Symmetry model E9CS

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Motivation 1:

Why do we consider the E9 group (more specifically the Coxeter element of this group)?
1) E9 is an affine group and thus has something to do with extension.
2) The extension is flat as the universe.
3) The key Coxeter element of the group produces symmetries involving our current standard model.

The fundamentals here:

https://en.wikipedia.org/wiki/Coxeter_group
https://de.wikipedia.org/wiki/Wurzelsystem
http://home.mathematik.uni-freiburg.de/soergel/Skripten/XXSPIEG.pdf

Symmetries which arise from the Coxeter element of the E9.

E9CS = SU(5) x SU(3) x SU(2) x U(1) x U(1) (SU(n) = Special unitary group, U(1) unitary group)

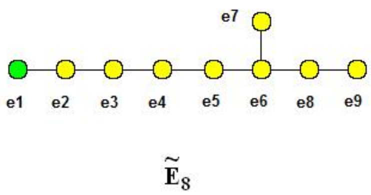
Pronounced E9Coxeter-Symmetry

evidently SU(5) x SU(3) x SU(2) x U(1) x U(1) ⊃ **$SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$** (Color charge, isospin, Hyper charge)

Write the symmetry in order to:

E9CS = $SU(5)_S$ x $U(1)_{Y2}$ x $U(1)_{Y1}$ x $SU(2)_L$ x $SU(3)_C$ (=Expansion x actual Standard Model)

Dynkin Diagram E9 (affine one point extension of group E8):



Derivative of the symmetries of E9CS from the invariants of the Coxeter elements E9:

The Coxeter element is the product of the generating reflections of E9.

Coxeterelement = e1.e2.e3.e4.e5.e6.e7.e8.e9

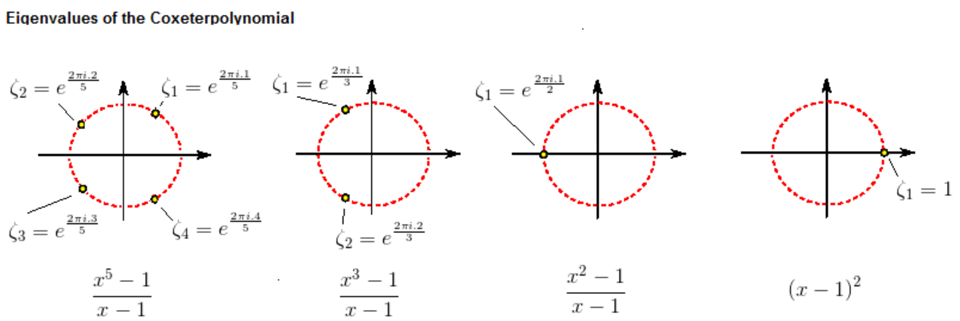
The Coxeterpolynom is the characteristic polynomial of Coxeter elements and has the form:

$$E_9(x) = \frac{x^5-1}{x-1}.\frac{x^3-1}{x-1}.\frac{x^2-1}{x-1}.(x-1)^2$$

$$E9CS = SU(5) \times SU(3) \times SU(2) \times U(1)^2$$

$E_9(x)$ ... *characteristical polynom of the coxeterelement of E9*

E9(x) is a polynom with terms of cyclotomic factors $Z_n = \frac{x^n-1}{x-1}$ for n>1 and (x-1) for n= 1.
The cyclotomic factors are the characteristical polynom of the A_{n-1} (which is the Dynkin diagram for the SU(n) Liegroup.See more here: https://en.wikipedia.org/wiki/Special_unitary_group).
So finally the symmetry space of the **Coxeterelement** is **SU(5) x SU(3) x SU(2) x U(1) x U(1)**



Eigenspace of the Coxeterpolynomial

$\mathbb{C}^4 \times \mathbb{C}^2 \times \mathbb{C}^1 \times \mathbb{C}^2$

Motivation 2:

What bring us the additional symmetries?
(1) These have the potential to describe new particles.
(2) These have the potential to describe the space and time.
(3) These have the potential to describe gravity.

Wish to analogously represent Graviton to the photon as a blend (Weinberg angle see <8>).

<1> The Idea

Light and gravitation just like photon and graviton have something in common.
Both are massless and propagate with the speed of light.

We know that light by the symmetry breaking 1: SU(2)xU (1)--> U(1) is described as a mixture.
So light is a part of the **electro-weak interactions.**

we consider analog gravity as a result of a further symmetry breaking
Symmetry breaking 2: SU(5) x U(1) x U(1) --> U(1)
Our extended standard model allows us this.

We will now like to assign our relevant SU(n)'s to algebras division (real numbers, complex numbers, ...).

$SU(1) \longleftrightarrow \mathbb{R}$
$SU(2) \longleftrightarrow \mathbb{C}$
$SU(3) \longleftrightarrow \mathbb{H}$
$SU(5) \longleftrightarrow \mathbb{O}$

This 4 divison algebras (real numbers, complex numbers, quaternions and octonions) develop through the doubling process
see more at https://de.wikipedia.org/wiki/Verdopplungsverfahren
Considering the dimensions of the SU(2) = 1,SU(3)= 2, SU(5) = 4 then this is double as well.

There appears to be a connection between the division algebras and the SU(n)'s (n = 2,3,5) which I hope is known in analytic geometry or another area.
I assume this connection warrants as simply as given.
Notes but no clear allocation can be found in this direction at Corinne A. Manogue and Tevian Dray, John Baez, etc.

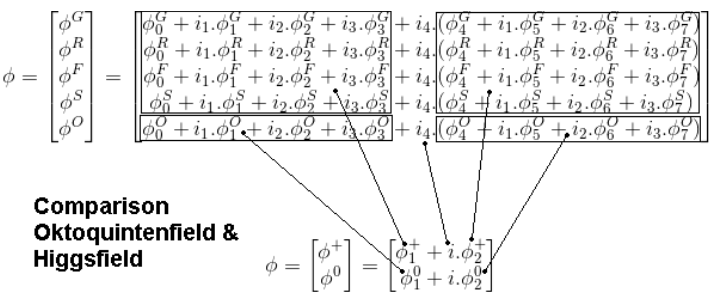
Therefore, we rely analogously on the **Higgsfield** (2 x complex = doublet)

$$\phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} = \begin{bmatrix} \phi_1^+ + i.\phi_2^+ \\ \phi_1^0 + i.\phi_2^0 \end{bmatrix}$$

<2> the Oktoquintenfield (5 x Oktonions= Quintett).

$$\phi = \begin{bmatrix} \phi^G \\ \phi^R \\ \phi^F \\ \phi^S \\ \phi^O \end{bmatrix} = \begin{bmatrix} \phi_0^G + i_1.\phi_1^G + i_2.\phi_2^G + i_3.\phi_3^G + i_4.\phi_4^G + i_5.\phi_5^G + i_6.\phi_6^G + i_7.\phi_7^G \\ \phi_0^R + i_1.\phi_1^R + i_2.\phi_2^R + i_3.\phi_3^R + i_4.\phi_4^R + i_5.\phi_5^R + i_6.\phi_6^R + i_7.\phi_7^R \\ \phi_0^F + i_1.\phi_1^F + i_2.\phi_2^F + i_3.\phi_3^F + i_4.\phi_4^F + i_5.\phi_5^F + i_6.\phi_6^F + i_7.\phi_7^F \\ \phi_0^S + i_1.\phi_1^S + i_2.\phi_2^S + i_3.\phi_3^S + i_4.\phi_4^S + i_5.\phi_5^S + i_6.\phi_6^S + i_7.\phi_7^S \\ \phi_0^O + i_1.\phi_1^O + i_2.\phi_2^O + i_3.\phi_3^O + i_4.\phi_4^O + i_5.\phi_5^O + i_6.\phi_6^O + i_7.\phi_7^O \end{bmatrix}$$

or written otherwise so that the equivalence to the Higgs field is clear (where i4 is pulled from)



$$\phi = \begin{bmatrix} \phi^G \\ \phi^R \\ \phi^F \\ \phi^S \\ \phi^O \end{bmatrix} = \begin{bmatrix} \phi_0^G + i_1.\phi_1^G + i_2.\phi_2^G + i_3.\phi_3^G + i_4.(\phi_4^G + i_1.\phi_5^G + i_2.\phi_6^G + i_3.\phi_7^G) \\ \phi_0^R + i_1.\phi_1^R + i_2.\phi_2^R + i_3.\phi_3^R + i_4.(\phi_4^R + i_1.\phi_5^R + i_2.\phi_6^R + i_3.\phi_7^R) \\ \phi_0^F + i_1.\phi_1^F + i_2.\phi_2^F + i_3.\phi_3^F + i_4.(\phi_4^F + i_1.\phi_5^F + i_2.\phi_6^F + i_3.\phi_7^F) \\ \phi_0^S + i_1.\phi_1^S + i_2.\phi_2^S + i_3.\phi_3^S + i_4.(\phi_4^S + i_1.\phi_5^S + i_2.\phi_6^S + i_3.\phi_7^S) \\ \phi_0^O + i_1.\phi_1^O + i_2.\phi_2^O + i_3.\phi_3^O + i_4.(\phi_4^O + i_1.\phi_5^O + i_2.\phi_6^O + i_3.\phi_7^O) \end{bmatrix}$$

$$\phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} = \begin{bmatrix} \phi_1^+ + i.\phi_2^+ \\ \phi_1^0 + i.\phi_2^0 \end{bmatrix}$$

This provides 40 degrees of freedom.

24 of which will be "spent" for our SU(5) tensor bosons for the 5th longitudinal spin degree of freedom (24 Goldstone bosons swallowed over gauge transformation) thus remain 16 left.

The S, F, R, G and H charges are the 5 charges of the SU (5) analogous to the 3 color charges of SU (3) and the 2 charges (+ .-) of SU (2).
The letters stand for S = See, F = feeling, R=smelling G = Taste and H = Hear
Calling therefore the charges of the SU (5) sense charges.
Note: These charges have (such as the color charges of quarks with color) nothing to do with the senses, but to give a name to the child for reference only.

We now want to look at the 16 (40-24 = 16) remaining degrees of freedom.

Make the following division for the 40 field components of the Oktoquinten field as a physical approach:
Take care that the division is not unique because for the left half 4 gray fields we can use 4.3.2.1= 24 Permutations of them in the orange area.
And for the left 4 charges we have five over 4 = 5 Permutations.
So at all we have 5 x 24 = 120 possible permutations.
On the Higgsfield we have 2 x 1= 2 permutations.

