INVERTED DYNAMICS

Accepted for publication in *Physics Essays*, Vol. **29**, No. **1** (March 2016)
http://www.ingentaconnect.com/content/pe/pe

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Abstract: In the field of faster-than-light motions a new theory is introduced: the inverted dynamics. As we will see, this theory is confirmed by the experiments MINOS, OPERA and Fermilab79. More precisely, in its framework, these three experiments give the same result for the muonic neutrino rest mass in the limit of the measurement uncertainties. This result is also well below the highest probable value of 170 keV experimentally determined by Assamagan. Furthermore, by the inverted dynamics, we are able to explain classically the tunnelling effect.

Keywords: *Superluminal Motions; Dynamics; Special Relativity; Tachyon; Tunneling Effect.*

1 Introduction

We all know that special relativity holds for velocities from 0 to the speed of light c, but beyond that limit it does not apply. According to the mainstream interpretation this fact means that any massive object cannot go faster than light. This is a reasonable point of view, anyway it is not the only one. Indeed, in some literature, see for example ref. [1], the particles[b] are divided into bradyons and tachyons. Bradyons are particles which travel slower than light and obey to special relativity, as tachyons are particles which travel faster than light[c]. At this point, what theory holds for tachyons? Again in ref. [1] you can see for example extended relativity.

In this work you will find another new theory for tachyons: the inverted dynamics. The laws and principles of inverted dynamics will be shown in section 2. In particular, we will introduce the dependence of the tachyonic inertial mass and linear momentum on the particle rest mass and superluminal speed. Then, from these two physical quantities and Newton's Second Law, we will derive the expression of the tachyonic total energy showing that the mass-energy equivalence in the superluminal world holds again. At the end of the paragraph we will give some simple examples indicating that every particle in the tachyonic state ($v > c$) tends to be teleported and so a tachyon can be defined as a particle which is experiencing the teleportation.

In section 3 we present some experimental confirmations of this theory. We will begin considering MINOS and OPERA[d]. In these two experiments the total energy and the velocity of superluminal muonic

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[b] From now on the term particle will mean any massive object.

[c] Commonly the term tachyon means a spacelike particle. More precisely, a spacelike particle is an object travelling faster than light which follows special relativity. Then, for this reason, a spacelike particle has imaginary mass and it is X-shaped.

To avoid misunderstandings, in inverted dynamics the term tachyon will have a different meaning. In fact, in this theoretical framework, the term tachyon means a pointlike particle (we are not interested in its shape and internal structure) with real mass which travels faster than light. Moreover, inverted dynamics describes the behavior of tachyons in our common laboratory frame and so, for this basic mechanical theory, the invariance or not of the speed of light even in a superluminal inertial reference frame has no consequence. Further considerations on this latter topic will be given in the conclusions.

[d] We will use the first version of the data released on the 22nd of September 2011.

neutrinos have been measured. From these data, using the inverted dynamics total energy equation, we will derive the value of the muonic neutrino rest mass and we will see that the measurements at five different energies give the same result in the limit of the experimental uncertainties or, in other words, all the five measurements are compatible with one another. We will also verify that the obtained value is in very good agreement with the experimentally known one. More precisely the measurements of MINOS and OPERA give for the muonic neutrino rest mass the value 6.7 ± 2.3 eV which is well below the maximum probable value of 170 keV measured by Assamagan et al. observing subluminal particles. Then we will deal with another experimental result: the tunnelling effect. This phenomenon is explained in the framework of quantum mechanics, but it cannot be explained using classical mechanics because, according to this latter one, negative values for the kinetic energy do not have any physical meaning. On the other hand, inverted dynamics foresees positive and negative values for the kinetic energy and, since they are functions of experimentally measurable quantities, it follows that they also have physical meaning. Now if we consider inverted dynamics as an extension in the superluminal world of Newton's physics, we see that the tunnelling effect is explained classically. In other words, a particle can stay under barrier travelling, in this way, faster than light. The superluminal speed of an α particle during its fly under ${}^{252}_{98}\mathrm{Cf}$ barrier will be calculated along with the time of tunnelling. Furthermore, since the α particle inside and outside the nucleus is subluminal and in the border region (under the barrier) is superluminal, the tunnelling effect provides an evidence of the fact that in nature it is possible to convert a bradyon into a tachyon and vice versa without any expense of energy. So, the tunnelling effect gives us an experimental example of a natural tachyonic converter. Then, following this way, we will suppose that it is possible to convert a starship at rest into a tachyon and we will study how the gravity affects the starship superluminal motion. This very ideal example has been included because it shows a very different picture with respect to what happens to normal rockets.

In section 3.3 we will consider Fermilab79. In this experiment the total energy and speed of muonic neutrinos have been measured. That is the same of MINOS and OPERA, but these data are presented separately because both subluminal and superluminal muonic neutrinos have been observed in this case. That allows to compare directly the predictions of special relativity and inverted dynamics. In fact, we will derive the muonic neutrino rest mass by the total energy equation of special relativity (the subluminal mass-energy equivalence) and, as we did for MINOS and OPERA, by the total energy equation of inverted dynamics (the superluminal mass-energy equivalence). We will see that the values given by the two theories are in very good agreement in the limit of the experimental uncertainties. Furthermore, we will see that they are also in very good agreement with the values derived from the data of MINOS and OPERA, again in the limit of the experimental uncertainties, and on top of that with the measurements of Assamagan.

In section 4 we will conclude with some considerations on what could be seen by an observer in the tachyonic state. More precisely, by a simple example and the experiment of Ayres et al. which has verified the validity of the time dilation equation of special relativity, we will see that we cannot say anything on what could happen to the spacetime of a teleported observer. In particular, having a lack of experimental data, we cannot say if the speed of light c stays invariant in a superluminal inertial reference frame as well. For this reason, inverted dynamics has been developed in a single inertial reference frame, our laboratory frame.

At the moment all the options are open.

2 Inverted dynamics: the teleportation theory

We consider a particle in uniform rectilinear motion in an inertial reference frame R, in practice our common laboratory frame. For simplicity, without loss of generality, we suppose that the particle moves parallel to the x axis of R and heads in the same direction as shown in figure 1.

