Proof of the Second Landau's Problem

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Contents

1 Definition of the Second Landau's Problem

Definition: Is there infinitely sequence of "simple twins" - primes (y_o), the difference between them being 2?

2 Algorithm for Proof of the Second Landau's Problem

2.1 Types of Pairs of Odd Numbers

The sequence of odd numbers $\{y\}$ consists of pairs of odd numbers of the following sequence after the separation of the sequence $\{3y\}$:

$$\left\{ y_n, (y_n + 2) \mid \frac{y_n}{3} \notin \mathbb{N}, \ \frac{y_n + 2}{3} \notin \mathbb{N} \right\}. \tag{1}$$

The sequence of pairs (1) consists only of "simple twins" after the separation $\{3y\}$.
Then "simple twins" are beat out by intersecting sequences of composite odd numbers (y_{comp}) of the following form:

$$\left\{ y_{on} y \mid y \geq y_{on}, \frac{y}{3} \notin \mathbb{N} \right\}. \tag{2}$$

After the successive interaction of the sequences (2), the pairs (1) will have the following form:

Scheme 1:

A		or	B		or	C		or	D	
y_{o1}	y_{o2}		y_o	y_{comp}		y_{comp}	y_o		y_{comp1}	y_{comp2}

Thus, after the selection of the sequence $\{3y\}$ in the infinite sequence of pairs (1) the frequency of the appearance of the sets $\{A\} = 100\%, \{B; C; D\} = 0\%$.

2.2 Pairs of the Sequence of Odd Numbers of the form 5y

Let's start in succession from the sequence of composite numbers $\{5y \mid y \geq 5,\ y/3 \notin \mathbb{N}\}$. The distribution of this sequence in pairs (1) is repeated in rows of five pairs according to the following scheme:

Scheme 2:

1 pair	2 pair	3 pair	4 pair	5 pair
B	A	C	A	A

The frequency of the appearance of the the set $\{B; C; D\}$ in the sequence of pairs (1) will now be determined according to *Scheme 2*:

$$\{B; C; D\} = 100\% \cdot \frac{2}{5} = 40\%. \tag{3}$$

2.3 Pairs of the Sequence of Odd Numbers of the form 7y

The distribution of the next sequence of composite numbers $\{7y \mid y \geq 7,\ y/3 \notin \mathbb{N}\}$ in pairs (1) is repeated in rows of seven pairs according to the following scheme:

Scheme 3:

1 pair	2 pair	3 pair	4 pair	5 pair	6 pair	7 pair
B	A	A	A	A	C	A

2.4 Pairs of the Sequence of Odd Numbers of the form 11y

The distribution of the next sequence of composite numbers $\{11y \mid y \geq 11,\ y/3 \notin \mathbb{N}\}$ in pairs (1) is repeated in rows of eleven pairs according to the following scheme:

Scheme 4:

1 pair	from 2 to 4 pairs	5 pair	from 6 to 11 pairs
B	A	C	A

2.5 Pairs of the Sequence of Odd Numbers of the form 13y

The distribution of the next sequence of composite numbers $\{13y \mid y \geq 13,\ y/3 \notin \mathbb{N}\}$ in pairs (1) is repeated in rows of thirteen pairs according to the following scheme:

Scheme 5:

1 pair	from 2 to 9 pairs	10 pair	from 11 to 13 pairs
B	A	C	A

2.6 Regularities of Knocking out Simple Twins from Pairs

According to (2.2)-(2.5), a separate knocking out "of simple twins " from pairs (1) by the sequences (2) has the following regularities:

1. The repetition of the appearance of y_{comp} from (2) in pairs (1) by rows with the number of pairs equal to y_{on};
2. Filling in a repeating rows with only one pair B and one pair C from *Scheme 1*; (4)
3. B and C are not located in adjacent pairs;
4. B is always located in the first pair.

Starting with (2.2) the knocking out of "simple twins" from pairs (1) by the sequences (2) is superimposed on the pairs already filled with the previous sequence (2) The regularity changes in this case. Repetition can now be represented by areas where the number of consecutively filled columns from right to left is y_{on}, and the number of rows filled from top to bottom is $y_{o(n-1)}$, where $y_{o(n-1)}$ is a prime number in the sequence of primes just before y_{on}. Now there is a knocking out of "simple twins" from pairs (1) according to the type D of *Scheme 1*, which means intersecting sequences.
Let's calculate the frequency of the appearance of the set $\{B; C; D\}$ in subsequent pairs (1) for (3) and the new sequence $\{7y \mid y \geq 7,\ y/3 \notin \mathbb{N}\}$ with allowance for intersecting sequences:

$$\{B; C; D\} = 100\% \cdot \left(\frac{2}{5} + \frac{2}{7} - \frac{4}{35}\right) = 100\% \cdot \frac{4}{7} \approx 57{,}1429\%. \tag{5}$$