ΦG	1	i1	i2	i3	i4	i5	i6	i7	
ΦR	1	i1	i2	i3	i4	i5	i6	i7	
ΦF	1	i1	i2	i3	i4	i5	i6	i7	
ΦS	1	i1	i2	i3	i4	i5	i6	i7	
ΦO	1	i1	i2	i3	i4	i5	i6	i7	Charge 0

Energy-Time
Momentum-Space
Shearstress\Energyflux
for the 4 neutral Bosons
for the 20 charged Bosons

Analogeous to the Higgspotential we declare a Potential on the Oktoquintenfield

<3> Potential over the Oktoquintenfield

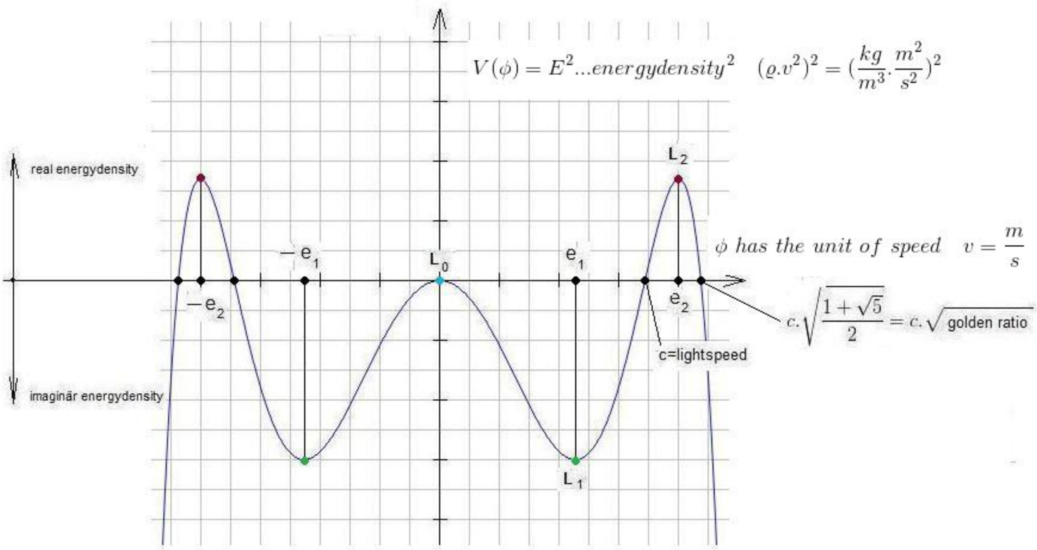
$$V(\phi) = \frac{\gamma^2}{2} \mid \phi \mid^2 + \frac{\mu^2}{4} \mid \phi \mid^4 + \frac{\lambda^2}{8} \mid \phi \mid^8 \quad with\ \phi\ \epsilon\ \mathbb{O}^5$$

$\gamma, \lambda\ \epsilon\ i.\mathbb{R}\ (imaginaer)\ and\ \ \mu\ \epsilon\ \mathbb{R}$

$\frac{\gamma^2}{2}$ $...momentumdensity^2 \quad (\varrho.v)^2 = (\frac{kg}{m^3}.\frac{m}{s})^2$

$\frac{\mu^2}{4}$ $...massdensity^2 \quad (\varrho)^2 = (\frac{kg}{m^3})^2$

$\frac{\lambda^2}{8}$ $...spindensity^2 \quad (\frac{\varrho}{v^2})^2 = (\frac{\frac{kg}{m^3}}{\frac{m^2}{s^2}})^2$



$The\ coefficients\ of\ the\ potential\ comes\ from\ selfinteractions.$
$Therefore\ we\ make\ the\ assumption\ that\ we\ have\ the\ following\ relation:$

$$C := \frac{-\mu^2}{\lambda^2} = (4.\frac{\gamma^2}{\mu^2})^2 \qquad \gamma^2, \lambda^2 < 0 \qquad \mu^2 > 0$$

$Then\ it\ follows\ by\ exact\ calculation\ that$

$$C = c^4.\varphi^2$$

$c...speed\ of\ light$
$\varphi...golden\ ratio\ = 1,6180...$

$The\ first\ mixingangle\ which\ comes\ from\ the\ minimum\ of$
$the\ Oktoquintenpotential\ is\ appr.\ equal\ to\ the$
$WEINBERG-ANGLE \approx 28,89^o$
$see\ <8>$

For aesthetic reasons we want keep in mind that for the coming formulars phi = absolut(phi).

We want that the second part of the Oktoqintenpotential is our quadratic vacuumenergydensity.

$$\frac{\mu^2}{4} \mid c \mid^4 = \frac{1}{4}.(\frac{\Lambda.c^4}{8\pi G})^2 = \frac{1}{4}.(\varrho_{vacuum}.c^2)^2$$

Λ...cosmological constant
ϱ_{vacuum}...vacuum massdensity
then with the relation $c^4.\varphi^2 \ = \frac{-\mu^2}{\lambda^2} = (4.\frac{\gamma^2}{\mu^2})^2$ *we get the potential as*

EINSTEIN − FORM

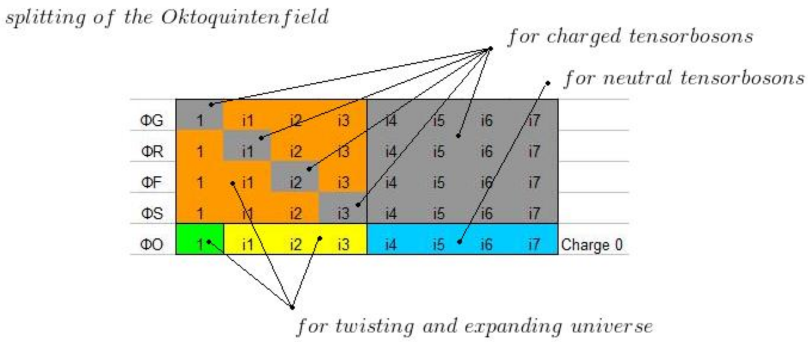
$$V(\phi) = (\frac{\Lambda.c^4}{8\pi G})^2.\frac{1}{8.\varphi^2}.\left(-\varphi^3.(\frac{\phi}{c})^2 + 2.\varphi^2.(\frac{\phi}{c})^4 - (\frac{\phi}{c})^8\right)$$

<4> Lagrangedensity of the Oktoquintenfield/Oktoquintenpotential

Hint:

I do the same steps as shown in this cooking recipe for the Higgsfield.

https://www.lsw.uni-heidelberg.de/users/mcamenzi/HD_Higgs.pdf



Analogous to the **electroweak** theory, we want to talk about a gravito super weak theory here.
The **electroweak** theory brings together the electrical with the weak interactions.
The **gravito-sensecharge** theory brings together the gravitational with the dark interactions.

Similar to the SU(3) Vectorbosons which are named Gluons we name our SU(5) Tensorbosons Repelions. The force between different charged Repelions is repulsiv because they are tensorbosons (2nd − order).

Similar to the Higgsfield we assign our Repelions to the Oktoquintenfield by the following scheme. The numbers are the sense charges (see < 2 >).
5 = See
4 = Feeling
3 = Smelling
2 = Taste
1 = Hear

ΦG	$2\bar{1}$	i1	i2	i3	$\bar{2}1$	$\bar{2}3$	$\bar{2}4$	$\bar{2}5$	2
ΦR	1	$3\bar{1}$	i2	i3	$\bar{3}2$	$\bar{3}1$	$\bar{3}4$	$\bar{3}5$	3
ΦF	1	i1	$4\bar{1}$	i3	$\bar{4}2$	$\bar{4}3$	$\bar{4}1$	$\bar{4}5$	4
ΦS	1	i1	i2	$5\bar{1}$	$\bar{5}2$	$\bar{5}3$	$\bar{5}4$	$\bar{5}1$	5
ΦO	1	i1	i2	i3	i4	i5	i6	i7	Charge 0
					$1\bar{1}$-$2\bar{2}$	$1\bar{1}$-$3\bar{3}$	$1\bar{1}$-$4\bar{4}$	$1\bar{1}$-$5\bar{5}$	

similar to the the higgsfield where the vacuumexpectation is

$$\phi_{vac} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

the vacuumexpectation of the Oktoquintenfield is (green, yellow, orange)

$$\phi_{vac} = v.\begin{pmatrix} 0 + i_1 + i_2 + i_3 \\ 1 + 0 \ + i_2 + i_3 \\ 1 + i_1 + 0 + i_3 \\ 1 + i_1 + i_2 + 0 \\ 1 + i_1 + i_2 + i_3 \end{pmatrix}$$

where $v = e_1$ ist the minimum of the Oktoquintenpotential and i_1, i_2 and i_3 the imaginaer quaternions.

$As\ mentioned\ in < 6 >\ point\ 3)\ we\ assume\ that\ the\ left\ 4\ charged\ bosons$
$decomposed\ to\ electrons, neutrinos\ and\ quarks\ and\ couple\ to\ the\ higgsfield\ instead.$
$Then\ the\ vacuumexpectation\ changes\ to:$

$$\phi_{vac} = v. \begin{pmatrix} 1+ i_1 + i_2 + i_3 \\ 1+ i_1 + i_2 + i_3 \\ 1+ i_1 + i_2 + i_3 \\ 1+ i_1 + i_2 + i_3 \\ 1+ i_1 + i_2 + i_3 \end{pmatrix}$$

STEP 1:Lorentzinvariant Lagrangedensity for the Oktoquintenfield

$$\mathcal{L}_\phi = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi^\dagger \phi)$$

$with\ \phi \in \mathbb{O}^5 \quad Octonions^5$

$The\ potential\ V\ is\ shown\ in\ < 3 >$

$\mathcal{T}_{ij}$	j ---->		$Generatores\ of\ the\ SU(5)$		
i ↓	$\begin{pmatrix}0&1&0&0&0\\1&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&-i&0&0&0\\i&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&1&0&0\\0&1&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&0&1&0\\0&0&0&0&0\\0&1&0&0&0\\0&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&0&0&1\\0&0&0&0&0\\0&0&0&0&0\\0&1&0&0&0\end{pmatrix}$
	$\begin{pmatrix}0&0&1&0&0\\0&0&0&0&0\\1&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&-i&0&0\\0&i&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&-i&0&0\\0&0&0&0&0\\i&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&1&0\\0&0&1&0&0\\0&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&1\\0&0&0&0&0\\0&0&1&0&0\end{pmatrix}$
	$\begin{pmatrix}0&0&0&1&0\\0&0&0&0&0\\0&0&0&0&0\\1&0&0&0&0\\0&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&0&-i&0\\0&0&0&0&0\\0&i&0&0&0\\0&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&-i&0\\0&0&i&0&0\\0&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&-i&0\\0&0&0&0&0\\0&0&0&0&0\\i&0&0&0&0\\0&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&1\\0&0&0&1&0\end{pmatrix}$
	$\begin{pmatrix}0&0&0&0&1\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\1&0&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&0&0&-i\\0&0&0&0&0\\0&0&0&0&0\\0&i&0&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&-i\\0&0&0&0&0\\0&0&i&0&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&-i\\0&0&0&i&0\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&-i\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\i&0&0&0&0\end{pmatrix}$
	$\begin{pmatrix}1&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&-1\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&1&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&-1\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&1&0&0\\0&0&0&0&0\\0&0&0&0&-1\end{pmatrix}$	$\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&1&0\\0&0&0&0&-1\end{pmatrix}$	

