Nonlinear Resonances Analysis of a RLC Series Circuit Modeled by a Modified Van der Pol Oscillator

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Abstract

In this paper, the nonlinear resonances analysis of a RLC series circuit modeled by a modified Van der Pol oscillator is investigated. After establishing of a new general class of nonlinear ordinary differential equation, a forced Van der Pol oscillator subjected to an inertial nonlinearity is derived. From this equation the multiple scales method is used to find the various resonant states. As analytical results primary resonance, sub-harmonic resonance of order 1/3 and super-harmonic resonance of order 3 are obtained. The steady-state solutions and theirs stabilities are determined. Numerical simulations display bistability, hysteresis, jump and bifurcation phenomena. The effects of different parameters on the system behavior are investigated and results are presented graphically and discussed.

Keywords: Nonlinear RLC circuit, Modified Van der Pol oscillator, Nonlinear resonances, Bistability, Hysteresis, jump and bifurcation.

1. Introduction

It is well known that the resonant electrical circuit composing most of the electric or electronic devices display a rich variety of nonlinear dynamical behavior [1-4]. Therefore, such electrical circuit is governed by nonlinear ordinary differential equations. The inherent nonlinearity source in resonant electrical systems arises from resistive, inductive and capacitive elements [4]. Given the importance of RLC circuit systems in many areas of physics and modern engineering applications, the understanding of nonlinear electrical properties becomes a necessity from theoretical and practical point of view. Therefore this requires of developing a mathematical model taking into account the nonlinear character of the three fundamental elements of the electrical circuits and proceeding to its numerical simulation before all optimization

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process and design [5-6]. In this perspective, mathematical modeling of nonlinear electrical systems was become a challenge for researchers in these three last decades. For instance, Kaufman and Roberts [7] have derived from RLC series circuit consisting of a nonlinear resistor and a nonlinear capacitor a class of nonlinear differential equations containing the Riccati's equation and Abel's equation of the first kind as a special case. Recently various nonlinear mathematical models of different complexities have been proposed in open literature for analyzing the nonlinear oscillations generated by RLC series circuit [8-13]. But it has been remarked that no simplest nonlinear oscillatory circuit taking simultaneously into consideration a nonlinear resistor and a nonlinear inductor has been developed for generating a class of nonlinear differential equations. Therefore this problematic is taken into account in this work for investigating a new class of nonlinear differential equations governing the dynamic of the nonlinear RLC series circuits and analyzing the nonlinear oscillations produced by such electrical circuit system.

The main objectives of this work consist firstly to derive a new general class of nonlinear differential equations governing the nonlinear dynamical behavior of a RLC series circuit with nonlinear resistor and nonlinear inductance and secondly to analyze the nonlinear oscillations that can arise in such electrical circuit through a nonlinear oscillator belonging to this general class.

In order to attain the fixed objectives in this paper, we generate at first sight, a class of nonlinear differential equations (Section 2) and we investigate the various resonant states of a nonlinear RLC series circuit by means of the multiple scales method (Section 3). We present afterward the numerical results and discussions (Section 4). Finally the conclusion of this research work will be drawn (Section 5).

2. Mathematical modeling

2.1. Formulation of problem

We consider an electrical circuit composed of a nonlinear resistor $R(i)$, a nonlinear inductance $L(i)$ and a linear capacitance C connected in series and driven by voltage source $E(t)$ as shown in Fig 1. From this figure, it concerns to build a new general class of nonlinear equations describing the dynamical behaviors of the nonlinear resonant circuits using Kirchoff's voltage and current laws.

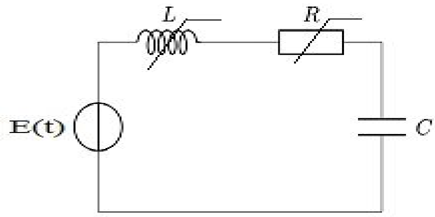


Fig.1. Nonlinear RLC series Circuit

2.2. New general class of mixed Liénard type equations

Applying Kirchoff's voltage and current laws at Fig.1, one obtain the following equation:

$$\frac{d\psi(i)}{dt} + U_R(i) + \frac{1}{C}q = E(t) \tag{1}$$

where $U_L = \frac{d\psi(i)}{dt} = L(i)i$, $U_R(i) = R(i)i$ and $U_C = \frac{q}{C}$ represent the voltage drop across the inductor, resistor and capacitor respectively. $E(t)$ denotes the applied voltage, q means the charge at the capacitance, $\psi(i)$ and $i(t)$ designate the flux and current respectively. The differentiation with respect to time of the equation (1) yields after some algebraic manipulations to the following equation:

$$\psi'(i)\frac{d^2 i}{dt^2} + \psi''(i)\left(\frac{di}{dt}\right)^2 + U_R'(i)\frac{di}{dt} + \frac{1}{C}i = \frac{d\,E(t)}{dt} \tag{2}$$

with $i = \frac{dq}{dt}$

The so obtained equation (2) with nonlinear functions describes the dynamic behavior of the nonlinear RLC series circuits. This equation is known as non-autonomous mixed Liénard-type equations [14] which will have many applications in modern electrical engineering, since it is possible to achieve an electronic circuit from a nonlinear differential equation [8].

Before all analysis of a nonlinear RLC series circuit under consideration, it is needed to make clear the nonlinear functions $\psi(i)$, $U_R(i)$ and the applied voltage $E(t)$ expressions. At this stage, it is very important to point out that various expressions of $\psi(i)$ and $U_R(i)$ are used in open literature for analyzing the dynamical response of nonlinear electrical circuits [15-17]. For this, we choose the expressions of these functions (section 3) in order to investigate the dynamic responses of the considered nonlinear resonant circuit.

3. Nonlinear resonances analysis

3.1. Mathematical problem

In this subsection one purpose to investigate the dynamic responses of a nonlinear RLC series circuit subjected to harmonic voltage source of the form: $E(t) = E_0 \sin(\omega t)$. For this purpose, we choose the following expressions for $\psi(i)$ and $U_R(i)$:

$$U_R(i) = R_1 i + R_3 i^3 \tag{3}$$

and

$$\psi(i) = L_1 i + L_3 i^3 \tag{4}$$

where R_1, R_3, L_1 and L_3 are constant parameters.