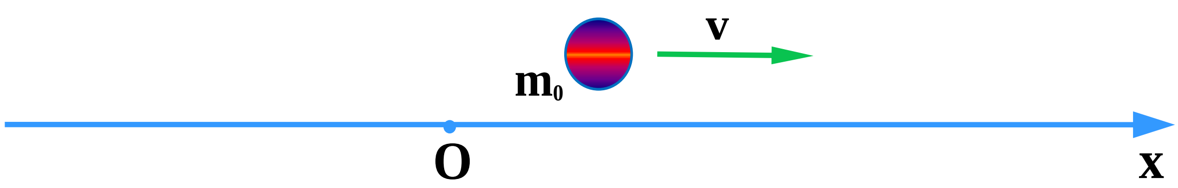

Figure 1 - *A particle of rest mass m_0 moving with constant speed v along the x axis of the laboratory reference frame.*

If the particle moves slower than light, then we will use to describe its dynamical behavior special relativity.
Instead if the particle moves faster than light, we will use the inverted dynamics to describe its dynamical behavior. According to this theory, by analogy with the bradyonic case, it is assumed that the mass of a superluminal particle depends on its velocity. More precisely, see Appendix A, in inverted dynamics it is supposed that the inertial mass of a tachyon that moves at constant speed $v > c$ is:

$$m = \frac{m_0 c^2}{(v-c)^2} \tag{1}$$

where m_0 is the particle rest mass.
Moreover, by analogy with the bradyonic case, it is also assumed that the linear momentum of a superluminal particle depends on its inertial mass and velocity. More precisely, in inverted dynamics it is supposed that the linear momentum of a tachyon that moves at constant speed $v > c$ is:

$$p = \frac{mc^2}{v-c} \tag{2}$$

Assuming the validity of Newton's Second Law even in the superluminal world, from equation (2) we have:

$$F = \frac{dp}{dt} = \frac{d}{dt}\left(\frac{mc^2}{v-c}\right) \tag{3}$$

where F is the force acting on the tachyon.
Now in inverted dynamics the infinitesimal work done by a force is defined as:

$$dL = F\,(dr - dr_c) \tag{4}$$

In fact, given an infinitesimal interval of time dt, the minimum length that superluminal particles can cover is $dr_c = c\,dt$ because c is the greatest lower bound for the allowed velocities tachyons can have. Conversely, the minimum length that subluminal particles can cover is $dr_0 = 0\,dt = 0$ because zero is the minimum velocity bradyons can have. So, in this latter case, the infinitesimal work done by a force is $dL = F\,(dr - dr_0) = F\,dr$, that is the well-known expression used in subluminal physics.
Then the power developed by a force is:

$$P = \frac{dL}{dt} = F\left(\frac{dr - dr_c}{dt}\right) = F\left(\frac{v\,dt - c\,dt}{dt}\right) = F\,(v - c)$$

or, from equation (3):

$$P = F\,(v - c) = \frac{d}{dt}\left(\frac{mc^2}{v - c}\right)(v - c)$$

Carrying out the calculations, after the substitution of relation (1) for the tachyonic inertial mass, we have:

$$P = \frac{d}{dt}\left(\frac{mc^2}{v - c}\right)(v - c) = \frac{d}{dt}\left[\frac{m_0 c^4}{(v - c)^3}\right](v - c) =$$

$$= -3\,\frac{m_0 c^4}{(v - c)^4}\,(v - c)\,\frac{dv}{dt} = -3\,\frac{m_0 c^4}{(v - c)^3}\,\frac{dv}{dt}$$

At this point, multiplying the above equation for the interval of time dt, we have that the infinitesimal work done by a force on a superluminal particle can also be written as:

$$dL = P\,dt = -3\,\frac{m_0 c^4}{(v - c)^3}\,dv$$

Since this last expression is equal to

$$d\left[\frac{3}{2}\,\frac{m_0 c^4}{(v - c)^2}\right] = -\frac{3}{2}\,\frac{2 m_0 c^4}{(v - c)^3}\,dv = -\frac{3 m_0 c^4}{(v - c)^3}\,dv$$

it follows that:

$$dL = d\left[\frac{3}{2}\,\frac{m_0 c^4}{(v - c)^2}\right] \qquad (5)$$

Now, assuming the validity of the conservation principle of energy in the superluminal world as well, we have that the work done on a tachyonic particle by a force has to be equal to its total energy variation. This means that $dL = dE$ and so relation (5) becomes:

$$dE = d\left[\frac{3}{2}\frac{m_0c^4}{(v-c)^2}\right]$$

In conclusion, putting the integration constant equal to zero in order to cover all the positive values[e], the total energy of a superluminal particle has the following form:

$$E = \frac{3}{2}\frac{m_0c^4}{(v-c)^2}$$

Now if we take a look at the trend of the tachyonic inertial mass, linear momentum and total energy, we will understand the reason why this theory has been called inverted dynamics.
For the first thing, relation (1) shows that the inertial mass of a tachyon goes to infinity when the velocity approaches to the speed of light, as it tends to zero when the velocity goes to infinity. This is a reverse behavior with respect to what happens for bradyons.
Indeed, bradyons have the lowest inertial mass when they are at rest, as they become infinitely massive when the velocity approaches to the speed of light. In other words, the bradyonic inertial mass has an increasing trend on the allowed velocity interval [0, c). Conversely the tachyonic inertial mass has a decreasing trend on the allowed velocity interval (c, +∞).
The same is true for the linear momentum. In fact, from special relativity we know that the bradyonic linear momentum has an increasing trend on the allowed velocity interval [0, c), while, see equation (2), the tachyonic linear momentum has a decreasing trend on the allowed velocity interval (c, +∞). Again, the two trends are reversed.
Finally, special relativity says that the bradyonic total energy has an increasing trend on the allowed velocity interval [0, c), as equation (6) of inverted dynamics says that the tachyonic total energy has a decreasing trend on the allowed velocity interval (c, +∞). Also in this case the trends are reversed.
Anyway, there is a similarity as well. In fact, for subluminal particles we have a mass-energy equivalence given by $E = mc^2$, and a similar relation holds for tachyons too. More precisely, using equations (1) and (6), the tachyonic total energy can be written synthetically as $E = 3/2\ mc^2$ giving, in this way, the mass-energy equivalence for superluminal particles. In other words, inertial mass and total energy differ for a multiplicative constant both in subluminal and superluminal world.
The tachyonic total energy trend is displayed in figure 2. From this figure we can see an important physical behavior. If we give more and more energy to a subluminal particle, its speed will rise without ever reaching the speed of light. On the other hand, if we give more and more energy to a superluminal particle, its speed will fall without ever reaching the speed of light again. Subluminal and superluminal states are two separated worlds. Moreover, it is clear that we have to supply energy to increase the speed of a bradyon, as for tachyons we have to take out energy.
Let us note that to increase the bradyonic speed of a very little amount near the speed of light we have to supply a lot of energy. Conversely, we have to take out a very little amount of energy, when tachyons are very far from the speed of light, to obtain a great velocity increase.