$W - Boson\ scheme$

$$\begin{pmatrix} W^{11} & W^{12} & W^{13} & W^{14} & W^{15} \\ W^{21} & W^{22} & W^{23} & W^{24} & W^{25} \\ W^{31} & W^{32} & W^{33} & W^{34} & W^{35} \\ W^{41} & W^{42} & W^{43} & W^{44} & W^{45} \\ W^{51} & W^{52} & W^{53} & W^{54} & \end{pmatrix}$$

$hint: \ W^{ij} = W^{ij}_\mu$

$We\ take\ a\ look\ on\ the\ symmetry$

$SU(5) \times\ U(1) \times U(1)$

$W^{ij} \qquad B^0 \qquad B^1$

$calculate\ covariant\ derivation$

$$D_\mu \phi = (\partial_\mu + \frac{i.g}{2}.\tau_{ij}.W^{ij}_\mu + \frac{i.g'}{2}.Id.^0 B^0_\mu + \frac{i.g''}{2}.Id.^1 B^1_\mu).\phi$$

$$\tau_{ij}.W^{ij}_\mu = \begin{pmatrix} W^{51} & W^{11} - i.W^{12} & W^{21} - i.W^{23} & W^{31} - i.W^{34} & W^{41} - i.W^{45} \\ W^{11} + i.W^{12} & W^{52} & W^{22} - i.W^{13} & W^{14} - i.W^{32} & W^{15} - i.W^{42} \\ W^{21} + i.W^{23} & W^{22} + i.W^{13} & W^{53} & W^{24} - i.W^{33} & W^{25} - i.W^{43} \\ W^{31} + i.W^{34} & W^{14} + i.W^{32} & W^{24} + i.W^{33} & W^{54} & W^{35} - i.W^{44} \\ W^{41} + i.W^{45} & W^{15} + i.W^{42} & W^{25} + i.W^{43} & W^{35} + i.W^{44} & -(W^{51} + W^{52} + W^{53} + W^{54}) \end{pmatrix}$$

$$Id^0 = Id^1 = \begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}$$

$and\ for\ example$

$$W_{12} = \frac{W^{11} - i.W^{12}}{\sqrt{2}}$$

$The\ boson\ which\ changes\ the\ charge\ from\ 1\ (hear)\ to\ 2\ (taste).$

Then

$$D_\mu \phi_{vac} = \frac{\phi_{vac}\cdot i}{2}\cdot\left[g\cdot\begin{pmatrix} W^{51} & \sqrt{2}\,W_{12} & \sqrt{2}\,W_{13} & \sqrt{2}\,W_{14} & \sqrt{2}\,W_{15} \\ \sqrt{2}\,W_{21} & W^{52} & \sqrt{2}\,W_{23} & \sqrt{2}\,W_{24} & \sqrt{2}\,W_{25} \\ \sqrt{2}\,W_{31} & \sqrt{2}\,W_{32} & W^{53} & \sqrt{2}\,W_{34} & \sqrt{2}\,W_{35} \\ \sqrt{2}\,W_{41} & \sqrt{2}\,W_{42} & \sqrt{2}\,W_{43} & W^{54} & \sqrt{2}\,W_{45} \\ \sqrt{2}\,W_{51} & \sqrt{2}\,W_{52} & \sqrt{2}\,W_{53} & \sqrt{2}\,W_{54} & -(W^{51}+W^{52}+W^{53}+W^{54}) \end{pmatrix} + \right.$$

$$\left. \begin{pmatrix} g'.B^0+g''.B^1 & 0 & 0 & 0 & 0 \\ 0 & g'.B^0+g''.B^1 & 0 & 0 & 0 \\ 0 & 0 & g'.B^0+g''.B^1 & 0 & 0 \\ 0 & 0 & 0 & g'.B^0+g''.B^1 & 0 \\ 0 & 0 & 0 & 0 & g'.B^0+g''.B^1 \end{pmatrix} \right] \cdot \begin{pmatrix} 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \end{pmatrix}$$

then

$$\begin{aligned}(D^\mu\phi_{vac})^\dagger(D_\mu\phi_{vac}) = \frac{\phi_{vac}^2}{4}.[\, & 4.(gW^{51}+g'B^0+g''B^1)^2 + 8g^2.(W_{12}W_{21}+W_{13}W_{31}+W_{14}W_{41}+W_{15}W_{51}) + something + \\ & 4.(gW^{52}+g'B^0+g''B^1)^2 + 8g^2.(W_{12}W_{21}+W_{23}W_{32}+W_{24}W_{42}+W_{25}W_{52}) + something + \\ & 4.(gW^{53}+g'B^0+g''B^1)^2 + 8g^2.(W_{13}W_{31}+W_{23}W_{32}+W_{34}W_{43}+W_{35}W_{53}) + something + \\ & 4.(gW^{54}+g'B^0+g''B^1)^2 + 8g^2.(W_{14}W_{41}+W_{24}W_{42}+W_{34}W_{43}+W_{45}W_{54}) + something + \\ & 4.(-g(W^{51}+W^{52}+W^{53}+W^{54})+g'B^0+g''B^1)^2 + 8g^2.(W_{15}W_{51}+W_{25}W_{52}+W_{35}W_{53}+W_{45}W_{54}) + something\,]\end{aligned}$$

hint : $W^{ij}=W^{ij}_\mu$ *and* $B^0=B^0_\mu$ *and* $B^1=B^1_\mu$

like the result of the Higgsfield we expect something like that :

$$(D^\mu\phi_{vac})^\dagger(D_\mu\phi_{vac}) = \frac{v^2}{8}.(g^2.(W^+)^2+g^2.(W^-)^2+(g^2+g'^2).Z_\mu.Z^\mu+0.A_\mu.A^\mu)$$

We have a lot of summands so we first want to take a look on the diagonal elements of the covariant derivation. In the Higgsfieldtheory we get as result the massive Z − Bosons and the Photon as a mixing of neutral W and B bosons.

We calculate the expression which is a symmetric bilinearform :

Momentumdensity − Matrix

$$\begin{pmatrix} W^{51} & W^{52} & W^{53} & W^{54} & B^0 & B^1 \end{pmatrix}\cdot\begin{pmatrix} 8g^2 & 4g^2 & 4g^2 & 4g^2 & 0 & 0 \\ 4g^2 & 8g^2 & 4g^2 & 4g^2 & 0 & 0 \\ 4g^2 & 4g^2 & 8g^2 & 4g^2 & 0 & 0 \\ 4g^2 & 4g^2 & 4g^2 & 8g^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20g'^2 & -20g'.g'' \\ 0 & 0 & 0 & 0 & -20g'.g'' & 20g''^2 \end{pmatrix}\cdot\begin{pmatrix} W^{51} \\ W^{52} \\ W^{53} \\ W^{54} \\ B^0 \\ B^1 \end{pmatrix}$$

linearly independent (upper 4×4 block) — *linearly dependent* (lower 2×2 block)

and compare it with the red area of the dynamic lagrangepart.

Someone can easy proof that is identical.

Then with diagonalizing the Momentumdensity − Matrix we get the following result :

$$(D^\mu\phi_{vac})^\dagger(D_\mu\phi_{vac}) = \frac{\phi_{vac}^2}{4}.[green\ area + red\ area + \underbrace{something}_{=0}]$$

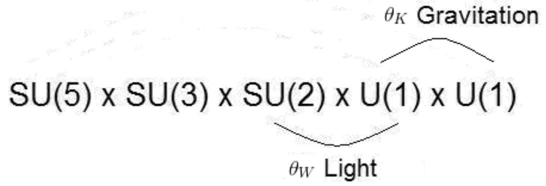
$$\begin{aligned}(D^\mu\phi_{vac})^\dagger(D_\mu\phi_{vac}) = \frac{\phi_{vac}^2}{4}.[\, & 8g^2.(|W_{12}|^2+|W_{21}|^2+|W_{13}|^2+|W_{31}|^2 + \\ & |W_{14}|^2+|W_{41}|^2+|W_{23}|^2+|W_{32}|^2 + \\ & |W_{24}|^2+|W_{42}|^2+|W_{34}|^2+|W_{43}|^2 + \\ & |W_{15}|^2+|W_{51}|^2+|W_{35}|^2+|W_{53}|^2 + \\ & |W_{25}|^2+|W_{52}|^2+|W_{45}|^2+|W_{54}|^2) + \\ & 8g^2.(|Z^0|^2+|Z^1|^2+|Z^2|^2+|Z^3|^2) + \\ & 20.(g'^2+g''^2).|\Gamma|^2 + \\ & 0.|G|^2\ ]\end{aligned}$$

U(1) Gravitation field (G)

eigenvalues of the Momentumdensity matrix ($8g^2$, $20.(g'^2+g''^2)$, 0)

