tag{83}$$

MORE GENERAL RELATIVITY

If the rate of increase of something is Φ then the derivative of something is Φ. Whatever it is, the derivative with respect to y is what will give Φ. Therefore, when looking for general relativity in the Cauchy–Riemann equations, we might reorient the complex plane as $z=\chi^5+ix^0$ (or it might be the property of our two spirals they their real and imaginary axes are swapped.) Then, using $\Psi=\psi+i\phi$ to get general relativity, we would use

$$\partial_y\,u=-\partial_x\,v\quad, \tag{84}$$

to obtain

$$\partial_y^3u=-\partial_y^2\partial_xv\qquad\Longrightarrow\qquad\omega^3\psi=-i\partial_y^2\partial_x\phi\quad. \tag{85}$$

Now two y derivatives contribute to the critical value as opposed to the two x derivatives which appeared in equations (65). We might consider say that the ordinary wavefunction ψ' of quantum mechanics is $\mathbb{C}$-valued and that Ψ is a $^\star\mathbb{C}$-valued wavefunction which describes a total state with ψ and ϕ in adjacent boxes of a spiral or one on each of the two expected spirals, or some such similar thing combining multiple levels of $\aleph$. If ϕ isn't a state in another box, but is rather the state of the box, then that would make it very easy to get the correct derivatives needed to confirm the hypothesis

$$\omega^3\psi=i\pi\Phi^2\phi\quad, \tag{86}$$

with some variant of the Cauchy–Riemann equations.

What does the other Cauchy–Riemann equation tell us? If

$$\partial_xu=\partial_yv\qquad\Longrightarrow\qquad\partial_x\psi=\partial_y\phi\quad, \tag{87}$$

then either ϕ doesn't depend on y or we need to add $x\equiv\chi^5$ to ψ. If ϕ doesn't depend on y, that breaks equation (85) so we need to add χ^5 to

$$\psi=e^{\mathbf{i}(\omega t-\vec{k}\cdot\vec{x})}\quad. \tag{88}$$

For instance, χ^5 is dimensionless so we could add it into the exponent and the use $\chi^5=0$ in $\mathcal{H}$ to get rid of it later. Whatever the answer is, there is still at least a fair amount to do to find Einstein's equation by analysis of first principles. It may even be the case that the Cauchy–Riemann equations have new subtleties in $^\star\mathbb{C}$ such that all of $\pi\Phi^2$ doesn't come from Ψ, but rather some or all of it comes from the adapted Cauchy–Riemann equations reliant on definitions of higher iterations of x and y in terms of their lower iterations. For instance, consider a relationship

$$\widetilde{y}^{\{2\}}(\widetilde{y}^{\{1\}})=\Phi\widetilde{y}^{\{1\}} \qquad \Longrightarrow \qquad \partial_y\widetilde{y}^{\{2\}}=\Phi \quad , \tag{89}$$

such that we would need to add integer labels to the partial derivative operators in the third derivative extension of the Cauchy–Riemann equations.

As a point of consistency, we showed in [27] that the energy is conserved when time in $\mathcal{H}_2$ is scaled by Φ^2 with respect to time in $\mathcal{H}_1$. Therefore, the convention $z=\chi^5+ix^0$, wherein time increases as y, is also the convention that conserves energy in [27]. Since Einstein's equation expresses conservation of energy, this will be a very important constraint! Furthermore, when time expands as y expands, that is the essentially the mechanism for MCM dark energy [7, 8, 11].

To get π out of the x derivative we might use the $\widetilde{y}=\tan^{-1}\big\{\tan\big[\tan^{-1}(x)\big]\big\}$ contraption to move sideways instead of straight up as we have done in figure 15. Earlier, we discussed how the inverse tangent can be used to generate $\theta\to\theta+\pi$ and we might add some more conformalism on top of that. Since $y_\pm\in[0,\infty)$ and $x\in[-\infty,\infty]$, and these are connected through $z=re^{i\theta}$, x should grow too but there is little reason require that is grows at the same rate as y in the transfinite region beyond $r=\infty$. Indeed, since we want the derivative of ϕ with respect to x to give π, and all of the domain structure of $\tan(x)$ is defined with π, there is a lot of reason to consider the case when infinity in the x-direction grows as π. The ratio of one Fibonacci number the previous one approaches Φ but the ratio of a diameter to a circumference is always π, so that is another clue about how to get $\partial_x\phi:=\pi\phi$. Indeed, the map from diameters to circumferences was the first concept in the MCM to capture the concept of infinite complexity [8, 11]. When that mechanism is synergized with the new material here, general relativity should shake out directly, more or less, hopefully. However it shakes out, gravity is probably like an accelerating rocket because it comes from the acceleration of $\mathcal{H}$ as x and y expand.

A STRING APPLICATION

We have shown how to get Φ out of $^\star\mathbb{C}$ without putting it in first. We have discussed at length the role of Φ the quantum and gravitational sectors of the MCM, and also how Φ is integral to the generation of the all-important, "God-given" coefficient of classical physics: $1/4\pi$. To finish this paper we will make an application to string theory which, as a 10D theory, we assume is connected to the MCM through $\dim(\Sigma^+\cup\Sigma^-)=10$. The physical boundary condition imposed by the MCM on $^\star\mathbb{C}$ is to put a topological singularity at $\chi^5_+=\Phi$. (To resolve the discrepancy about whether it is x^0 or χ^5 that grows by Φ, let the two cases pertain to our two spirals.) Use the singularity to define a string boundary condition such that the amplitude of the vibration on the string is constrained to vanish at $\chi^5_+=\Phi$. Furthermore, set a string boundary condition, one associated with $\psi(\infty)=0$, that the amplitude of the vibration vanishes at the conformal point $\pi/2\sim\infty$. When the amplitude is so constrained, no amplitude at $\pi/2$ or Φ along a string of length π (the conformal coordinate $\widetilde{y}_\pm\in(-\pi/2,\pi/2)$), we induce a quantized spectrum of vibrational modes on the string as required to do quantum theory with strings. Also note, when we model the black/white hole throughput with $\psi\to e^{i\pi}\psi=-\psi$, that is very nearly the operation needed to change the sign of $\widetilde{y}^{\{1\}}_+$ so that it may become $y^{\{2\}}_-$. The largest mode allowed by these boundary conditions will have wavelength

$$\lambda=2\times\big(\Phi-\pi/2\big)\approx 0.09 \quad . \tag{90}$$

which is small compared to the length $L=\pi$ of the string. We have

$$\frac{\lambda}{L}=\frac{2\big(\Phi-\pi/2\big)}{\pi}\approx 3\% \quad . \tag{91}$$

This value is interesting because 3% is also the discrepancy between Φ^2 and e, and between Φ and $\pi/2$. Regarding the former, when e^{ix} is an eigenfunction of ∂_x, we get the self-similarity like that afforded to doubly orthogonal levels of $\aleph$ whose relative phase Φ^2 is the magical ingredient to physics in the MCM.

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