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[e] These are all the possible total energy values experimentally measurable.

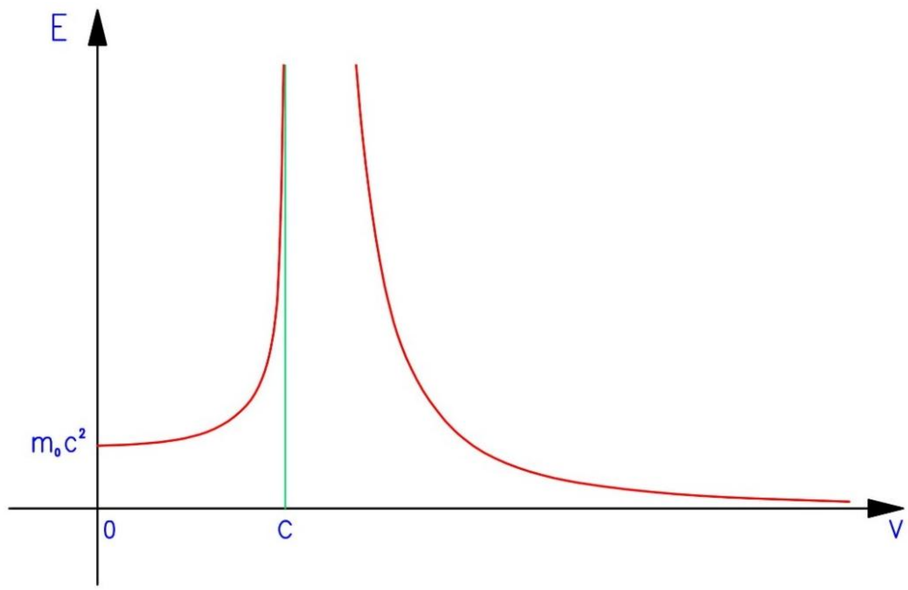

Figure 2 - *The total energy of subluminal and superluminal particles as a function of their velocity* v.

Let us note that to increase the bradyonic speed of a very little amount near the speed of light we have to supply a lot of energy. Conversely, we have to take out a very little amount of energy, when tachyons are very far from the speed of light, to obtain a great velocity increase.
For example, we consider a starship weighing 1000 t which travels at the speed of 290000 km/sec. Since in the subluminal state special relativity rules, the total energy of the starship is:

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1000000 \times 299792458^2}{\sqrt{1-\left(\frac{290000000}{299792458}\right)^2}} \ \mathrm{kg}\ \frac{m^2}{s^2} = 3.5454 \cdot 10^{23}\ \mathrm{J}$$

Now if the starship travels at the speed of 290100 km/sec, its total energy will be:

$$E = \frac{1000000 \times 299792458^2}{\sqrt{1-\left(\frac{290100000}{299792458}\right)^2}} \ \mathrm{kg}\ \frac{m^2}{s^2} = 3.5633 \cdot 10^{23}\ \mathrm{J}$$

Hence to increase the starship speed of 100 km/sec in one second, starting from 290000 km/sec, we have to supply the following power:

$$(3.5633 - 3.5454) \cdot 10^{23}\ \frac{J}{s} = 1.79 \cdot 10^{21}\ \mathrm{W}$$

That is an incredible amount of power: nearly two trillion of GW.
At the speed of 290100 km/sec we can reach Proxima Centauri, which is 4.24 light years far from the Earth, in:

$$t = \frac{4.24 \times 9.4607 \cdot 10^{15}}{290100000}\ \mathrm{ly}\ \frac{m}{\mathrm{ly}}\ \frac{s}{m} = 1.38 \cdot 10^{8}\ \mathrm{s} \cong 4.4\ \mathrm{y}$$

To sum up, in the subluminal world the space trip is very expensive: huge amounts of power and long times of flight to reach, as we have seen in the example above, our nearest star. But in the superluminal world everything is completely different.
For the first thing, by definition, a tachyonic starship travels faster than light. For example, a starship moving at the speed of $1.84 \cdot 10^{16}$ km/s, a speed much greater than the speed of light, can be teleported to the red dwarf star Proxima Centauri in:

$$t = \frac{4.24 \times 9.4607 \cdot 10^{15}}{1.84 \cdot 10^{19}} \ \text{ly} \ \frac{\text{m}}{\text{ly}} \ \frac{\text{s}}{\text{m}} = 2.2 \cdot 10^{-3} \ \text{s}$$

while it will be teleported to Andromeda Galaxy, which is 2.5 million light years far from the Earth, in:

$$t = \frac{2.5 \cdot 10^{6} \times 9.4607 \cdot 10^{15}}{1.84 \cdot 10^{19}} \ \text{ly} \ \frac{\text{m}}{\text{ly}} \ \frac{\text{s}}{\text{m}} = 1285 \ \text{s} \cong 21 \ \text{min}$$

