NP?=EXP

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Abstract

We only point out that the work of algorithmic algebra community is not enough, at least so far.

I. Proposition

In [SM13], authors presents an algorithm named ANewDsc claiming in *Corollary 27* of efficient computation time. The n in that corollary is the given polynomial's degree. Thus, if the polynomial is given in the form of $567 * x * x * x * x * x + 872 * x * x * x + 12$, then indeed, we have a polynomial time algorithm for the problem at hand. But what if, the polynomial is given like $x^{123456789} + 2 * x^7 - 5$. To the best of our knowledge, we have not devised an algorithm with reasonable time bound.

Definition

RealRootCount = { (P, k) | polynomial P given in the latter (described above) way has exactly k distinct real roots }

Proposition

$$\text{RealRootCount} \leq_p \text{ExistentialRealThy}$$

where ExistentialRealThy is the definitive problem of existential first-order theory. Details can be found in [WikiExistRealThy]

We can use the expressive capability of existential first-order theory to build things up like $y = x * x$, $z = y * y$, $u = z * x + y + y + y + x + x + 1 + 1$, etc. Note that in the RealRootCount we have the exponentiation symbol, but in the theory considered here, we only have 0, 1, +, * and existential quantifier. For large coefficient, we utilize the same idea.

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i. Conclusion

We have to ask ourselves whether the question in the title is significant.

References

[SM13] , Michael Sagraloff, Kurt Mehlhorn, Computing Real Roots of Real Polynomials, 2013

[WikiExistRealThy] , The free encyclopedia, `https://en.wikipedia.org/wiki/Existential_theory_of_the_reals`