Momentumdensity Quadrats

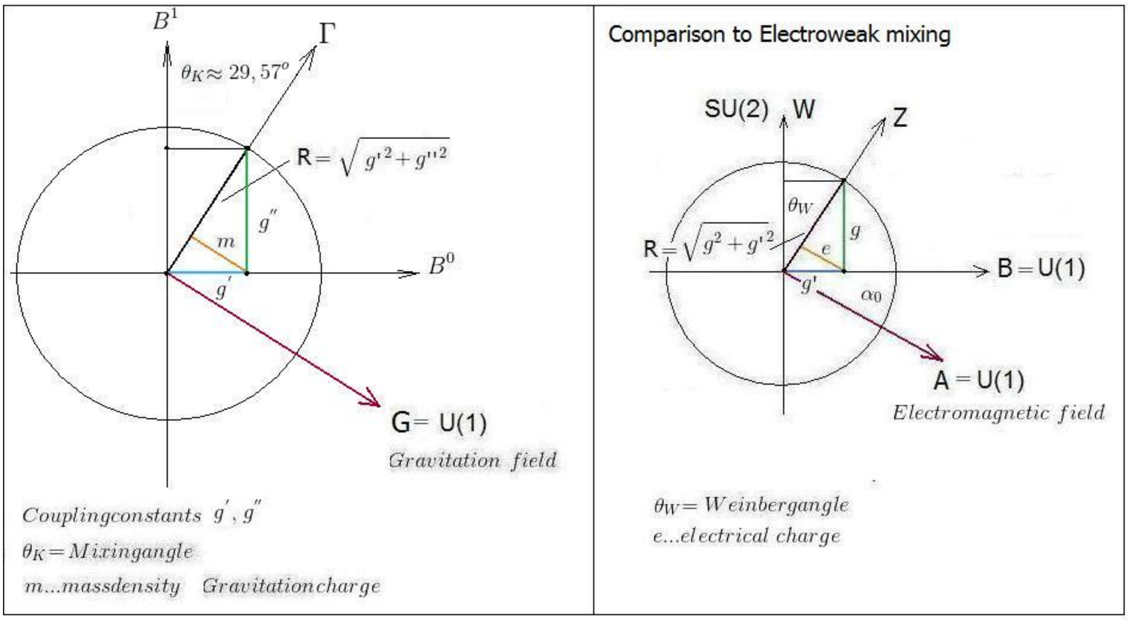
The couplingangles α_1 *and* α_2 *comes from the extremalvalues of the Oktoquintenpotential.*
As calculated in < 8 > *we have :*



Mixingangles

$\theta_W \approx \alpha_1 \qquad (\theta_W = Weinbergangle)$
$\theta_K \approx 29,57^o$
$\alpha_1 \approx 28,89^o$
$\alpha_2 \approx 121,60^o$

$$g^{'} = R.sin(\alpha_1)$$
$$g^{''} = R.sin(\alpha_2)$$



Then the Graviton and the $\Gamma - Boson$ *is a mixing :*

$$\begin{pmatrix}\Gamma_\mu \\ G_\mu\end{pmatrix} = \begin{pmatrix}cos(\theta_K) & -sin(\theta_K) \\ sin(\theta_K) & cos(\theta_K)\end{pmatrix} \cdot \begin{pmatrix}B^0_\mu \\ B^1_\mu\end{pmatrix}$$

The Impulsdensity and therefore the massdensity and therefore the mass of the W and Z Bosons are equal because they have the same couplingconstant g.

The relation of the Γ *particlemass to the* $W - Bosonmass$ *is :*

$$\frac{M_\Gamma}{M_W} = \frac{20.(g'^2 + g''^2)}{8g^2} = \frac{5.(g'^2 + g''^2)}{2g^2}$$

We develope around the vacuumexpectation ϕ_{min} *to see the interactionterms.*

$$\phi = \frac{\phi_{min}}{c} \cdot (c+H) \cdot \begin{pmatrix} 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \end{pmatrix}$$

then with $\dfrac{\phi_{min}}{c} = \beta$ *and* $c =$ *speed of light*

$$(D^{\mu}\phi)^{\dagger}(D_{\mu}\phi) = 20\beta^2.(\partial^{\mu}H).(\partial_{\mu}H) +$$

$$2\beta^2 g^2 c^2.(|\, W_{\bar{1}2}\,|^2 + |\, W_{\bar{2}1}\,|^2 + ...) + \qquad W - Boson\ Momentumdensity$$

$$2\beta^2 g^2 c^2.(|\, Z^0\,|^2 + |\, Z^1\,|^2 + |\, Z^2\,|^2 + |\, Z^3\,|^2) + \qquad Z - Boson\ Momentumdensity$$

$$2\beta^2 g^2.(2cH + H^2).(W_{\mu\bar{1}2}.W^{\mu}_{\bar{2}1} + ...) + \qquad H - W\ Interaction$$

$$2\beta^2 g^2.(2cH + H^2).(Z^0_{\mu}.Z^{0\mu} + Z^1_{\mu}.Z^{1\mu} + Z^2_{\mu}.Z^{2\mu} + Z^3_{\mu}.Z^{3\mu}) \qquad H - Z\ Interaction$$

$$5\beta^2.(g'^2 + g''^2).(c^2 + 2cH + H^2).|\Gamma|^2$$

$\Gamma - Boson\ Momentumdensity$ $\qquad H - \Gamma\ Interaction$

In our Lagrange we have massdensities instead of mass!
So how can we talk about particles with mass?

One possibility is to take a look on Planckunits.
More concrete the W−Bosons could be Planckparticles with some special properties.

. *Planckparticles have Planckmass*
. *Planckparticles are something like the smallest Blackhole*

What can we say about the geometrical shape of such a particle?

We assume that a Clifford−Torus is the right object and will proof this.
Then it follows that a Planckparticle is an object with dimension $=2$.
This explains the holographical principle because a Planckparticle then is a two − dimensional object.

The Oktoquintenpotential has two (classes of) nontrivial zeropoints on $|\,\phi\,| = c$ *and* $|\,\phi\,| = c.\sqrt{\varphi}$ *with* φ*...golden ratio and c..speed of light.*
This means the Energydensity there is zero!
Let us assume that nature prefers this two states.
The unit of ϕ *is speed.*
Multiplying it with $\sqrt{\dfrac{c}{G.\hbar}}$ *results a curvature* $\quad \phi.\sqrt{\dfrac{c}{G.\hbar}} = \dfrac{1}{r_{\phi}}$
Then for $\phi = c$ *we get as curvature*

$$\sqrt{\frac{c^3}{G.\hbar}} = \frac{1}{r_c} = \frac{1}{l_p}$$

and for $\phi = c.\sqrt{\varphi}$ *we get as curvature*

$$\sqrt{\varphi}.\sqrt{\frac{c^3}{G.\hbar}} = \frac{\sqrt{\varphi}}{r_c} = \frac{\sqrt{\varphi}}{l_p}$$

With this two radii l_p *and* $l'_p = \dfrac{l_p}{\sqrt{\varphi}}$ *we want to define a special Clifford − Torus.*

A Clifford − Torus is defined as $\mathbb{T}^2 = S^1_a \times S^1_b \ \subset\ S^3_{\sqrt{a^2+b^2}} \subset \mathbb{R}^4$ *where*
$S^1 = 1 -$ *Sphere and* a, b *are the radii of the spheres.*
Then we write our special Clifford − Torus as

$$\mathbb{T}^2_p = S^1_{l_p} \times S^1_{l'_p}$$

and name it Planck − Torus

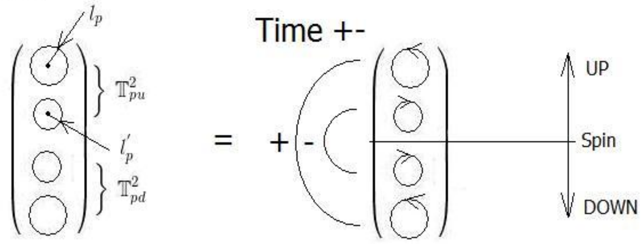
Then the Planck − Torus $\mathbb{T}^2_p$ *lays in* $S^3_{\sqrt{l_p^2 + l_p'^2}} = S^3_{l_p.\sqrt{\varphi}}$

This is the $3 -$ *Sphere with radius* $l_p.\sqrt{\varphi}$

Motivated by our results in < 6 > we are thinking about two such Planck − Tori

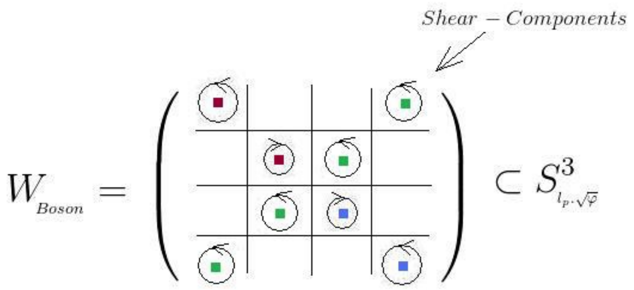
$\mathbb{T}^2_{pu}$ *and* $\mathbb{T}^2_{pd}$

Visual



To generate an 2 − dimensional object with spin = 2 we have to connect the coordinatecycles like in < 6.4 > .

schematic in our second curvaturetensor for the double Cliffordtorus



This construction has 3 different Tori $\subset S^3_{l_p \cdot \sqrt{\varphi}}$

$\mathbb{T}^2_{pu}$	$\mathbb{T}^2_{pd}$	$\mathbb{T}^2_{ps}$
$r = l_p$	$r = \frac{l_p}{\sqrt{\varphi}}$	$r = l_p \cdot \sqrt{\frac{\varphi}{2}}$
$r = \frac{l_p}{\sqrt{\varphi}}$	$r = l_p$	$r = l_p \cdot \sqrt{\frac{\varphi}{2}}$

The W_{Boson} *has Planckmass then it follows that the frequency is :*

$$\omega_p = \frac{1}{t_p} = \sqrt{\frac{c^5}{\hbar.G}}$$

resting W_{Boson} *:*

$$W_{Boson} = \begin{pmatrix} l_p.e^{-i\omega_p t} & 0 & 0 & l_p \cdot \sqrt{\frac{\varphi}{2}}.e^{-i\omega_p t} \\ 0 & \frac{l_p}{\sqrt{\varphi}} \cdot e^{i\omega_p t} & l_p \cdot \sqrt{\frac{\varphi}{2}}.e^{-i\omega_p t} & 0 \\ 0 & l_p \cdot \sqrt{\frac{\varphi}{2}}.e^{-i\omega_p t} & \frac{l_p}{\sqrt{\varphi}} \cdot e^{i\omega_p t} & 0 \\ l_p \cdot \sqrt{\frac{\varphi}{2}}.e^{-i\omega_p t} & 0 & 0 & l_p.e^{-i\omega_p t} \end{pmatrix}$$

Properties of W_{Boson}

1) *is a Tensorboson*
2) *lies in a 3 − Sphere*
3) *is build by 3 Clifford Tori*
4) *Is a flat two dimensional object*