Now from relation (6) we have that the total energy of the starship travelling at the superluminal speed of $1.84 \cdot 10^{16}$ km/s is:

$$E = \frac{3}{2} \frac{m_0 c^4}{(v - c)^2} = \frac{3}{2} \frac{1000000 \times 299792458^4}{(1.84 \cdot 10^{19} - 299792458)^2} \ \text{kg} \ \frac{\text{m}^4}{\text{s}^4} \ \frac{\text{s}^2}{\text{m}^2} \cong 36 \ \text{J}$$

Doing a negative work of nearly 36 J on the starship, this latter one can be left with only $1 \cdot 10^{-10}$ J of total energy. So, spending the power emitted by a common fluorescent tube, 36 W, the starship in one second will reach the following amazing speed obtained from the inversion of relation (6):

$$v = \sqrt{\frac{3}{2} \frac{m_0 c^4}{E}} + c = c \left(\sqrt{\frac{3}{2} \frac{m_0 c^2}{E}} + 1 \right) =$$

$$= 299792458 \times \left(\sqrt{\frac{3}{2} \frac{1000000 \times 299792458^2}{1 \cdot 10^{-10}}} + 1 \right) \frac{\text{m}}{\text{s}} = 1.1 \cdot 10^{22} \ \frac{\text{km}}{\text{s}}$$

In this way the starship will be teleported to Andromeda Galaxy almost immediately. It will take about:

$$t = \frac{2.5 \cdot 10^{6} \times 9.4607 \cdot 10^{15}}{1.1 \cdot 10^{25}} \ \text{ly} \ \frac{\text{m}}{\text{ly}} \ \frac{\text{s}}{\text{m}} = 2.1 \cdot 10^{-3} \ \text{s}$$

Unlike what happens for bradyons, in the superluminal world the space trip is very quick and, if you grant me the term, is surely energy saving. But the 36 watts of power are not essential. We can also travel for free. In fact, we consider a particle at rest: its total energy is $E = m_0 c^2$. Now if the particle is converted into a tachyon, for the conservation principle of the total energy, it will be $E = m_0 c^2$ again, where E has become the tachyonic total energy. Although we have not spent any amount of energy the particle, being in the tachyonic state, now moves faster than light. We will see in section 3.2 an example in nature of conversion from bradyon to tachyon at zero energy expense when we will talk about the tunnelling effect. From the inversion of relation (6), the speed of the converted particle initially at rest is:

$$v = \sqrt{\frac{3}{2}\,\frac{m_0 c^4}{E}} + c = c\left(\sqrt{\frac{3}{2}\,\frac{m_0 c^2}{m_0 c^2}} + 1\right) = c\left(\sqrt{\frac{3}{2}} + 1\right) \tag{7}$$

or, in number,

$$v = 299792458 \times \left(\sqrt{\frac{3}{2}} + 1\right)\ \frac{m}{s} = 666961733\ \frac{m}{s}$$

Equation (7) also shows that the conversion velocity of a particle initially at rest is independent from its mass. So, starting from our bedroom, we will be teleported from Hamburg to Singapore, which are about 10165 km far apart, in:

$$t = \frac{10165000}{666961733}\ m\,\frac{s}{m} = 0.015\ s$$

Then if we want to go to Sydney which is 6315 km far from Singapore, the teleported travel will take:

$$t = \frac{6315000}{666961733}\ m\,\frac{s}{m} = 0.009\ s$$

And finally, we will return in our bedroom in Hamburg, which is 16300 km far from Sydney, in:

$$t = \frac{16300000}{666961733}\ m\,\frac{s}{m} = 0.024\ s$$

In 5 hundredths of a second, we can go around the world by an instant trip.
At the end of this paragraph we note another important thing.
A positive force acting on the tachyon in figure 1 will increase its total energy because the work done on it is positive according to the law $dL = F\,(v - c)\,dt$ that we have seen at the beginning of this paragraph. From relation (6) we see that a total energy increase is equivalent to a tachyonic velocity decrease. This is in agreement with relation (3) where a positive force is equivalent to a tachyonic speed drop. Hence a force acting in the same direction of the particle motion slows the tachyon down. On the other hand, a force acting in the opposite direction, a negative force, speeds the tachyon up for what we have just said. Even in this case the behavior of a particle which follows the inverted dynamics is reversed compared to what happens with normal particles ruled by special relativity. In fact, we must push the tachyon to brake it and inversely we must counteract the tachyon motion to make it faster as we will see, in this latter case, at the end of section 3.2 where the gravity force will be taken into account.
In conclusion, the teleportation is not any sort of device that magically makes things disappear and then reappear instantly in another place far away. The considerations and the very simple examples given above should make it clear that the teleportation is one of the two possible states of the matter: the subluminal state and the superluminal state (the teleportation state). In each state the particles have different features and behaviors because they follow different physical laws: the ones of special relativity for subluminal particles and the ones of inverted dynamics for superluminal particles. In other words, special relativity is the classical mechanical description applied to the subluminal world, as inverted dynamics is the postulated classical mechanical description applied to the superluminal world.

3 Experimental results

3.1 Tachyon hunting: MINOS and OPERA

Like any theory, the inverted dynamics must be experimentally verified. To do that we will use the third version of the data from MINOS experiment, see ref. [2], submitted to arXiv on the 31st of August 2007 and the first version of the data from OPERA experiment, see ref. [3], submitted to arXiv on the 22nd of September 2011 (see Appendix C for some considerations on this data).
In these two experiments muonic neutrinos were detected. Their total energy and the deviation from the expected travel time were measured. In particular the deviation from the expected travel time δt is the difference between the time of flight t_c for muonic neutrinos travelling at the speed of light and the time of flight $t_c - \delta t$ experimentally observed. The results are listed in table 1 where E is the observed muonic neutrino total energy, δt is the deviation from the expected travel time, σ_r is the random error (statistical error) in the best evaluation of δt and σ_s is the systematic error in the evaluation of the same quantity.