Let's calculate the frequency of the appearance of the set $\{B; C; D\}$ in subsequent pairs (1) for (5) and the new sequence $\{11y \mid y \geq 11,\ y/3 \notin \mathbb{N}\}$ with allowance for intersecting sequences:

$$\{B; C; D\} = 100\% \cdot \left(\frac{4}{7} + \frac{2}{11} - \frac{6}{77}\right) = 100\% \cdot \frac{52}{77} \approx 67{,}5325\%. \tag{6}$$

Let's calculate the frequency of the appearance of the set $\{B; C; D\}$ in subsequent pairs (1) for (6) and the new sequence $\{13y \mid y \geq 13,\ y/3 \notin \mathbb{N}\}$ with allowance for intersecting sequences:

$$\{B; C; D\} = 100\% \cdot \left(\frac{52}{77} + \frac{2}{13} - \frac{102}{1001}\right) = 100\% \cdot \frac{104}{143} \approx 72,7272\%. \tag{7}$$

2.7 Expression for the Set $\{B; C; D\}$

Let's represent the expressions (5), (6) and (7) in a different form.
Let's expression (5) as follows:

$$\{B; C; D\} = 100\% \cdot \left(\frac{2}{5} + \frac{2 - \frac{4}{5}}{7}\right). \tag{8}$$

Let's expression (6) as follows:

$$\{B; C; D\} = 100\% \cdot \left(\frac{4}{7} + \frac{2 - \frac{6}{7}}{11}\right). \tag{9}$$

Let's expression (7) as follows:

$$\{B; C; D\} = 100\% \cdot \left(\frac{\frac{52}{7}}{11} + \frac{2 - \frac{102}{77}}{13}\right). \tag{10}$$

According (8), (9) and (10), the expression for the frequency of the appearance of the set $\{B; C; D\}$ in the sequence of subsequent pairs (1) under the action of (2) can be represented as follows:

$$\{B; C; D\} = 100\% \cdot \left(\frac{K_{y_{on}}}{y_{o(n-1)}} + \frac{2 - R_{y_{on}}}{y_{on}}\right), \tag{11}$$

where, without delving into the formula, we can distinguish the following conditions:

$$\begin{aligned}
&1.\quad K_{y_{on}} \notin \mathbb{N};\\
&2.\quad y_{o(n-1)} - K_{y_{on}} > 2, \quad (y_{o(n-2)} - K_{y_{o(n-1)}}) \leq (y_{o(n-1)} - K_{y_{on}});\\
&3.\quad \frac{K_{y_{o(n-1)}}}{y_{o(n-2)}} < \frac{K_{y_{on}}}{y_{o(n-1)}};\\
&4.\quad R_{y_{on}} \notin \mathbb{N};\\
&5.\quad 0 < R_{y_{on}} < 2, \quad \frac{2 - R_{y_{o(n-1)}}}{y_{o(n-1)}} < \frac{2 - R_{y_{on}}}{y_{on}};\\
&6.\quad \frac{K_{y_{o(n-1)}}}{y_{o(n-2)}} + \frac{2 - R_{y_{o(n-1)}}}{y_{o(n-1)}} < \frac{K_{y_{on}}}{y_{o(n-1)}} + \frac{2 - R_{y_{on}}}{y_{on}}.
\end{aligned} \tag{12}$$

2.8 New Definition of the Second Landau's Problem

From expression (11) follows a new definition of the **Second Landau's Problem**:
New definition: Is it possible that with the successive filling of pairs (1) with the next sequence (2), the frequency of the appearance of the set $\{B; C; D\}$ in expression (11) reaches 100%?

2.9 Proof of the Second Landau's Problem by contradiction

Let's assume that when the pairs (1) are successively filled with some sequence (2) in expression (11), the frequency of appearance of the set $\{B; C; D\} = 100\%$. Then:

$$1 - \frac{K_{y_{on}}}{y_{o(n-1)}} = \frac{2 - R_{y_{on}}}{y_{on}}. \tag{13}$$

But in the sequence of primes:

$$y_{o(n-1)} < y_{on}. \tag{14}$$

According to condition 5 in (12):

$$\frac{2 - R_{y_{on}}}{y_{on}} < \frac{2}{y_{on}}. \tag{15}$$

According to condition 2 in (12):

$$1 - \frac{K_{y_{on}}}{y_{o(n-1)}} > \frac{2}{y_{o(n-1)}}. \tag{16}$$

Because of (14), (15) and (16), expression (13) becomes invalid and takes the following form:

$$1 - \frac{K_{y_{on}}}{y_{o(n-1)}} > \frac{2 - R_{y_{on}}}{y_{on}}. \tag{17}$$

According to (17), when pairs (1) are filled with the next sequence (2), always:

$$\{B; C; D\} < 100\%. \tag{18}$$

Consequently, the sequence of "simple twins" is infinite.

Publications: http://samlib.ru/editors/b/bezymjannyj_a/w2.shtml