$We\ set\ l_p = 1\ for\ easier\ handling.$

$The\ class\ of\ all\ Clifford-Tori\ in\ S^3_{\sqrt{\varphi}}\ then\ is$

$$C_a := \left\{ (z_1, z_2) \in \mathbb{C}^2 : |z_1| = \sqrt{\varphi}.\sqrt{\frac{1+a}{2}} \quad |z_2| = \sqrt{\varphi}.\sqrt{\frac{1-a}{2}} \right\}$$

for a real parameter $a \in (-1, 1)$. This is an embedded surface in $S^3_{\sqrt{\varphi}}$ with mean curvature constant equal to

$$H_a := \frac{2a}{\sqrt{\varphi}.\sqrt{1-a^2}}$$

$Then\ our\ three\ Clifford-Tori\ \mathbb{T}^2_{pu}, \mathbb{T}^2_{pd}\ and\ \mathbb{T}^2_{ps}\ can\ be\ written\ as$

$$\mathbb{T}^2_{pu} = C_a \quad with\ a = \frac{2-\varphi}{\varphi} \quad and$$

$$\mathbb{T}^2_{pd} = C_{-a}$$

$$\mathbb{T}^2_{ps} = C_0$$

$Then\ the\ mean\ curvature\ of\ \mathbb{T}^2_{pu}\ is$

$$H_a = H_{\frac{2-\varphi}{\varphi}} = \sqrt{\frac{2-\varphi}{\varphi}}$$

$and\ the\ mean\ curvature\ of\ \mathbb{T}^2_{pd}\ is$

$$H_{-a} = H_{\frac{\varphi-2}{\varphi}} = -\sqrt{\frac{2-\varphi}{\varphi}} = -H_a$$

$and\ the\ mean\ curvature\ of\ \mathbb{T}^2_{ps}\ is$

$$H_0 = 0$$

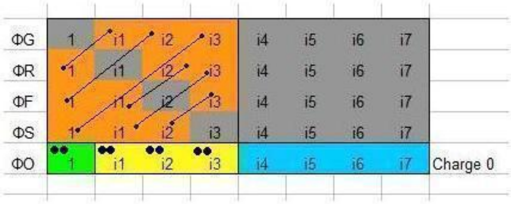
$The\ shape\ of\ the\ W-Boson\ then\ is:$

$$topology\ W_{Boson} = \left\{ C_{\frac{\varphi-2}{\varphi}}, C_0, C_{\frac{2-\varphi}{\varphi}} \right\} \subset S^3_{\sqrt{\varphi}}$$

<5> Curvaturetensors by the Oktoquintenfield

The construction comes from multiplications (symmetric to the diagonal) by 2 degrees of freedome (complex subspaces).
With this construction the tensor is symmetric in the diagonal.

$$\phi = \begin{bmatrix} \phi^G \\ \phi^R \\ \phi^F \\ \phi^S \\ \phi^O \end{bmatrix} = \begin{bmatrix} \phi_0^G + i_1.\phi_1^G + i_2.\phi_2^G + i_3.\phi_3^G + i_4.\phi_4^G + i_5.\phi_5^G + i_6.\phi_6^G + i_7.\phi_7^G \\ \phi_0^R + i_1.\phi_1^R + i_2.\phi_2^R + i_3.\phi_3^R + i_4.\phi_4^R + i_5.\phi_5^R + i_6.\phi_6^R + i_7.\phi_7^R \\ \phi_0^F + i_1.\phi_1^F + i_2.\phi_2^F + i_3.\phi_3^F + i_4.\phi_4^F + i_5.\phi_5^F + i_6.\phi_6^F + i_7.\phi_7^F \\ \phi_0^S + i_1.\phi_1^S + i_2.\phi_2^S + i_3.\phi_3^S + i_4.\phi_4^S + i_5.\phi_5^S + i_6.\phi_6^S + i_7.\phi_7^S \\ \phi_0^O + i_1.\phi_1^O + i_2.\phi_2^O + i_3.\phi_3^O + i_4.\phi_4^O + i_5.\phi_5^O + i_6.\phi_6^O + i_7.\phi_7^O \end{bmatrix}$$



c...speed of light
G...Gravitationconstant
ħ...Planckconstant

symmetric Curvature Tensor

$$C_{em} = \frac{c}{G.\hbar}. \begin{bmatrix} \phi_0^O.\phi_0^O & i_1.\phi_0^R.\phi_1^G & i_2.\phi_0^F.\phi_2^G & i_3.\phi_0^S.\phi_3^G \\ i_1.\phi_0^R.\phi_1^G & -\phi_1^O.\phi_1^O & i_3.\phi_1^F.\phi_2^R & -i_2.\phi_1^S.\phi_3^R \\ i_2.\phi_0^F.\phi_2^G & i_3.\phi_1^F.\phi_2^R & -\phi_2^O.\phi_2^O & i_1.\phi_2^S.\phi_3^F \\ i_3.\phi_0^S.\phi_3^G & -i_2.\phi_1^S.\phi_3^R & i_1.\phi_2^S.\phi_3^F & -\phi_3^O.\phi_3^O \end{bmatrix}$$

10 independent fields.

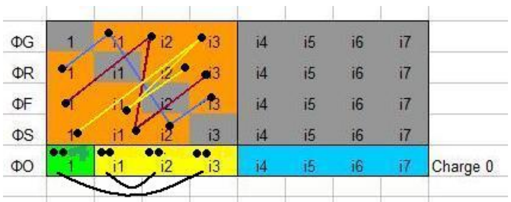
remark :
for $\phi = c$ we get as curvature the plank curvature
which is the reciprocal of the planck area.
The value of the curvature is :

$$0,34 \times 10^{70} \ \frac{1}{m^2}$$

Second CURVATURE TENSOR from the Oktoquintenfield (generates a spinpotential)

The construction comes from multiplications by 4 degrees of freedome (quaternionic subspaces).
With this construction the tensor is symmetric in both diagonals.

$$\phi = \begin{bmatrix} \phi^G \\ \phi^R \\ \phi^F \\ \phi^S \\ \phi^O \end{bmatrix} = \begin{bmatrix} \phi_0^G + i_1.\phi_1^G + i_2.\phi_2^G + i_3.\phi_3^G + i_4.\phi_4^G + i_5.\phi_5^G + i_6.\phi_6^G + i_7.\phi_7^G \\ \phi_0^R + i_1.\phi_1^R + i_2.\phi_2^R + i_3.\phi_3^R + i_4.\phi_4^R + i_5.\phi_5^R + i_6.\phi_6^R + i_7.\phi_7^R \\ \phi_0^F + i_1.\phi_1^F + i_2.\phi_2^F + i_3.\phi_3^F + i_4.\phi_4^F + i_5.\phi_5^F + i_6.\phi_6^F + i_7.\phi_7^F \\ \phi_0^S + i_1.\phi_1^S + i_2.\phi_2^S + i_3.\phi_3^S + i_4.\phi_4^S + i_5.\phi_5^S + i_6.\phi_6^S + i_7.\phi_7^S \\ \phi_0^O + i_1.\phi_1^O + i_2.\phi_2^O + i_3.\phi_3^O + i_4.\phi_4^O + i_5.\phi_5^O + i_6.\phi_6^O + i_7.\phi_7^O \end{bmatrix}$$



$$C_{spin} = \frac{1}{G.\hbar.c}. \begin{bmatrix} -\phi_0^O.\phi_0^O.\phi_3^O.\phi_3^O & -C & B & -A \\ -C & \phi_1^O.\phi_1^O.\phi_2^O.\phi_2^O & -A & B \\ B & -A & \phi_1^O.\phi_1^O.\phi_2^O.\phi_2^O & -C \\ -A & B & -C & -\phi_0^O.\phi_0^O.\phi_3^O.\phi_3^O \end{bmatrix}$$

$$A = \phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R \quad B = \phi_0^F.\phi_2^G.\phi_1^S.\phi_3^R \quad C = \phi_0^R.\phi_1^G.\phi_2^S.\phi_3^F$$

5 independent fields A,B,C and two in the diagonal (blue and yellow).

So finally we get three derivation- or curvaturetensors of the Oktoquintenpotential for twisted spacetime excitation

0-th Curvaturetensor

ϕ_0^O	$i_1.\phi_1^G$	$i_2.\phi_2^G$	$i_3.\phi_3^G$
ϕ_0^R	$i_1.\phi_1^O$	$i_2.\phi_2^R$	$i_3.\phi_3^R$
ϕ_0^F	$i_1.\phi_1^F$	$i_2.\phi_2^O$	$i_3.\phi_3^F$
ϕ_0^S	$i_1.\phi_1^S$	$i_2.\phi_2^S$	$i_3.\phi_3^O$

ϕ_i unit is speed m/s

1-Curvaturetensor Cem em = energy-momentum

$\phi_0^O\cdot\phi_0^O$	$i_1.\phi_0^R.\phi_1^G$	$i_2.\phi_0^F.\phi_2^G$	$i_3.\phi_0^S.\phi_3^G$
$i_1.\phi_0^R.\phi_1^G$	$-\phi_1^O\cdot\phi_1^O$	$i_3.\phi_1^F.\phi_2^R$	$-i_2.\phi_1^S.\phi_3^R$
$i_2.\phi_0^F.\phi_2^G$	$i_3.\phi_1^F.\phi_2^R$	$-\phi_2^O.\phi_2^O$	$i_1.\phi_2^S.\phi_3^F$
$i_3.\phi_0^S.\phi_3^G$	$-i_2.\phi_1^S.\phi_3^R$	$i_1.\phi_2^S.\phi_3^F$	$-\phi_3^O\cdot\phi_3^O$

This tensor is up to a constant equal to the energy-momentum tensor.

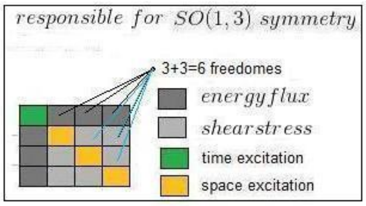
$Energydensity:$

$$\frac{E_{i,j}}{m^3} = \phi_i.\phi_j.\frac{c}{G.\hbar}.\frac{c^4}{8.\pi.G} = \phi_i.\phi_j.\frac{c^2}{K.l_p^2}$$

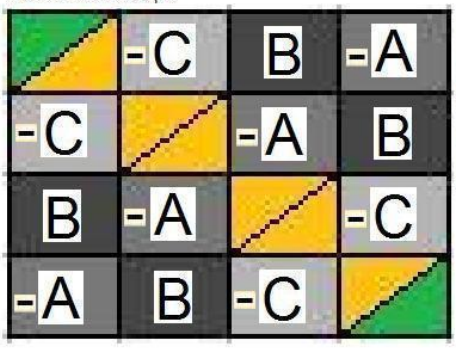
c...speed of light
G...Gravitationconstant
$\hbar$...Planckconstant
K..Einsteinconstant
l_p^2...Planckarea

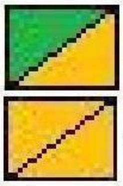
the vacuumexcitation :

$$\phi_i^{o2} = \Lambda.\frac{G.\hbar}{c} \quad i = 0,1,2,3$$



2-Curvaturetensor Cspin





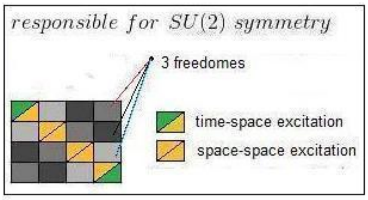
$= -\phi_0^O.\phi_0^O.\phi_3^O.\phi_3^O$

$= \phi_1^O.\phi_1^O.\phi_2^O.\phi_2^O$

$A = \phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R$

$B = \phi_0^F.\phi_2^G.\phi_1^S.\phi_3^R$

$C = \phi_0^R.\phi_1^G.\phi_2^S.\phi_3^F$



<6> Extension of the ART by the second curvaturetensor

The Oktoquintenpotential has two symmetric curvaturetensors.This motivates us to extend the Einstein Equation.

$$Vacuumsymmetry = \begin{pmatrix} SO(1,3) \\ SU(2) \end{pmatrix} = \begin{pmatrix} rotations, boosts \\ spin \end{pmatrix} \qquad \begin{pmatrix} real\ energydensity \\ imaginaer\ energydensity \end{pmatrix}$$

I think this shows that the GR (General Relativity) has to be extended by an imaginäry part (spinpart) to be a consistent quantumtheorie.