EXPERIMENT	E (GeV)	δt (ns)	σ_r (ns)	σ_s (ns)
MINOS	3	126	32	64
OPERA	13.9	53.1	18.8	7.4
OPERA	17.0	60.7	6.9	7.4
OPERA	28.1	60.3	13.1	7.4
OPERA	42.9	67.1	18.2	7.4

Table 1 – *MINOS and OPERA experimental results (see ref. [2] and ref. [3]).*

Now from this data we can derive the muonic neutrino velocity v. More precisely, we will calculate the more useful expression v/c – 1. We have:

$$\frac{v}{c} - 1 = \frac{d}{t_c - \delta t}\frac{t_c}{d} - 1 = \frac{t_c - t_c + \delta t}{t_c - \delta t} = \frac{\delta t}{t_c - \delta t} \tag{8}$$

where d is the distance used for the time of flight measurement. In the case of MINOS experiment, it is $d = 734298.6$ m and $t_c = 2449356$ ns (see ref. [2]). For OPERA experiment, the time of flight is measured over the distance $d = 731278$ m, as t_c is:

$$t_c = \frac{d}{c} = \frac{731278}{299792458}\ \text{m}\frac{\text{s}}{\text{m}} = 2439280.8\ \text{ns}$$

For example, using the data in table 1 for muonic neutrinos at 13.9 GeV, it follows from equation (8) that:

$$\frac{v}{c} - 1 = \frac{53.1}{2439280.8 - 53.1} = 2.18 \cdot 10^{-5}$$

while the uncertainties associated to this term are calculated from the uncertainties in the measurement of δt:

$$\sigma_r = \frac{18.8}{2439280.8 - 18.8} = 0.77 \cdot 10^{-5}$$

$$\sigma_s = \frac{7.4}{2439280.8 - 7.4} = 0.30 \cdot 10^{-5}$$

Doing the same thing for the other data in table 1, we have the results listed in table 2:

EXPERIMENT	E (GeV)	v/c – 1	σ_r	σ_s
MINOS	3	$5.1 \cdot 10^{-5}$	$1.3 \cdot 10^{-5}$	$2.6 \cdot 10^{-5}$
OPERA	13.9	$2.18 \cdot 10^{-5}$	$0.77 \cdot 10^{-5}$	$0.30 \cdot 10^{-5}$
OPERA	17.0	$2.49 \cdot 10^{-5}$	$0.28 \cdot 10^{-5}$	$0.30 \cdot 10^{-5}$
OPERA	28.1	$2.47 \cdot 10^{-5}$	$0.54 \cdot 10^{-5}$	$0.30 \cdot 10^{-5}$
OPERA	42.9	$2.75 \cdot 10^{-5}$	$0.75 \cdot 10^{-5}$	$0.30 \cdot 10^{-5}$

Table 2 – *MINOS and OPERA* v/c – 1 *results and associated uncertainties.*

From the third column of table 2 we see that all the observed muonic neutrinos travel faster than light and so they should follow the inverted dynamics laws. In particular, their total energy should be given by equation (6) in section 2, where this physical quantity is written as a function of the muonic neutrino rest mass and its superluminal speed. Since in MINOS and OPERA experiments the total energy and the velocity were measured, from equation (6) we can derive the experimental value of the muonic neutrino rest mass. In other words, if equation (6) is verified, all the experimental results will give the same value for the muonic neutrino rest mass in the limit of the measurement uncertainties. To see that, we begin deriving the expression of the muonic neutrino rest mass from equation (6):

$$m_0c^2 = \frac{2}{3} E \left(\frac{v}{c} - 1\right)^2 \tag{9}$$

Putting the best estimates of E and v/c – 1 in relation (9), we will get the best estimate of m_0c^2. Like any experimental value, the best estimate of the muonic neutrino rest mass is determined with some uncertainty brought by the term v/c – 1 (to simplify the calculations we will suppose that the uncertainty in the total energy measurement is negligible). The total error in the best estimate of this latter term will be calculated starting from random and systematic errors associated to its measurement.

Because the method for combining random and systematic errors is not completely clear, many scientists leave the two error components separate (see section 4.6 in ref. [4]). This was not done for MINOS experiment where the scientists have calculated the total error as well (random and systematic errors are combined in quadrature as you can see in ref. [2]). In OPERA experiment instead (see ref. [3]) the random component and the systematic component of the total uncertainty have been left separated and so we will have to combine them on our own to get the total error. At this point, we can combine in the proper way random and systematic errors only when we know the statistical distribution underlying the systematic inaccuracy of an experimental setup. In fact, by appropriate mathematical calculations, we can correctly estimate the total uncertainty at some confidence level as explained in ref. [5] where you

can find a general dissertation on the topic according to two possible particular points of view. However, the statistics behind the systematic uncertainty of the OPERA experimental setup is not very clear and so, to be sure, we will combine random and systematic errors directly.
The simple sum of the two components to get the total uncertainty is surely a rough method but, in this case, a safer one since, at least, it provides a reasonable estimate. More sophisticated, skilled and accurate estimates will be left to the experts of the field. For example, we consider the experimental results for muonic neutrinos at 13.9 GeV. From table 2 we have that the total error associated to the term v/c – 1 is $\sigma_t = \sigma_r + \sigma_s = 0.77 \cdot 10^{-5} + 0.30 \cdot 10^{-5} = 1.07 \cdot 10^{-5}$.
In the same way we can get the total errors for muonic neutrinos at the other energies, as you can see in table 3 (for MINOS experiment we have used the scientist estimate reported in ref. [2]).