Substituting Eqs.(3) and (4) into Eq.(2), we obtain after few mathematical operations the following equation

$$\left(L_1 + 3L_3 i^2\right)\frac{d^2 i}{dt^2} + 6L_3 i\left(\frac{di}{dt}\right)^2 + (R_1 + 3R_3 i^2)\frac{di}{dt} + \frac{1}{C}i = E_0\omega\cos(\omega t) \tag{5}$$

Now, using $i = x\, i_0$ and $\tau = \frac{1}{\sqrt{L_1 C}}t$, where i_0, τ and x denote the normalization current and the dimensionless variables respectively, Eq.(5) becomes

$$\left(1 + \alpha x^2\right)x'' + 2\alpha\, x x'^2 + \left(Q^{-1} + \beta x^2\right)x' + x = F_0 \cos(\Omega\tau) \tag{6}$$

with $\alpha = \frac{3L_3}{L_1}i_0^2, Q^{-1} = R_1\sqrt{\frac{C}{L_1}}, \beta = \frac{3R_3}{R_1 Q}i_0^2, F_0 = \frac{E_0\omega L_1 C^2}{i}$ and $\Omega = \omega\sqrt{L_1 C}$.

It is very important to show that when the dimensionless parameter $\alpha = 0$, Eq. (6) becomes $x'' + \left(Q^{-1} + \beta x^2\right)x' + x = F_0\cos(\Omega\tau)$, which represents the famous Van der Pol oscillator intensively studied in the open literature in context of various problems. Therefore Eq.(6) represents the generalized Van der Pol oscillator.

Now, in view of importance of the mixed Liénard-type equations in electronic areas and other branches of sciences, the problem here is to investigate the various resonant states of the equation (6) by applying the multiple scales method.

3.2. Primary resonance state

In this case of oscillation, the detuning parameter σ and the excitation frequency Ω are related according to $\Omega = 1 + \varepsilon\sigma$. In order to apply the multiple scales method, it is necessary to introduce into Eq.(6) the small perturbation parameter $0 \prec \varepsilon \prec 1$. That making, Eq.(6) becomes

$$x'' + x = \varepsilon\left(-\alpha x^2 x'' - 2\alpha\, x x'^2 - Q^{-1}x' - \beta x^2 x' + F_0 \cos(\Omega\tau)\right) \tag{7}$$

with $\alpha = \varepsilon\alpha$, $Q^{-1} = \varepsilon Q^{-1}$, $\beta = \varepsilon\beta$ and $F_0 = \varepsilon F_0$

Eq.(7) is known as weakly nonlinear equation. From this equation, we define the fast time scale $T_0 = \tau$, which associates with changes occurring at the frequencies 1 and Ω and the slow time scales $T_1 = \varepsilon\tau$ which associates with modulations in the amplitude and phase caused by nonlinearity. In terms of the new time scales, the first and second time derivative become

$$\begin{aligned} \frac{d}{d\tau} &= D_0 + \varepsilon D_1 \\ \frac{d}{d\tau} &= D_0 + 2\varepsilon D_0 D_1 \end{aligned} \tag{8}$$

where $D_n = \dfrac{\partial}{\partial T_n}$

Now, we begin to assume that the approximate solution of Eq.(7) can be written in the following form:

$$x(\tau,\varepsilon) = x_0(T_0,T_1) + \varepsilon x_1(T_0,T_1) \tag{9}$$

Substituting Eqs.(8) and (9) into Eq.(7) and equating coefficients of like powers of ε, we get

$$D_0^2 x_0 + x_0 = 0 \tag{10}$$

$$D_0^2 x_1 + x_1 = -2D_0 D_1 x_0 + \alpha x_0^3 - 2\alpha x_0 (D_0 x_0)^2 - Q^{-1} D_0 x_0 - \beta x_0^2 D_0 x_0 + F_0 \cos(\Omega T_0) \tag{11}$$

The solution of Eq.(10) can be expressed in the complex form:

$$x_0(T_0,T_1) = A(T_1)e^{iT_0} + \overline{A}(T_1)e^{-iT_0} \tag{12}$$

where $\overline{A}(T_1)$is the complex conjugate of $A(T_1)$, and $A(T_1)$ is the complex amplitude function which can be determined by eliminating the secular terms at the higher levels of approximation equation.

Inserting Eq.(12) into Eq.(11), yields to

$$D_0^2 x_1 + x_1 = \left(-2iD_1 A + \alpha A^2 \overline{A} - iQ^{-1} A - i\beta A^2 \overline{A} + \frac{F_0}{2} e^{i\sigma T_1} \right) e^{iT_0} + c.c + NST \tag{13}$$

where NST denotes the terms does not produce secular terms and c.c designates the complex conjugate terms.

Now eliminating the secular terms from Eq.(13) and introducing the amplitude $A(T_1)$by the following polar form:

$$A(T_1) = \frac{1}{2} a(T_1) e^{i\theta(T_1)} \tag{14}$$

we obtain after separating real and imaginary parts the following modulation equations:

$$D_1 a = -\frac{1}{2} Q^{-1} a - \frac{1}{8} \beta a^3 + \frac{1}{2} F_0 \sin\phi \tag{15}$$

$$D_1 \phi = \sigma + \frac{1}{8} \alpha a^2 + \frac{1}{2a} F_0 \cos\phi \tag{16}$$

where $\phi = \sigma T_1 - \theta$

To determine the steady-state solution, we put $D_1 a = D_1 \phi = 0$into Eqs.(15) and (16). Thus we obtain

$$\frac{1}{2} F_0 \sin\phi = \frac{1}{2} Q^{-1} a + \frac{1}{8} \beta a^3 \tag{17}$$

$$\frac{1}{2} F_0 \cos\phi = -a\sigma - \frac{1}{8} \alpha a^3 \tag{18}$$

Eqs.(17) and (18) show that there no trivial solution at a=0. For non-trivial solution (a≠ 0), eliminating $\phi$from these equations yields to

$$\sigma = -\frac{\alpha}{8} a^2 \pm \frac{1}{2} \sqrt{\left(\frac{F_0}{a} \right)^2 - \left(Q^{-1} + \frac{\beta}{4} a^2 \right)^2} \tag{19}$$

Eq.(19) represents the frequency-response equation for primary resonance.