So finally we expect something like **GR+i.GR°** where GR° is the spinpart.

with the two curvaturetensors C_{em} *and* C_{spin} *we can define following equation :*

$$Real(C_{em}) + \frac{i}{\varphi}.C_{spin} = \frac{8.\pi.G}{c^4}.(T_{\mu\nu} + \frac{i}{\varphi}.S_{\mu\nu})$$

where the real part is the GR and the imaginaer part is GR^O

GR...General Relativity
GR^O...Spinextension of GR
φ...golden ratio

the operator Real(A) is defined by

$$Real\left(\begin{pmatrix} a_{0,0} & i_1.a_{0,1} & i_2.a_{0,2} & i_3.a_{0,3} \\ i_1.a_{1,0} & a_{1,1} & i_3.a_{1,2} & i_2.a_{1,3} \\ i_2.a_{2,0} & i_3.a_{2,1} & a_{2,2} & i_1.a_{2,3} \\ i_3.a_{3,0} & i_2.a_{3,1} & i_1 2.a_{3,2} & a_{3,3} \end{pmatrix}\right) = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}$$

the reversing $Real^{-1}$ *is :*

$$Real^{-1}\left(\begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}\right) = \begin{pmatrix} a_{0,0} & i_1.a_{0,1} & i_2.a_{0,2} & i_3.a_{0,3} \\ i_1.a_{1,0} & a_{1,1} & i_3.a_{1,2} & i_2.a_{1,3} \\ i_2.a_{2,0} & i_3.a_{2,1} & a_{2,2} & i_1.a_{2,3} \\ i_3.a_{3,0} & i_2.a_{3,1} & i_1 2.a_{3,2} & a_{3,3} \end{pmatrix}$$

i_1, i_2, i_3*...imaginaer quaternions*

more detailed with the two curvaturetensors of the oktoquintenfield:

$$\frac{8.\pi.G}{c^4}.(T_{\mu\nu} + \frac{i}{\varphi}.S_{\mu\nu}) = Real(C_{em}) + \frac{i}{\varphi}.C_{spin} =$$

$$= \frac{c}{G.\hbar}.\left[\begin{pmatrix} \phi_0^0.\phi_0^0 & \phi_0^R.\phi_1^G & \phi_0^F.\phi_2^G & \phi_0^S.\phi_3^G \\ sym. & -\phi_1^0.\phi_1^0 & \phi_1^F.\phi_2^R & -\phi_1^S.\phi_3^R \\ sym. & sym. & -\phi_2^0.\phi_2^0 & \phi_2^S.\phi_3^F \\ sym. & sym. & sym. & -\phi_3^0.\phi_3^0 \end{pmatrix} + \frac{i}{c^2\varphi}.\begin{pmatrix} -\phi_0^0.\phi_0^0.\phi_3^0.\phi_3^0 & -\phi_0^R.\phi_1^G.\phi_2^S.\phi_3^F & \phi_0^F.\phi_2^G.\phi_1^S.\phi_3^R & -\phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R \\ sym. & \phi_1^0.\phi_1^0.\phi_2^0.\phi_2^0 & -\phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R & \phi_0^F.\phi_2^G.\phi_1^S.\phi_3^R \\ sym. & sym. & \phi_1^0.\phi_1^0.\phi_2^0.\phi_2^0 & -\phi_0^R.\phi_1^G.\phi_2^S.\phi_3^F \\ sym. & sym. & sym. & -\phi_0^0.\phi_0^0.\phi_3^0.\phi_3^0 \end{pmatrix}\right]$$

10 different products — *generates Poincare group* $\mathbb{R}^{1,3} \rtimes O(1,3)$

5 different products; *sym.* and the red products are redundant — $2 \times S^1 \times S^1 \times SU(2) = 2 \times \mathbb{T}^2 \times SU(2)$ *with* $\mathbb{T}^2 = Torus$

$\mathbb{T}^2$*...Clifford − Torus*
This flat torus is a subset of the unit 3 − sphere S^3.
The Clifford torus divides the 3 − sphere into two congruent solid tori.
The Clifford − Torus embedded in S^3 *becomes a minimal surface.*

The second curvaturetensor Cspin is determinded by the first curevaturetensor Cem because its components are a mix of the components of Cem.

The vacuumpart of the extended Einstein equation then is:

$$Vacuum = \Lambda.g + i.\Lambda^o.g^o$$

Λ*...cosmological constant*
Λ^o*...second cosmological constant*
Λ *and* Λ^o *comes from the Oktoquintenpotential (see picture).*
$g = \eta$ *comes from the first curvaturetensor*
$g^o = \eta^o$ *comes from the second curvaturetensor*

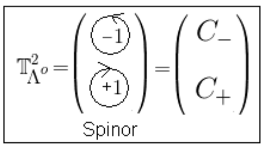
then

hyperbolic (1,3) — ultrahyperbolic (2,2)

$$Vacuum\ Energydensity = \frac{c^4}{8\pi G}.[\Lambda.\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + i.\Lambda^o.\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}]$$

$\begin{pmatrix} (-1) \\ (+1) \end{pmatrix}$ $\mathbb{T}^2_{up}$; $\begin{pmatrix} (+1) \\ (-1) \end{pmatrix}$ $\mathbb{T}^2_{down}$

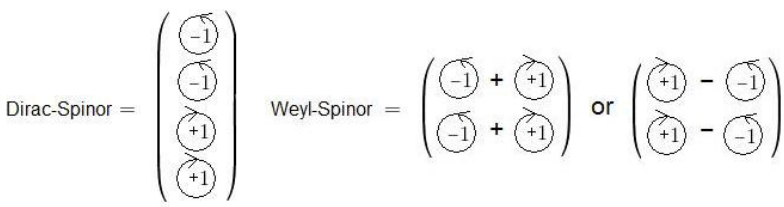
generates flat expanding spacetime = *generates 2 × spinning Torus* $\mathbb{T}^2_{\Lambda^o} = S^1 \times S^1$ *flat Clifford − Torus*

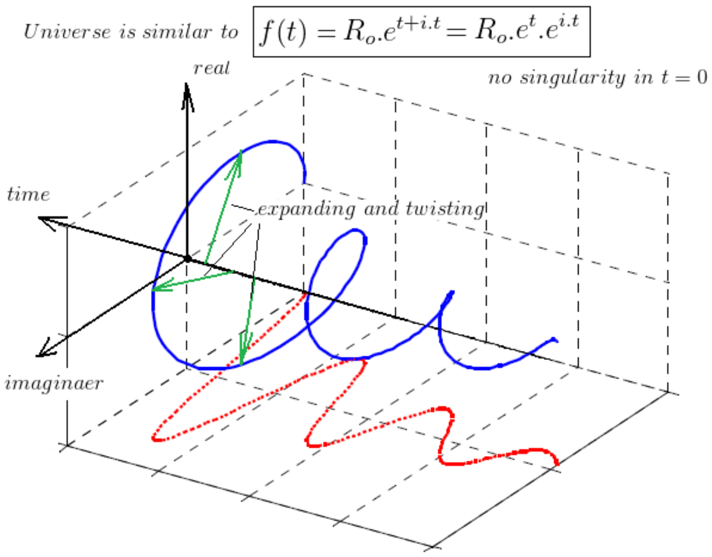


generates flat twisting and expanding vacuum

Hint : Λ^o *comes from the third part of the Oktoquintenpotential.*

$\Lambda^o = \frac{\Lambda}{\varphi}$ *with* φ*...golden ratio*





The question now is how does the spin of particles act on the second curvaturetensor?
The second curvaturetensor has 5 different exciteable values.
D_o, D_i *(D outer, D inner) in the diagonal and* A, B, C *out off the diagonal.*
A, B, C *appears 4 times in the tensor and* D_o, D_i *2 times.*
$16\ components = 4.A + 4.B + 4.C + 2.D_o + 2.D_i$
Now we want to assign the fields A, B, C, D_o, D_i *to the different spins of particles* $0, \frac{1}{2}, 1, \frac{3}{2}, 2.$

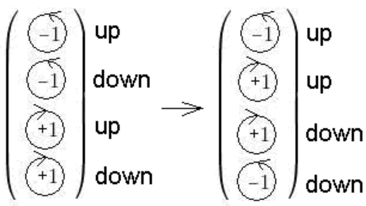
$-D_o$	$-C$	B	$-A$
$-C$	D_i	$-A$	B
B	$-A$	D_i	$-C$
$-A$	B	$-C$	$-D_o$

Example free Diracparticle

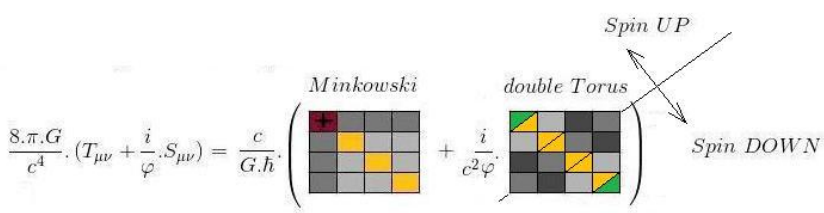
First we want to rearrange the Diracspinor by a matrix.
This makes the relation between Diracparticles like electrons and the curvaturetensors better visible.