EXPERIMENT	**E (GeV)**	**v/c – 1**	σ_t
MINOS	3	$5.1 \cdot 10^{-5}$	$2.9 \cdot 10^{-5}$
OPERA	13.9	$2.18 \cdot 10^{-5}$	$1.07 \cdot 10^{-5}$
OPERA	17.0	$2.49 \cdot 10^{-5}$	$0.58 \cdot 10^{-5}$
OPERA	28.1	$2.47 \cdot 10^{-5}$	$0.84 \cdot 10^{-5}$
OPERA	42.9	$2.75 \cdot 10^{-5}$	$1.05 \cdot 10^{-5}$

Table 3 – *Total uncertainties associated to the MINOS and OPERA* v/c – 1 *results.*

Now, using relation (9), our best estimate for the muonic neutrino rest mass is:

$$m_0c^2 = \frac{2}{3} \times 13.9 \cdot 10^9 \times (2.18 \cdot 10^{-5})^2 \ \text{eV} = 4.4 \ \text{eV}$$

where we have used the OPERA data at 13.9 GeV in table 3.
The problem, then, is to find the uncertainty in m_0c^2 that results from the known uncertainty in v/c – 1. From table 3 we see that the term v/c – 1 probably lies between the value $2.18 \cdot 10^{-5} - 1.07 \cdot 10^{-5}$ and the value $2.18 \cdot 10^{-5} + 1.07 \cdot 10^{-5}$, that is $1.11 \cdot 10^{-5} \le v/c - 1 \le 3.25 \cdot 10^{-5}$. Since the largest probable value of v/c – 1 is $3.25 \cdot 10^{-5}$, the largest probable value for m_0c^2 will be:

$$m_0c^2 = \frac{2}{3} \times 13.9 \cdot 10^9 \times (3.25 \cdot 10^{-5})^2 \ \text{eV} = 9.8 \ \text{eV}$$

The smallest probable value of m_0c^2 is given in the same way using this time the smallest probable value for v/c – 1:

$$m_0c^2 = \frac{2}{3} \times 13.9 \cdot 10^9 \times (1.11 \cdot 10^{-5})^2 \ \text{eV} = 1.1 \ \text{eV}$$

So, according to the OPERA data at 13.9 GeV, the muonic neutrino rest mass probably lies in the range 1.1 ÷ 9.8 eV.
Doing the same thing for the other experimental data in table 3, we get the results in table 4: the best estimate and the interval of values where the muonic neutrino rest mass probably lies.

EXPERIMENT	E (GeV)	m_0c^2 (eV)	Range (eV)
MINOS	3	5.2	1.0 ÷ 12.8
OPERA	13.9	4.4	1.1 ÷ 9.8
OPERA	17.0	7.0	4.1 ÷ 10.7
OPERA	28.1	11.4	5.0 ÷ 20.5
OPERA	42.9	21.6	8.3 ÷ 41.3

Table 4 – *Probable values of the muonic neutrino rest mass derived from MINOS and OPERA results.*

From table 4 we see that the discrepancies between the five experimental results are insignificant because the margins of error overlap with one another. In other words, there is an interval of muonic neutrino rest mass values common to all the five measurements. Therefore, these measurements are consistent with inverted dynamics law for the total energy of superluminal particles in equation (6). Let us note that the two best estimates, at comparable total energies, of the muonic neutrino rest mass obtained with two different experimental equipment, MINOS at 3 GeV and OPERA at 13.9 GeV, are nearly the same. Now we compare the results in table 4 with the accepted experimental interval of values where the muonic neutrino rest mass probably lies.

As you can see in ref. [6], measuring the positive muon momentum in the pion decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$ Assamagan et al. have found that the muonic neutrino rest mass, with a confidence level of 90 %, is in the range 0 ÷ 170000 eV. Then it follows that the experimental results from MINOS and OPERA are in very good agreement with this latter one. In fact, from the last column of table 4, the ranges of possible muonic neutrino rest mass values completely overlap with the interval measured by Assamagan et al. since they are well below 170 keV, that is the greatest probable value for the muonic neutrino rest mass. So the results obtained dealing with superluminal particles and inverted dynamics are consistent with the results obtained dealing with subluminal particles and special relativity. In conclusion the inverted dynamics law for the total energy of a superluminal particle is experimentally verified and, indirectly, the expressions for the superluminal inertial mass, the superluminal momentum and the superluminal work that led to it; see equations (6), (2), (3) and (4) respectively.

At the end of this section we will combine the five separate measurements to give a single best estimate of the muonic neutrino rest mass. This single best estimate will be the weighted average.

Let us note that the weighted average, see section 7.2 of ref. [4], is a reliable estimate when all the measurements are governed by the Gauss distribution. As we have seen previously, the statistics behind the systematic error of OPERA measurements is not very clear and so the use of the weighted average is not fully justified. Anyway, this method will give us a not so unrealistic single experimental value for the muonic neutrino rest mass and its associated uncertainty. Once again, more detailed and skilled evaluations will be left to the experts of the field. Following ref. [4], the weighted average of the muonic neutrino rest mass is:

$$m_0c^2 = \frac{\sum k_i \,(m_0c^2)_i}{\sum k_i} \tag{10}$$

where the sums are over i = 1,…, 5 and the weights k_i are the reciprocal squares of the corresponding uncertainties[f], that is:

$$k_i = \frac{1}{\sigma_{mi}^2}$$

Furthermore, the uncertainty in m_0c^2 is:

$$\sigma = \frac{1}{\sqrt{\sum k_i}} \qquad (11)$$

where, again, the sum runs over all of the five measurements. In table 5 are listed the values for σ_{mi} and k_i calculated using the data in table 4.

EXPERIMENT	E (GeV)	m_0c^2 (eV)	σ_m (eV)	k (eV^{-2})
MINOS	3	5.2	5.9	0.029
OPERA	13.9	4.4	4.4	0.052
OPERA	17.0	7.0	3.3	0.092
OPERA	28.1	11.4	7.8	0.016
OPERA	42.9	21.6	16.5	0.004

Table 5 – *Mean total errors and weights for MINOS and OPERA* v/c − 1 *results.*

Then from equations (10) and (11) it follows that:

$$m_0c^2 = \frac{1.2924}{0.193}\ \text{eV} = 6.7\ \text{eV}$$

$$\sigma = \frac{1}{\sqrt{0.193}}\ \text{eV} = 2.3\ \text{eV}$$

To sum up, our single best estimate of the muonic neutrino rest mass obtained from the five measurements of MINOS and OPERA is:

$$m_0c^2 = 6.7 \pm 2.3\ \text{eV}$$

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[f] Since in all five measurements the range of possible muonic neutrino rest mass values is not centered around the best estimate, we will define the total error σ_m as the mean between the right side and the left side of possible values. For example, we consider the results in table 4 obtained with MINOS experiment. The total error is

$$\sigma_m = \frac{(5.2 - 1.0) + (12.8 - 5.2)}{2}\ \text{eV} = 5.9\ \text{eV}$$

The total error defined in this way is only a crude evaluation of the real one, but at least it gives us the magnitude of the uncertainty associated to our best estimate of the muonic neutrino rest mass.