In order to analyze the stability of the non-trivial fixed points of modulation equations (15) and (16), we let:

$$a = a_0 + a_1, \ \ \phi = \phi_0 + \phi_1 \tag{20}$$

where a_0 and ϕ_0 represent the non-trivial solutions and a_1 and ϕ_1 are assumed to be infinitesimal. Substituting Eqs.(20) into Eqs.(15) and (16) and keeping only linear terms in the perturbation quantities, we get the following variational equations describing the stability of the steady-state solution:

$$a_1^{'} = -\left(\frac{Q^{-1}}{2} + \frac{3\beta}{8}a_0^2\right)a_1 + \frac{F_0}{2}\cos(\phi_0)\phi_1 \tag{21}$$

$$\phi_1^{'} = -\left(\frac{\alpha}{4}a_0 - \frac{F_0}{2a_0^2}\cos(\phi_0)\right)a_1 - \frac{F_0}{2a_0}\sin(\phi_0)\phi_1 \tag{22}$$

Eqs.(21)and (22) admit solutions of the form $(a_1, \phi_1) = (c_1, c_2)e^{\lambda T_1}$ provided that:

$$\lambda = \frac{p_1}{2} \pm \frac{1}{2}\sqrt{p_1^2 + 4p_2} \tag{23}$$

with

$$p_1 = -\left(\frac{Q^{-1}}{2} + \frac{3\beta}{8}a_0^2 + \frac{F_0}{2a_0}\sin\phi_0\right) \text{ and } p_2 = \frac{F_0}{2}\left(\frac{\alpha}{4}a_0 - \frac{F_0}{2a_0^2}\cos\phi_0\right)\cos\phi_0 - \frac{F_0}{2a_0}\left(\frac{Q^{-1}}{2} + \frac{3\beta}{8}a_0^2\right)\sin\phi_0$$

Therefore the steady state solution is unstable when p_2 is greater than zero.

3.3. Super-harmonic and sub-harmonic resonances

In this case, we consider a hard resonant excitation, that is to say F_0=0(1). In this situation, Eqs.(10) and (11) are modified to

$$D_0^2 x_0 + x_0 = F_0 \cos(\Omega T_0) \tag{24}$$

$$D_0^2 x_1 + x_1 = -2D_0 D_1 x_0 + \alpha x_0^3 - 2\alpha x_0 (D_0 x_0)^2 - Q^{-1} D_0 x_0 - \beta x_0^2 D_0 x_0 \tag{25}$$

The solution of Eq.(24) is

$$x_0(T_0, T_1) = A(T_1)e^{iT_0} + \Lambda e^{i\Omega T_0} + cc \tag{26}$$

where $\Lambda = \dfrac{F_0}{2(1-\Omega^2)}$ with $\Omega \neq 1$. Substituting Eq.(26) into Eq.(25) we get:

$$
\begin{aligned}
D_0^2 x_1 + x_1 = &\left[-2iD_1 A - 4\alpha\Lambda^2\left(\Omega^2 - \Omega\right)A + \alpha A\left(A\overline{A} + 2\Lambda^2\right) - iQ^{-1}A - 2\beta A\left(A\overline{A} + \Lambda^2\right) + i\beta A^2\overline{A}\right]e^{iT_0} \\
&+\left[-\alpha\Lambda^3\left(2\Omega^2 - 3\Omega\right) + 2\alpha\Lambda\Omega A\overline{A} - iQ^{-1}\Lambda\Omega - i\beta\Lambda\Omega\left(\Lambda^2 + 2A\overline{A}\right)\right]e^{i\Omega T_0} \\
&+[\alpha\Lambda^3\left(2\Omega^2 + \Omega\right) - i\beta\Lambda^3\Omega]e^{3i\Omega T_0} \\
&+[\alpha\Lambda\left(4 - 3\Omega\right) - i\beta\Lambda\left(\Omega - 2\right)]\overline{A}^2 e^{i(\Omega-2)T_0} \\
&+[2\alpha\Lambda^2\left(\Omega^2 - \Omega\right) + \alpha\Lambda^2 - i\beta\Lambda^2\left(2\Omega - 1\right)]\overline{A}e^{i(2\Omega-1)T_0} + cc + NST
\end{aligned} \tag{27}
$$

Eq.(27) shows two possibles cases of resonances such as $3\Omega = 1$ and $\Omega = 3$. We treat in following section these cases of resonances.

3.3.1. Super-harmonic resonance ($3\Omega = 1$)

In this case of oscillation, the detuning parameter σ and the dimensionless excitation frequency can be written according to the following relationship

$$3\Omega = 1 + \varepsilon\sigma \tag{28}$$

Inserting Eq.(28) into Eq.(27) we obtain

$$D_0^2 x_1 + x_1 = W(T_1)e^{iT_0} + cc + NST \tag{29}$$

where

$$W(T_1) = -2iD_1 A + \alpha A\left(A\overline{A} + \frac{26}{9}\Lambda^2\right) - iQ^{-1}A - 2\beta A\left(\Lambda^2 + A\overline{A}\right) + i\beta A^2\overline{A} + \left(\frac{5\alpha\Lambda^3}{9} - i\frac{\beta\Lambda^3}{3}\right)e^{i\sigma T_1}$$

Eliminating the secular terms from Eq.(29) and introducing the expression of $A(T_1)$ given by Eq.(14) into $W(T_1) = 0$, we obtain after some mathematical operations the following modulation equations:

$$D_1 a = -\frac{Q^{-1}}{2}a + \frac{\beta}{8}a^3 - \frac{\beta\Lambda^3}{3}\cos\phi + \frac{5\alpha\Lambda^3}{9}\sin\phi \tag{30}$$

$$D_1\phi = \sigma + \frac{\alpha}{72}\left(9a^2 + 104\Lambda^2\right) - \frac{\beta}{4}\left(4\Lambda^2 + a^2\right) + \frac{5\alpha\Lambda^3}{9a}\cos\phi + \frac{\beta\Lambda^3}{3a}\sin\phi \tag{31}$$

where $\phi = \sigma T_1 - \theta$ $\Lambda = \dfrac{9F_0}{16}$. To determine the steady-state solution, we let $D_1 a = D_1\phi = 0$. Thus we get

$$\frac{5\alpha\Lambda^3}{9}\sin\phi-\frac{\beta\Lambda^3}{3}\cos\phi=\frac{Q^{-1}}{2}a-\frac{\beta}{8}a^3 \tag{32}$$

$$\frac{\beta\Lambda^3}{3}\sin\phi+\frac{5\alpha\Lambda^3}{9}\cos\phi=-a\sigma-\frac{\alpha}{72}\left(9a^3+104\Lambda^2 a\right)+\frac{\beta}{4}\left(4\Lambda^2 a+a^3\right) \tag{33}$$