$$\psi_D^G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} .\psi_D$$

Dirac-Spinor =



<6.1> The complete curvature Tensor for a free Spin 0 particle:



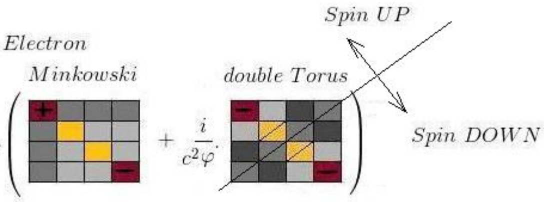
$$= \frac{c}{G.\hbar}.\left[\begin{pmatrix} \phi_0^{0\,2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \frac{i}{c^2\varphi}.\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\right]$$

excited
φ*...golden ratio*

<6.2> The complete curvature Tensor for a free standstill Electron is:

Curvature by an not moving Electron

$$\frac{8.\pi.G}{c^4}.\left(T_{\mu\nu}+\frac{i}{\varphi}.S_{\mu\nu}\right)=\frac{c}{G.\hbar}.\left(\text{Minkowski} + \frac{i}{c^2\varphi}.\ \text{double Torus} \right)$$



$$=\frac{c}{G.\hbar}.\left[\begin{pmatrix}\phi_0^{0\,2} & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -\phi_3^{0\,2}\end{pmatrix}+\frac{i}{c^2\varphi}.\begin{pmatrix}-\phi_0^{0\,2}.\phi_3^{0\,2} & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -\phi_0^{0\,2}.\phi_3^{0\,2}\end{pmatrix}\right]$$

pressure on a random spacedirection z

excited

φ...golden ratio

Our experience with electrons say us that they have a attractive gravity.
To get a attractive gravity we need a positiv pressure.
To get a positiv pressure we are forced to let ϕ_j^0 be imaginaer.

$\phi_j^0 = i.\omega_j^0$ *and* $\phi_0^0=\omega_0^0$

for $j=1,2,3$

we get the Spacetime and Torus curvature by an electron

$$\frac{8.\pi.G}{c^4}.\left(T_{\mu\nu}+\frac{i}{\varphi}.S_{\mu\nu}\right)=\frac{c}{G.\hbar}.\left[\begin{pmatrix}\omega_0^{0\,2} & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \omega_3^{0\,2}\end{pmatrix}+\frac{i}{c^2\varphi}.\begin{pmatrix}\omega_0^{0\,2}.\omega_3^{0\,2} & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \omega_0^{0\,2}.\omega_3^{0\,2}\end{pmatrix}\right]$$

positive pressure on a random spacedirection z
ω real

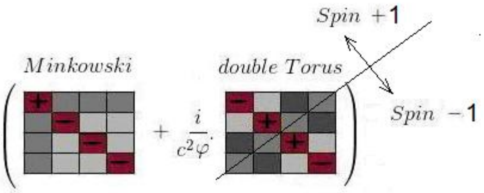
the Spacetime and Torus curvature by a positron is

$$\frac{8.\pi.G}{c^4}.\left(T_{\mu\nu}+\frac{i}{\varphi}.S_{\mu\nu}\right)=\frac{c}{G.\hbar}.\left[\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & \omega_1^{0\,2} & 0 & 0\\ 0 & 0 & \omega_2^{0\,2} & 0\\ 0 & 0 & 0 & 0\end{pmatrix}+\frac{i}{c^2\varphi}.\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & \omega_1^{0\,2}.\omega_2^{0\,2} & 0 & 0\\ 0 & 0 & \omega_1^{0\,2}.\omega_2^{0\,2} & 0\\ 0 & 0 & 0 & 0\end{pmatrix}\right]$$

positive pressure
therefore a positron is falling down on earth

<6.3> The complete curvature Tensor for a Photon (massless vectorboson) is:

$$\frac{8.\pi.G}{c^4}.\left(T_{\mu\nu}+\frac{i}{\varphi}.S_{\mu\nu}\right)=\frac{c}{G.\hbar}.\left(\text{Minkowski} + \frac{i}{c^2\varphi}.\ \text{double Torus} \right)$$



$$=\frac{c}{G.\hbar}.\left[\begin{pmatrix}\phi_0^{0\,2} & 0 & 0 & 0\\ 0 & -\phi_1^{0\,2} & 0 & 0\\ 0 & 0 & -\phi_2^{0\,2} & 0\\ 0 & 0 & 0 & -\phi_3^{0\,2}\end{pmatrix}+\frac{i}{c^2\varphi}.\begin{pmatrix}-\phi_0^{0\,2}.\phi_3^{0\,2} & 0 & 0 & 0\\ 0 & \phi_1^{0\,2}.\phi_2^{0\,2} & 0 & 0\\ 0 & 0 & \phi_1^{0\,2}.\phi_2^{0\,2} & 0\\ 0 & 0 & 0 & -\phi_0^{0\,2}.\phi_3^{0\,2}\end{pmatrix}\right]$$

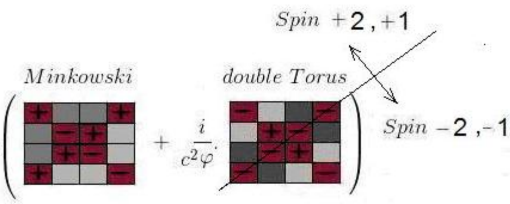
trace=0

excited

φ...golden ratio

<6.4> The complete curvature Tensor for a Graviton (massless tensorboson) is:

$$\frac{8.\pi.G}{c^4}.\left(T_{\mu\nu}+\frac{i}{\varphi}.S_{\mu\nu}\right)=\frac{c}{G.\hbar}.\left(\text{Minkowski} + \frac{i}{c^2\varphi}.\ \text{double Torus} \right)$$



$$=\frac{c}{G.\hbar}.\left[\begin{pmatrix}\phi_0^0.\phi_0^0 & 0 & 0 & \phi_0^S.\phi_3^G\\ 0 & -\phi_1^0.\phi_1^0 & \phi_1^F.\phi_2^R & 0\\ 0 & \phi_1^F.\phi_2^R & -\phi_2^0.\phi_2^0 & 0\\ \phi_0^S.\phi_3^G & 0 & 0 & -\phi_3^0.\phi_3^0\end{pmatrix}+\frac{i}{c^2\varphi}.\begin{pmatrix}-\phi_0^0.\phi_0^0.\phi_3^0.\phi_3^0 & 0 & 0 & -\phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R\\ 0 & \phi_1^0.\phi_1^0.\phi_2^0.\phi_2^0 & -\phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R & 0\\ 0 & -\phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R & \phi_1^0.\phi_1^0.\phi_2^0.\phi_2^0 & 0\\ -\phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R & 0 & 0 & -\phi_0^0.\phi_0^0.\phi_3^0.\phi_3^0\end{pmatrix}\right]$$

trace=0

Getting a closed form for the extended General Relativity EGR.

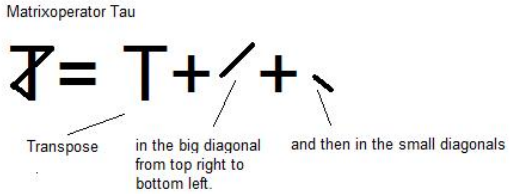
we know that our energy − momentum curvaturetensor

$$Real\left(C_{em}\right) = R_{\mu\nu} - \frac{R}{2}.g_{\mu\nu} + \Lambda.g_{\mu\nu} \quad and\ that$$

C_{spin} *is defined by multiplication of tensorelements of* C_{em}

The question now is how can we express C_{spin} *analogeous to* C_{em} *above as terms of Riemann − Geometrie?*

For that we define the operator for 4 x 4 matrices or tensors:



$$\begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} -> \begin{pmatrix} a_{3,3} & a_{2,3} & a_{1,3} & a_{0,3} \\ a_{3,2} & a_{2,2} & a_{1,2} & a_{0,2} \\ a_{3,1} & a_{2,1} & a_{1,1} & a_{0,1} \\ a_{3,0} & a_{2,0} & a_{1,0} & a_{0,0} \end{pmatrix} -> \begin{pmatrix} a_{3,3} & a_{2,3} & a_{1,3} & a_{1,2} \\ a_{3,2} & a_{2,2} & a_{0,3} & a_{0,2} \\ a_{3,1} & a_{3,0} & a_{1,1} & a_{0,1} \\ a_{2,1} & a_{2,0} & a_{1,0} & a_{0,0} \end{pmatrix}$$

$$A \qquad\qquad\qquad\qquad A^{\tau}$$

and a

special simple matricesmultiplication
$C = A.B$
with

$$c_{i,j} = \begin{cases} +a_{i,j}.b_{i,j} & \text{if } i = j \\ -a_{i,j}.b_{i,j} & \text{if } i \neq j \end{cases}$$

then it is easy to see that

$$(A+B)^{\tau} = A^{\tau} + B^{\tau}$$

with

$$C_{em} = \frac{c}{G.\hbar}.\begin{pmatrix} \phi_0^0.\phi_0^0 & i_1.\phi_0^R.\phi_1^G & i_2.\phi_0^F.\phi_2^G & i_3.\phi_0^S.\phi_3^G \\ sym. & -\phi_1^0.\phi_1^0 & i_3.\phi_1^F.\phi_2^R & -i_2.\phi_1^S.\phi_3^R \\ sym. & sym. & -\phi_2^0.\phi_2^0 & i_1.\phi_2^S.\phi_3^F \\ sym. & sym. & sym. & -\phi_3^0.\phi_3^0 \end{pmatrix}$$

and

$$C_{spin} = \frac{1}{G.\hbar.c}.\begin{pmatrix} -\phi_0^0.\phi_0^0.\phi_3^0.\phi_3^0 & -\phi_0^R.\phi_1^G.\phi_2^S.\phi_3^F & \phi_0^F.\phi_2^G.\phi_1^S.\phi_3^R & -\phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R \\ sym. & \phi_1^0.\phi_1^0.\phi_2^0.\phi_2^0 & -\phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R & \phi_0^F.\phi_2^G.\phi_1^S.\phi_3^R \\ sym. & sym. & \phi_1^0.\phi_1^0.\phi_2^0.\phi_2^0 & -\phi_0^R.\phi_1^G.\phi_2^S.\phi_3^F \\ sym. & sym. & sym. & -\phi_0^0.\phi_0^0.\phi_3^0.\phi_3^0 \end{pmatrix}$$

it follows that

$$\boxed{C_{spin} = \frac{G.\hbar}{c^3}.Real\left(C_{em}\right).Real\left(C_{em}\right)^{\tau} = l_p^2.Real\left(C_{em}\right).Real\left(C_{em}\right)^{\tau} \quad l_p...Plancklength}$$