3.2 The tunnelling effect: a natural tachyonic converter

We consider a particle of rest mass m_0 and total energy E, then its kinetic energy is by definition:

$$T = E - m_0c^2 \tag{12}$$

When we consider subluminal particles, from special relativity we know that $m_0c^2 \le E < +\infty$ and so, using relation (12), we have that $0 \le T < +\infty$. On the other hand, when we consider superluminal particles, from relation (6) of inverted dynamics it follows that $0 < E < +\infty$ which gives, using relation (12) again, $-m_0c^2 < T < +\infty$. We see that, unlike what happens for subluminal particles, tachyons can also have negative kinetic energy experimentally measurable. In fact, the total energy and the particle rest mass are physical quantities that usually could be measured given the proper experimental setup.
Now we determine the superluminal speed of a tachyon as a function of its kinetic energy. To do that we substitute the expression for the tachyonic total energy, see relation (6), in relation (12) and we move the term m_0c^2 to the other member. It follows:

$$\frac{3}{2}\frac{m_0c^4}{(v-c)^2} = T + m_0c^2$$

Developing the calculations further we can write:

$$\frac{c^2}{(v-c)^2} = \frac{2}{3}\frac{T + m_0c^2}{m_0c^2}$$

Now reversing the last equality, we have:

$$\frac{(v-c)^2}{c^2} = \frac{3}{2}\frac{m_0c^2}{T + m_0c^2}$$

$$(v-c)^2 = c^2\ \frac{3}{2}\frac{m_0c^2}{T + m_0c^2}$$

Then we apply the square root to both members, that gives:

$$v - c = c\sqrt{\frac{3}{2}\frac{m_0c^2}{T + m_0c^2}}$$

In conclusion the superluminal speed of a tachyon as a function of its kinetic energy is:

$$v = c\left(1 + \sqrt{\frac{3}{2}\frac{m_0c^2}{T + m_0c^2}}\right) \tag{13}$$

You notice that for $T \to -m_0c^2$ (the greatest lower bound of the tachyonic kinetic energy) the tachyon speed $v \to +\infty$ (the least upper bound of the tachyon speed), as for $T \to +\infty$ (the least upper bound of the

tachyonic kinetic energy) the tachyon speed $v \rightarrow c^+$ (the greatest lower bound of the tachyon speed). Even in this case we have a reversed behavior with respect to special relativity. In fact, considering subluminal particles, when $T \rightarrow 0$ (the minimum bradyonic kinetic energy) the bradyon speed $v \rightarrow 0$ (the minimum speed of bradyons), as for $T \rightarrow +\infty$ (the least upper bound of the bradyonic kinetic energy) the bradyon speed $v \rightarrow c^-$ (the least upper bound of the bradyon speed).
This introduction shows that it is possible to explain the penetration of a step potential barrier by a particle in the theoretical framework of inverted dynamics.
Indeed, we consider a bradyon incident upon a step potential barrier where, see figure 3, region I is the region before the barrier, region II is the barrier region and region III is the region beyond the barrier. Furthermore, Q is the kinetic energy of the bradyon, V_0 is the height of the step potential barrier and we assume that $Q < V_0$. Now, following the classical physics, we define the total energy of a particle as the sum of its kinetic energy, its potential energy and its rest mass energy. So, in region I we will have that $E_I = T_I + V_I + m_0c^2 = Q + 0 + m_0c^2$, as in region II it will be $E_{II} = T_{II} + V_{II} + m_0c^2 = T_{II} + V_0 + m_0c^2$. For the conservation of the total energy, when we go from region I to region II it must be $E_I = E_{II}$, that is, using the identities just written above, $Q = T_{II} + V_0$ Then it follows that $T_{II} = Q - V_0 < 0$. As we have seen at the beginning of this section the region under the barrier is forbidden to subluminal particles, but not to tachyons. In fact, when $-m_0c^2 < Q - V_0$ the particle can fly under the barrier at superluminal speed after a change of state to the tachyonic world. So the entry of the step potential barrier works as a tachyonic converter from bradyon to tachyon at zero expense of energy because this latter one is conserved during the change.

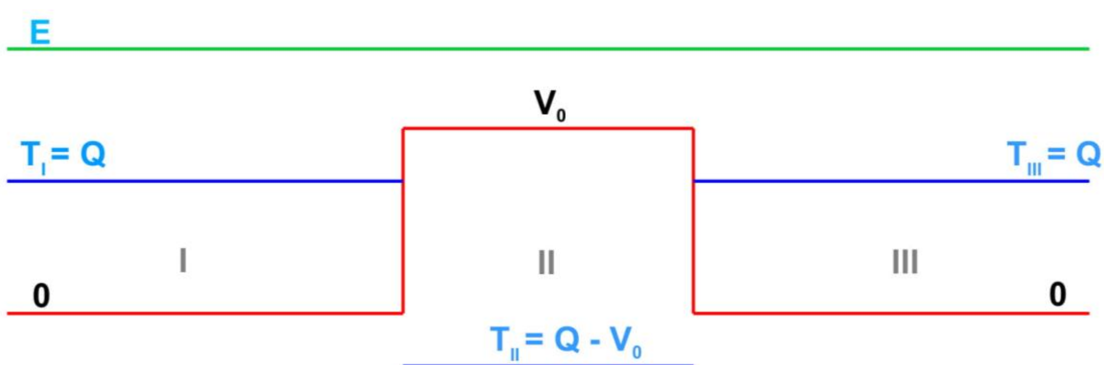

Figure 3 - *A subluminal particle incident upon a step potential barrier. The kinetic energy of the particle is less than the height of the barrier.*