Eqs.(32) and Eq.(33) show that there are no trivial solution at a=0. Eliminating ϕ from these two equations, the solution of super-harmonic resonance of order 3 is given by the following equation:

$$\sigma=\frac{18\beta-\alpha}{72}\left(4\Lambda^2+a^2\right)-\frac{\alpha}{18}\left(25\Lambda^2+2a^2\right)\pm\sqrt{\left(\frac{5\alpha\Lambda^3}{9a}\right)^2+\left(\frac{\beta\Lambda^3}{3a}\right)^2-\left(\frac{Q^{-1}}{2}-\frac{\beta}{8}a^2\right)^2} \tag{34}$$

To analyze the stability of the non-trivial fixed points of modulation equations (30) and (31) we substitute Eqs.(20) into (30) and (31) and keeping only linear terms in the perturbation quantities, we get:

$$a_1^{'}=J_1a_1+J_2\phi_1 \tag{35}$$

$$\phi_1^{'}=J_3a_1+J_4\phi_1 \tag{36}$$

with $J_1=\frac{Q^{-1}}{2}-\frac{\beta}{8}a_0^2$, $J_2=\frac{\beta\Lambda^3}{3}\sin\phi_0+\frac{5\alpha\Lambda^3}{9}\cos\phi_0$, $J_3=\frac{(\alpha-2\beta)a_0}{4}-\frac{5\alpha\Lambda^3}{9a_0^2}\cos\phi_0-\frac{\beta\Lambda^3}{3a_0^2}\sin\phi_0$

and $J_4=\frac{\beta\Lambda^3}{3a_0}\cos\phi_0-\frac{5\alpha\Lambda^3}{9a_0}\sin\phi_0$

Eqs.(35) and (36) admit solution of the form $(a_1,\phi_1)\propto(\gamma_1,\gamma_2)e^{\lambda T_1}$ where γ_1 and γ_2 are constants, provided that

$$\lambda=\frac{1}{2}(J_1+J_4)\pm\frac{1}{2}\sqrt{(J_1+J_4)^2+4(J_2J_3-J_1J_4)} \tag{37}$$

Consequently, the steady state solution is unstable when $J_2J_3-J_1J_4>0$.

3.3.2. Sub-harmonic resonance of order $\frac{1}{3}$

To treat this case of nonlinear resonance, we introduce the detuning parameter σ according to

$$\Omega=3+\varepsilon\sigma \tag{38}$$

into Eq.(27), we get

$$D_0^2 x_1 + x_1 = Z(T_1)e^{iT_0} + cc + NST \tag{39}$$

where

$$Z(T_1) = -2iD_1 A + \alpha A\left(A\overline{A} + \frac{26}{9}\Lambda^2\right) - iQ^{-1}A - 2\beta A\left(\Lambda^2 + A\overline{A}\right) + i\beta A^2\overline{A} - \Lambda\overline{A}^2\left(5\alpha + i\beta\right)e^{i\sigma T_1}$$

Replacing the expression of amplitude $A(T_1)$ given by Eq.(14) into $Z(T_1)$ and eliminating the secular terms from Eq.(39), we obtain after separating real and imaginary parts, the following modulation equations:

$$D_1 a = -\frac{Q^{-1}a}{2} + \frac{\beta a^3}{8} - \frac{\beta\Lambda a^2}{4}\cos\phi - \frac{5\alpha\Lambda a^2}{4}\sin\phi \tag{40}$$

$$D_1\phi = \sigma + \frac{3\alpha}{8}\left(a^2 - 88\Lambda^2\right) - \frac{3\beta}{4}\left(4\Lambda^2 + a^2\right) - \frac{15a\alpha\Lambda}{4}\cos\phi + \frac{3a\beta\Lambda}{4}\sin\phi \tag{41}$$

where $\phi = \sigma T_1 - 3\theta$ $\Lambda = -\frac{F_0}{16}$. In order to determine the steady-state solution, it is enough to set $D_1 a = D_1\phi = 0$. Thus, we obtain

$$\frac{\beta\Lambda a}{4}\cos\phi + \frac{5\alpha\Lambda a}{4}\sin\phi = -\frac{Q^{-1}}{2} + \frac{\beta a^2}{8} \tag{42}$$

$$\frac{5a\alpha\Lambda}{4}\cos\phi - \frac{a\beta\Lambda}{4}\sin\phi = \frac{1}{3}\sigma + \frac{\alpha}{8}\left(a^2 - 88\Lambda^2\right) - \frac{\beta}{4}\left(4\Lambda^2 + a^2\right) \tag{43}$$

From Eqs.(42) and (43) we observe that there are no trivial solution at a=0. Then eliminating the oscillation phase from these two equations we get after some algebraic manipulations the following frequency response equation for sub-harmonic resonance of order 1/3:

$$\sigma = \frac{6\beta - 3\alpha}{8}\left(4\Lambda^2 + a^2\right) - \frac{69\alpha\Lambda^2}{2} \pm 3\sqrt{\left(\frac{5a\alpha\Lambda}{4}\right)^2 + \left(\frac{\beta\Lambda a}{4}\right)^2 - \left(\frac{\beta}{8}a^2 - \frac{Q^{-1}}{2}\right)^2} \tag{44}$$

In order to analysis the stability of the non-trivial solution, we follow the steps used in the preceding subsection. Thus, we obtain the following variational equations describing the stability of the steady-state solutions:

$$a_1' = M_1 a_1 + M_2\phi_1 \tag{45}$$

$$\phi_1' = M_3 a_1 + M_4\phi_1 \tag{46}$$

with $M_1 = -\frac{Q^{-1}}{2} + \frac{3\beta}{8}a_0^2 - \frac{5a_0\alpha\Lambda}{2}\sin\phi_0 - \frac{a_0\beta\Lambda}{2}\cos\phi_0$, $M_2 = \frac{a_0^2\beta\Lambda}{4}\sin\phi_0 - \frac{5a_0^2\alpha\Lambda}{4}\cos\phi_0$,
$M_3 = \frac{3(\alpha-2\beta)a_0}{4} - \frac{15\alpha\Lambda}{4}\cos\phi_0 + \frac{3\beta\Lambda}{4}\sin\phi_0$ and $M_4 = \frac{3a_0\beta\Lambda}{4}\cos\phi_0 + \frac{15a_0\alpha\Lambda}{4}\sin\phi_0$

Eqs.(45) and (46) admit solutions of the form $(a_1, \phi_1) \propto (\Gamma_1, \Gamma_2)e^{\lambda T_1}$ where Γ_1 and Γ_2 are constants, provided that

$$\lambda = \frac{1}{2}(M_1 + M_4) \pm \frac{1}{2}\sqrt{(M_1 + M_4)^2 + 4(M_2M_3 - M_1M_4)}$$
(37)

Therefore, the steady state solution is unstable if and only if $M_2M_3 - M_1M_4 > 0$.