$with$

$$Real\,(C_{em}) = R_{\mu\nu} - \frac{R}{2}.g_{\mu\nu} + \Lambda.g_{\mu\nu} = G_{\mu\nu} + \Lambda.g_{\mu\nu} = K_{\mu\nu}$$

and

$$Real\,(C_{em}) + \frac{i}{\varphi}.C_{spin} = \frac{8.\pi.G}{c^4}.(T_{\mu\nu} + \frac{i}{\varphi}.S_{\mu\nu})$$

$we\ get\ the\ final\ compact\,result\ for\ the\ extension\ of\ General\ Relativity\ by$

$$\boxed{K_{\mu\nu} + \frac{i}{\varphi}.l_p^2.K_{\mu\nu}.K_{\mu\nu}^{\text{₹}} = \frac{8.\pi.G}{c^4}.(T_{\mu\nu} + \frac{i}{\varphi}.S_{\mu\nu})}$$

$with$

$$S_{\mu\nu} = l_p^2.T_{\mu\nu}.T_{\mu\nu}^{\text{₹}}...Spintensor$$

$$K_{\mu\nu} = R_{\mu\nu} - \frac{R}{2}.g_{\mu\nu} + \Lambda.g_{\mu\nu}$$

$$K_{\mu\nu}^{\text{₹}} = R_{\mu\nu}^{\text{₹}} - \frac{R}{2}.g_{\mu\nu}^{\text{₹}} + \Lambda.g_{\mu\nu}^{\text{₹}}$$

$\varphi...golden\ ratio$

The real part is the known General Relativity.
The imaginaer part is the Spinextension of GR.

Hint : The Energy–Stress tensor is still symmetric with or without Spin!

The question now is what is it good for?
In the same way as energydensity, momentumdensity aso. warps spacetime spin or spindensity warps a $\mathbb{T}^2$ *Torus.*

Take care that the multiplication of the tensor is not the normal tensormultiplication or matricesmultiplication.
It is the above defined simple multiplication Cij=+/- Aij.Bij.

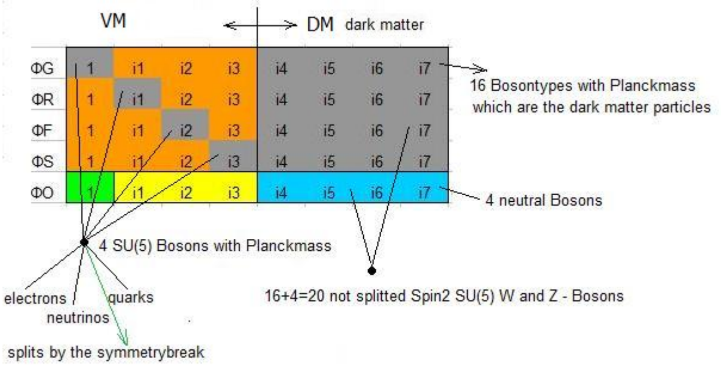
<7> Candidates for the dark matter in the universe.

The Oktoquintefield can be divided into 2 areas (left and right).
The left one has 4 SU(5) Bosons which are over the timespace (curvature) fields.

I suppose that the Planckparticles with planckmass come into being by the Symmetriebreak **SU(5) x U(1) x U(1) -->U(1)** and the decomposition (Protons,electrons,...) of it comes from the other Symmetriebreak **SU(2)xU(1) --> U(1)** which only acts on the left half of the Oktoquintenfield.

I suppose that the 4 W-Bosons in the left half (4 of the 20 charged SU(5) Bosons) split into protons,electrons and so on and the other SU(5) Bosons in the right half keep planckparticles.

So the left W-Bosons are the reason for the visible matter and the right W- and Z-Bosons are the reason vor the dark matter.



As seen in **<4>** we get a particle Lambda which is also a additional candidate for dark matter!

<8> Some important points of the Oktoquintenpotential

To get the maxima, minima and the zeropoints of the potential we have to substitute $z = \phi^2$ *it is enough (because of symmetry) to take a look on the positive* ϕ'*s.*
and solve the cubic equations in the bracket

$$V(\sqrt{z}) = z.(\frac{\gamma^2}{2} + \frac{\mu^2}{4}z + \frac{\lambda^2}{8}z^3) \text{ and}$$

$$V'(\sqrt{z}) = \sqrt{z}.(\gamma^2 + \mu^2.z + \lambda^2.z^3)$$

We will make it short and write the results.
First the Zeropoints :

$$z_1 = u + v = -\sqrt{\frac{2C}{3}}(\sqrt[3]{\frac{\sqrt{27} - i.\sqrt{5}}{\sqrt{32}}} + \sqrt[3]{\frac{\sqrt{27} + i.\sqrt{5}}{\sqrt{32}}})$$
$$z_2 = \epsilon_1.u + \epsilon_2.v$$
$$z_3 = \epsilon_2.u + \epsilon_1.v$$

$C = c^4.\varphi^2$
c...*speed of light*
φ...*golden ratio*

Where $\epsilon_1 = -\frac{1}{2} + i.\frac{\sqrt{3}}{2}$ *and* $\epsilon_2 = -\frac{1}{2} - i.\frac{\sqrt{3}}{2}$
then

$$z_1 = -0,990839414 \times 2.\sqrt{\frac{2C}{3}}$$

$$z_2 = 0,378466979 \times 2.\sqrt{\frac{2C}{3}}$$

$$z_3 = 0,612372435 \times 2.\sqrt{\frac{2C}{3}}$$

then the zeropoints are

$$\phi_1 = -0,995409169 \times \sqrt[4]{\frac{8.C}{3}}$$

$$\phi_2 = 0,615196699 \times \sqrt[4]{\frac{8.C}{3}}$$

$$\phi_3 = 0,782542290 \times \sqrt[4]{\frac{8.C}{3}}$$

$$\phi_2 = c = 0,615196699 \times \sqrt[4]{\frac{8.C}{3}} = sin(37,966214178) \times \sqrt[4]{\frac{8.C}{3}}$$

$$\alpha_c = 37,966214178^o$$

$$\phi_3 = c.\sqrt{\varphi} = 0,78254229 \times \sqrt[4]{\frac{8.C}{3}} = sin(128,506061932) \times \sqrt[4]{\frac{8.C}{3}}$$

$$\alpha_{c.\sqrt{\varphi}} = 128,506061932^o$$

Then the Maxima and the Minima :

$$z_1 = u + v = -\sqrt{\frac{C}{3}}.(\sqrt[3]{\frac{\sqrt{37} - i.\sqrt{27}}{\sqrt{64}}} + \sqrt[3]{\frac{\sqrt{37} + i.\sqrt{27}}{\sqrt{64}}})$$
$$z_2 = \epsilon_1.u + \epsilon_2.v$$
$$z_3 = \epsilon_2.u + \epsilon_1.v$$

Finally we have two positiv results :

$$z_{min} = 0,233475630 \times 2.\sqrt{\frac{C}{3}} \text{ and}$$

$$z_{max} = 0,725352944 \times 2.\sqrt{\frac{C}{3}}$$

and one negative
$$z_3 = -(z_{max} + z_{min})$$

Then because of $z = \phi^2$

$$\phi_{min} = 0,483193160 \times \sqrt[4]{\frac{4.C}{3}} \text{ and}$$

$$\phi_{max} = 0,8516765489 \times \sqrt[4]{\frac{4.C}{3}}$$

In cubic equations the real zeropoints comes from the $cos(\alpha)$ *or from* $sin(90-\alpha)$ *of angles (see https://en.wikipedia.org/wiki/Cubic_function).*

Then for ϕ_{min} *we get an angle* α_{min} *:*

$$\phi_{min} = 0,483193160 \times \sqrt[4]{\frac{4.C}{3}} = sin(28,894160846) \times \sqrt[4]{\frac{4.C}{3}}$$

$\alpha_{min} = 28,894160846$ *degrees is very near to the Weinbergangle*

$$sin^2(\alpha_{min}) = sin^2(28,894160846) = 0,233475630$$

and for ϕ_{max} *we get an angle*

$$\phi_{max} = 0,8516765489 \times \sqrt[4]{\frac{4.C}{3}} = sin(121,605508985) \times \sqrt[4]{\frac{4.C}{3}}$$

$\alpha_{max} = 121,605508985$ *degrees*

$\phi_{min} = 0,483193160 \times \sqrt[4]{\frac{4.C}{3}}$ *and*

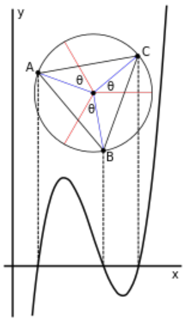
$\phi_{max} = 0,8516765489 \times \sqrt[4]{\frac{4.C}{3}}$

In cubic equations the real zeropoints comes from the $cos(\alpha)$ *or from* $sin(90-\alpha)$ *of angles (see https://en.wikipedia.org/wiki/Cubic_function).*

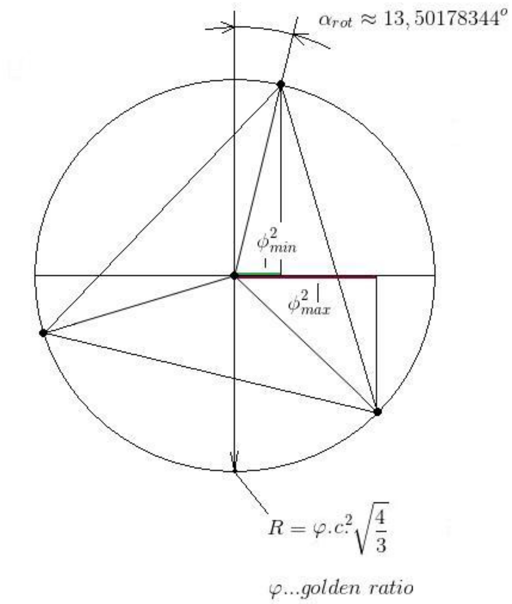
Our second extreme value L_1 *is at* e_1
With the relation above we can calculate the third extreme value L_2.

$\phi_{max} = 1,762600... \times e_1$

Geometric interpretation of the roots (zeropoints) in cubic equations with 3 real zeropoints



graphical zeropoints of the derivation of the (radicaled $\phi^2 = z$*) Oktoquintenpotential*



Conclusions

Dark Energy comes from the Oktoquintenpotential (the second term in the potential).
Dark Matter could be the W , Z Bosons of the SU(5) Symmetry and the Lambda Boson from the mixing.