4. Numerical Results and discussions

This section presents numerical solutions of the frequency-response equations. Frequency response equations (19), (34) and (44) are nonlinear algebraic equations in the amplitude (a). The steady-state equations and stability conditions are solved numerically. Solid/dotted lines designate the stable/unstable solution branches respectively. The numerical solutions are plotted in a group of Figs.2-10, which represent the variation of the amplitude (a) versus the detuning parameter (σ). Figs.2-3 present the frequency-response curves of the primary resonance.

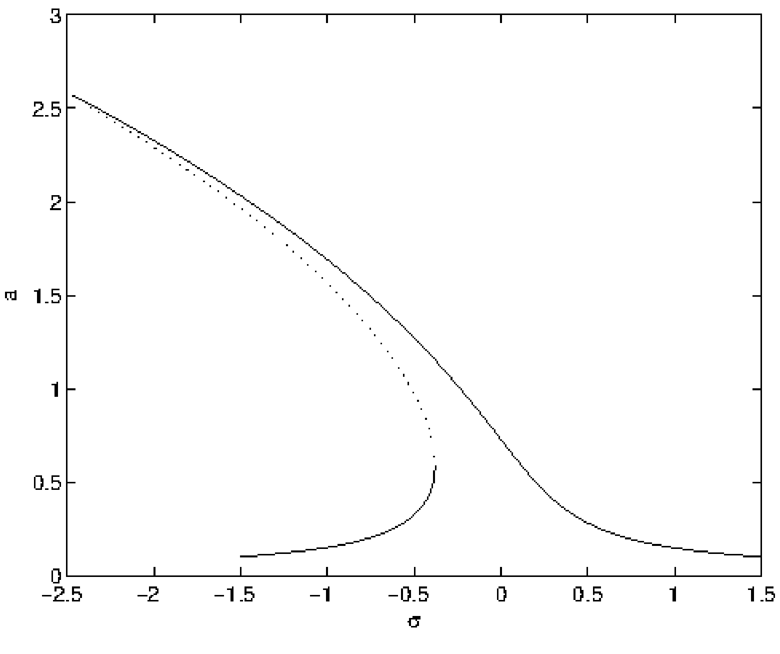
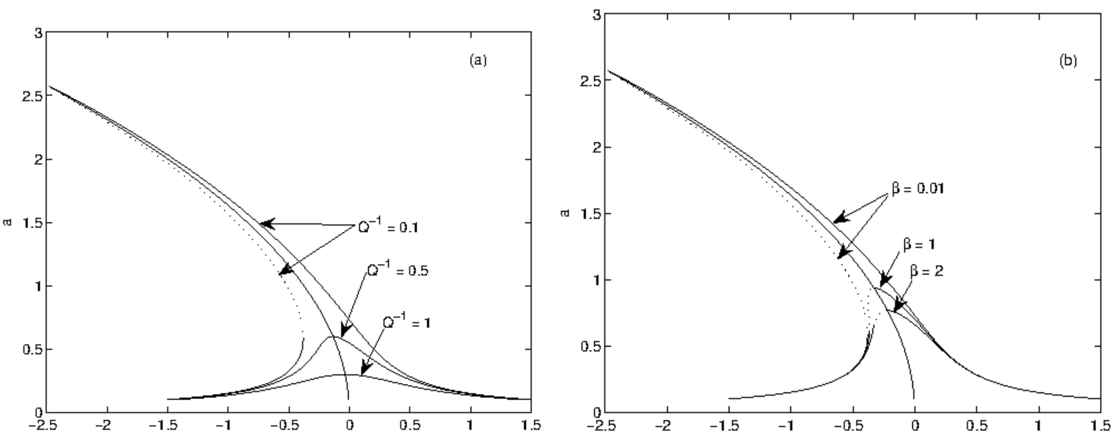


Figure 2. The frequency response curves of the primary resonance solution for the parameters: $\alpha = 3, \beta = 0.01$, $Q^{-1} = 0.1$ and $F_0 = 0.3$

In Fig.2 we observe that the response amplitude has two branches which bent to the left showing the softening behavior. The upper branch has stable solution while the lower branch presents unstable and stable solutions. Therefore, we conclude that there appears in this system, the jump and bifurcation phenomena. When the linear damping factor (the inverse of quality factor) increases, the maximum amplitude of the response decreases considerably and the instability region disappears as shown in Fig.3(a). In the same way, increasing of nonlinear damping coefficient (van der Pol coefficient) decreases considerably the instability domain and the maximum amplitude of the response (see Fig.3(b)). In Fig.3(c), when the inertial nonlinearity α decreases and takes the values 2, we always note the softening behavior. Moreover the maximum amplitude of the response stays unchanged but its location value increases. However the hardening behavior is obtained when α takes the value -3. In such situation, we can conclude that there appear in the system under consideration the jump and bifurcation phenomena. In Fig.3(d) we note that when F_0 decreases the maximum amplitude of the response decreases highly as well as the instability region. The system displays always the softening behavior and therefore it appears in such system the jump and bifurcation phenomena.



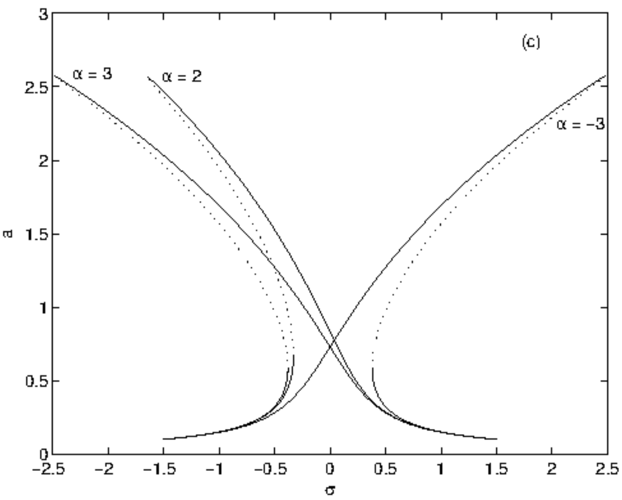


Figure 3. Frequency response curves exhibiting the effect of: (a) Q^{-1}; (b) β; (c) α and F_0 on the primary resonance solution

Figures 4-6 represent the frequency-response curves for super-harmonic resonance of order three. In Fig.4 we observe that the response amplitude has multi-valued curves which bent to the right showing that there appears in the system the hardening behavior. The multi-valued curves consists of two stables branches. Then bistability phenomenon is obtained.

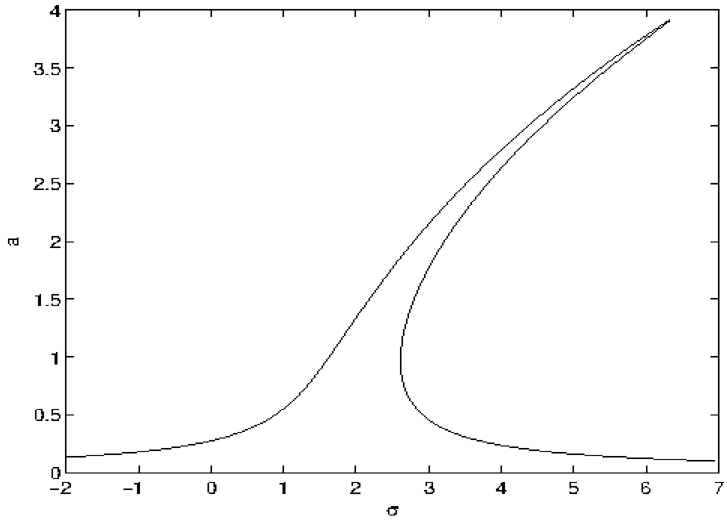
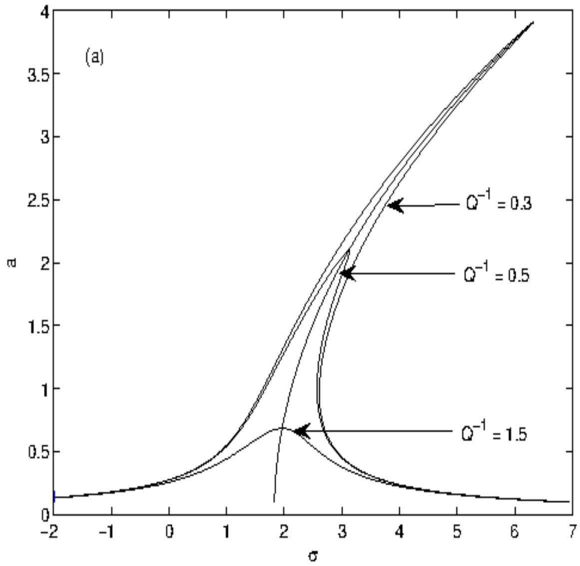
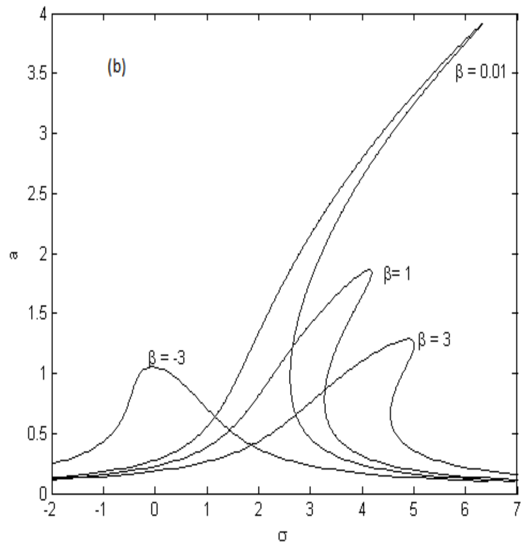


Figure 4.Typical frequency response curves of the super-harmonic resonance solution for the parameters: $\alpha = 3, \beta = 0.01$, $Q^{-1} = 0.3$ and $F_0 = 1.2$

As in Fig.3 (a), the same variations of linear damping factor Q^{-1} is obtained in Fig.5(a). In Fig.5(b), when β increases we note an important displacement of the curves towards the right with respect to typical frequency response curves shown in Fig.4. Moreover the maximum amplitude of the response of these curves decreases widely. For certain negative values of β, the softening behavior is observed in the system. In Fig5(c) we notice that the system exhibits the softening behavior when α becomes negative. The upper and lower branches have stable solutions. When α increases the maximum amplitude decreases highly and the corresponding curves move to the right with respect to typical frequency response curves with $\alpha = -3$. For positive values of α hardening behavior is obtained and the maximum amplitude of the response decreases with decreasing of α (Fig.5(d)). Fig.6 shows the variation of excitation amplitude F_0. In this figure, we note equally the same effects produced by α in Fig.5(d).





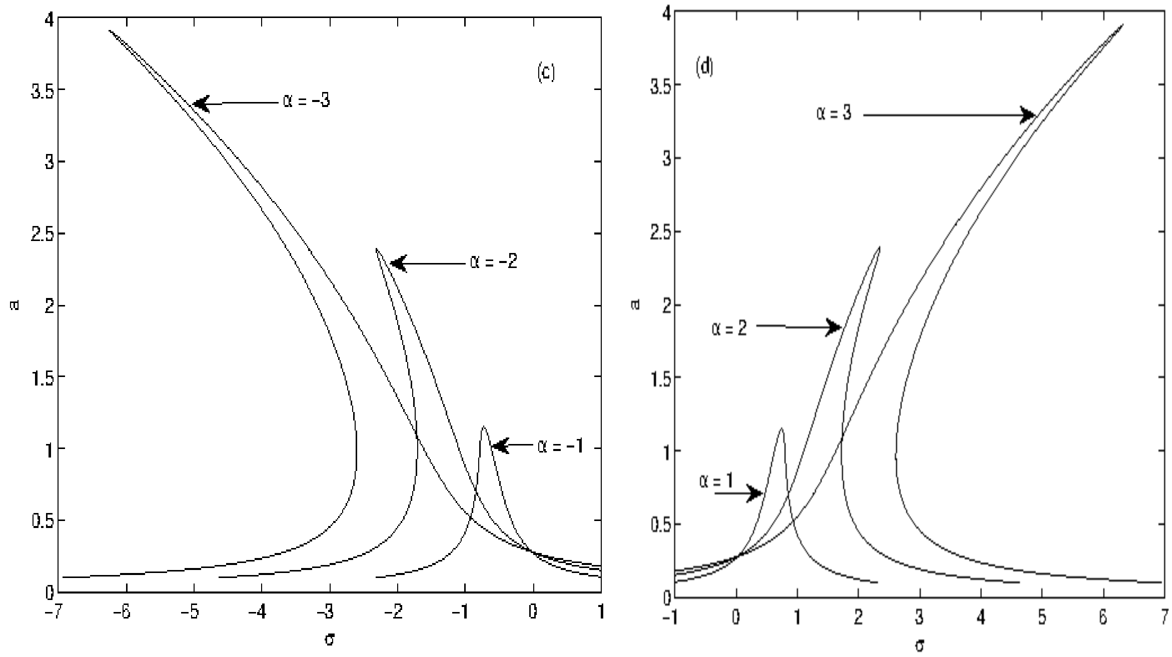


Figure 5. Frequency response curves illustrating the effects of: (a) Q^{-1}; (b) β and (c,d) α on the 3:1super-harmonic resonance solution

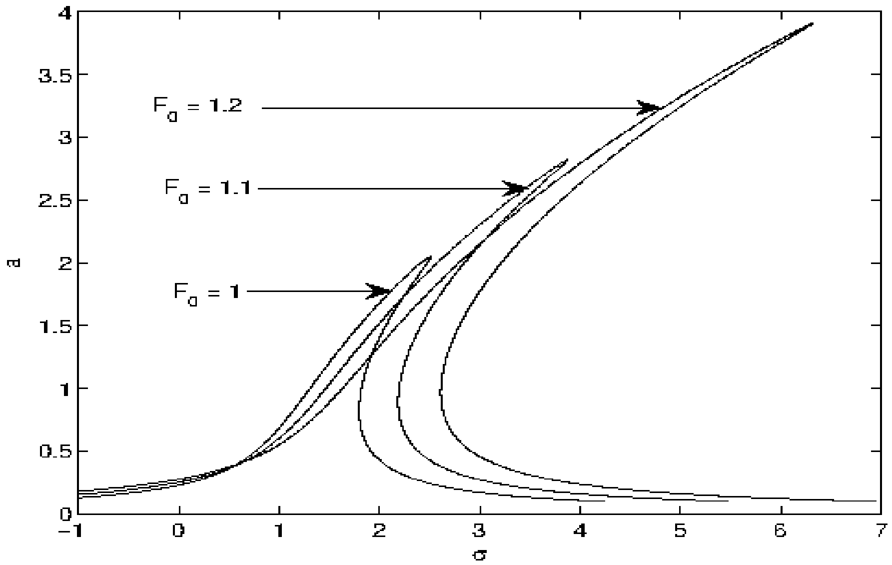


Figure 6. Frequency response curves showing the effects of F_0 on the 3:1super-harmonic resonance solution

Figures7-9 represent the frequency-response curves of the sub-harmonic resonance of order 1/3. In Fig.7 we note that the response amplitude is obtained for positive values of detuning parameter. This solution is oval and it is composed of two branches which bent to the right showing the hardening behavior. The upper and lower branches have stable and unstable solutions and there exist two saddle node bifurcations.

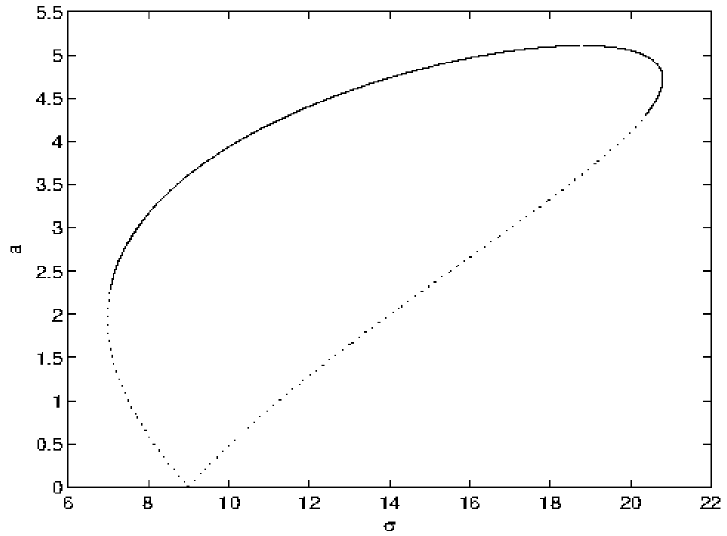


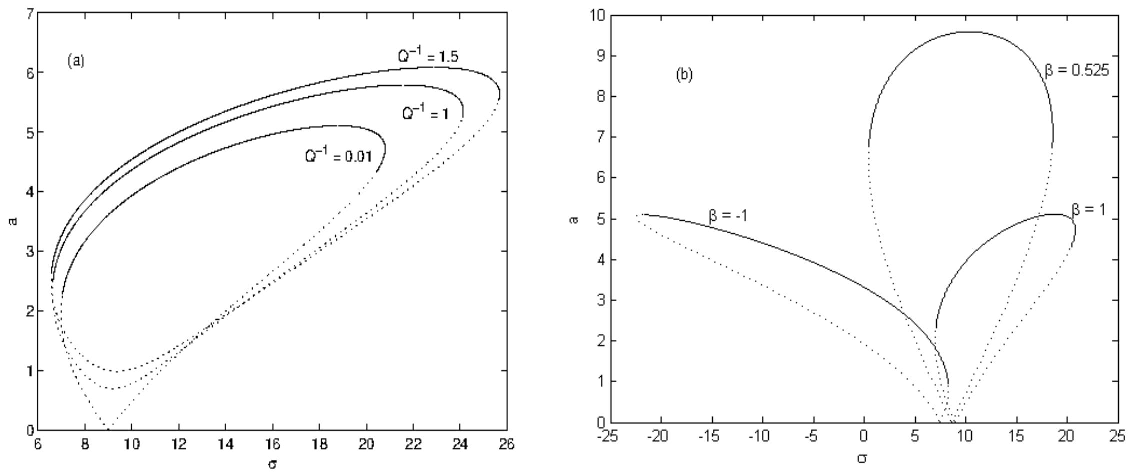
Figure 7. Typical frequency response curves of the 1/3 sub-harmonic resonance solution for the parameters: $\alpha = 1, \beta = 1$, $Q^{-1} = 0.01$ and $F_0 = 8$

When the linear damping factor Q^{-1} increases, the maximum amplitude as well as instability domain increase with appearance of two saddle node bifurcations (Fig.8(a)). In Fig.8(b), when β decreases and takes the value 0.525 the response amplitude has oval and it is symmetric about $\sigma = 8.75$. For this response amplitude the upper and lower branches have stable and unstable solutions. Therefore, there exist two saddle node bifurcations in the system. When $\beta = -1$, softening behavior appears in the system under study. The upper branch is stable and unstable solutions while the lower branch has unstable solution. Then it exists in the system the bifurcation phenomenon. When α increases positively, the minimum and maximum amplitudes increase. With $\alpha = 2.03$, the response amplitude is symmetric about $\sigma = 18.75$ and it presents two saddle node bifurcations. When α takes the values 2.5 the two branches of the amplitude response bent to the left showing the softening behavior. The upper and lower branches have stable and unstable solutions with appearance of two saddle node bifurcations (Fig.8(c)). For negative values of α hardening behavior is observed with high values of detuning parameter σ .We also note the presence of two saddle node bifurcations in the system (Fig8(d)). We conclude that some positive values of α, the system may switch from the softening behavior to the hardening behavior and vice versa. But for negative values softening behavior is only obtained.

In Fig.9, we observe that when F_0 increases the minimum amplitude and maximum amplitude increase with increasing of the instability domain. There also exist two saddle node bifurcations.

5. Conclusion

In this work the nonlinear resonances analysis of a nonlinear RLC series circuit modeled by modified Van der Pol oscillator is investigated. The multiple scales method is used to investigate the various resonant states of the system. It is found that the system modeled by modified Van der Pol oscillator presents three resonant states such as primary resonance, sub-harmonic resonance of order 1/3 and super-harmonic resonance of order 3. The steady-state solutions and their stabilities in each resonance are solved numerically. The obtained numerical results display that the system presents hardening or softening behavior, bistability, hysteresis, jump and bifurcation phenomena. The effects of different parameters of the system on the each steady-state response are studied. It is found that the parameters of the system can be used to control the amplitude of the nonlinear response of the system. Finally, we can point out that the coefficients of inertial non-linearity and van der pol respectively are responsible for the hardening or softening behavior of the system in the three resonance states and the sub-harmonic and super-harmonic resonances obtained.



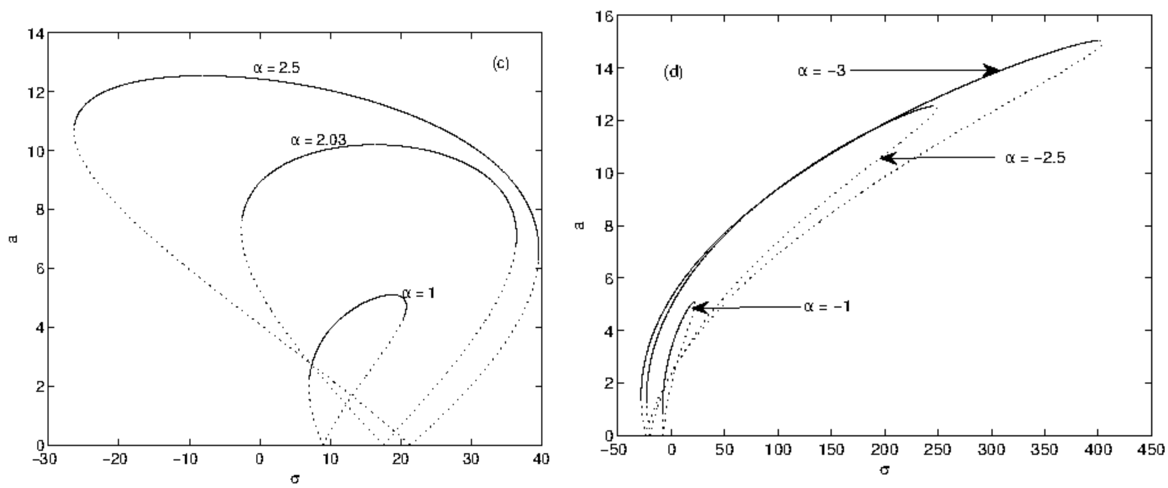


Figure 8. Frequency response curves showing the variations of: (a) Q^{-1}; (b) β and (c, d) α on the sub-harmonic resonance of order 1/3 solution

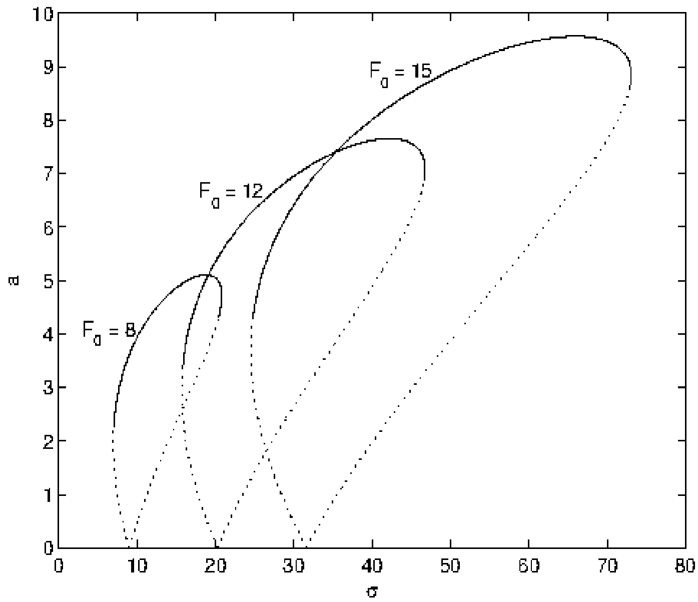


Figure 9. Frequency response curves exhibiting the effect of F_0 on the sub-harmonic resonance of order 1/3 solution

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