Finally, in the passage from region II to region III, always using the conservation of the total energy, it has to be $E_{II} = E_{III}$. Since the total energy $E_{II} = T_{II} + V_{II} + m_0c^2 = Q - V_0 + V_0 + m_0c^2 = Q + m_0c^2$ and the total energy $E_{III} = T_{III} + V_{III} + m_0c^2 = T_{III} + 0 + m_0c^2 = T_{III} + m_0c^2$ it follows that $T_{III} = Q$. As usually observed, at the exit of the barrier the tachyon change state again returning to be a bradyon with the same initial kinetic energy. Hence the step potential barrier gives an example of tachyonic converter in both ways: from bradyon to tachyon and from tachyon to bradyon.
Now an experimental confirmation of the tunnelling effect can be found considering the α decay. In the following we will give a short description of the α-decay model taking as guideline the section 8.4 in

ref. [7]; moreover, for more details on the α decay you can see chapter 8 of the same reference.
According to the quantum mechanical theory developed in 1928 by Gamow, Gurney and Condon, a preformed α particle moves in a spherical region inside the daughter nucleus. A plot of the potential energy between the α particle and the residual nucleus for various distances between their centers is shown in figure 4. The horizontal line Q is the disintegration energy that will be equal to the kinetic energy of the α particle outside the daughter nucleus neglecting the recoil energy of this latter one.
Note that the Coulomb potential is extended inward to a radius d and then cut off. The radius d is taken as the sum between the radius of the residual nucleus and the radius of the α particle.
There are three regions to consider. In the spherical region $r < d$ we are inside the nucleus in a potential well of depth W, where W is a positive number. The subluminal α particle moves in this region with kinetic energy Q + W, but it cannot escape because the annular-shell region $d < r < f$ forms a potential barrier. This region is forbidden to subluminal α particles and so they cannot reach the permitted region $r > f$ outside the barrier. A subluminal α particle in the spherical potential well sharply reverses its motion every time it tries to pass beyond the distance $r = d$. However, since experimentally α-unstable nuclei are seen to decay emitting this kind of particle, there is a chance of leakage or tunnelling through such a barrier. Indeed, such nuclei do not decay immediately and so the subluminal α particle within the nucleus must present itself again and again at the barrier surface until, for the law of large numbers, it is finally converted into a tachyon and penetrates. At the exit of the barrier the superluminal α particle returns to the subluminal state, as indicated by many measurements, moving with kinetic energy Q.

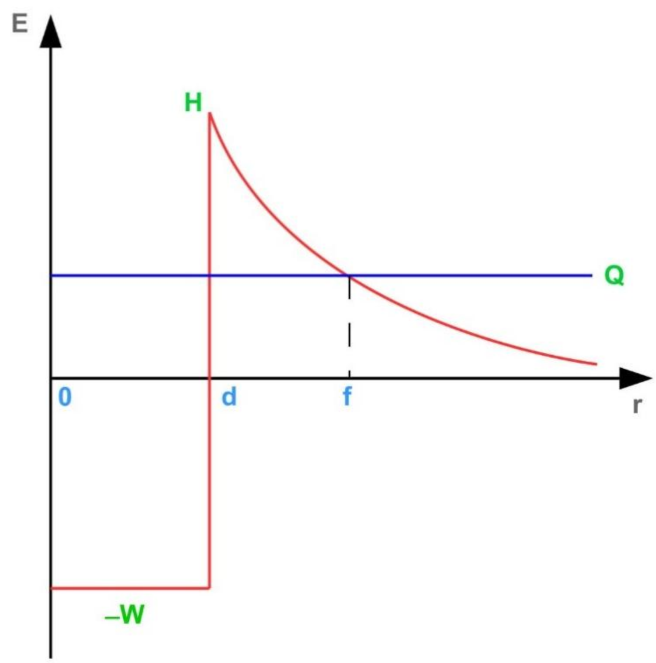

Figure 4 - *Potential energy of an α particle as a function of its separation from the daughter nucleus.*

Now the Coulomb barrier of figure 4 has height H, at $r = d$, where:

$$H = \frac{1}{4\pi\varepsilon_0}\frac{z\tilde{Z}e^2}{d} \tag{14}$$

In this expression the α particle has charge ze and the daughter nucleus, which provides the Coulomb repulsion, has charge $\tilde{Z}e = (Z - z)\,e$, being Ze the electric charge of the α-unstable nucleus. Considering the α decay of ${}^{252}_{98}Cf$, we calculate first the value of d.

Since the nuclear radius varies with mass number as $R_0 \cdot A^{1/3}$, with $R_0 \cong 1.2$ fm, the nuclear radius of the α particle, ${}^{4}_{2}He$, is $1.2 \times 4^{1/3} = 1.9$ fm, while the nuclear radius of the residual heavy fragment ${}^{248}_{96}Cm$ is $1.2 \times 248^{1/3} = 7.5$ fm. Summing the two nuclear radii we have that d = 1.9 + 7.5 fm = 9.4 fm which substituted in relation (14) gives:

$$H = \frac{1}{4 \times 3.14 \times 8.85 \cdot 10^{-12}} \frac{2 \times 96 \times (1.602 \cdot 10^{-19})^2}{9.4 \cdot 10^{-15}} \frac{Nm^2}{C^2} \frac{C^2}{m} =$$

$$= 4.7 \cdot 10^{-12}\ J = \frac{4.7 \cdot 10^{-12}}{1.602 \cdot 10^{-19}}\ J\ \frac{eV}{J} = 29.3 \cdot 10^{6}\ eV = 29.3\ MeV$$

Because the measured kinetic energy of the α particle emitted in the decay of ${}^{252}_{98}Cf$ ($t_{1/2} = 2.645$ y) is measured to be Q = 6.1 MeV, it follows that the tachyonic kinetic energy of the α particle under the barrier at r = d is:

$$Q - H = (6.1 - 29.3)\ MeV = -23.2\ MeV$$

which is well above the greatest lower bound equal to −3.7 GeV (the rest mass of the α particle in energy units). Finally, the α particle will come out from the barrier at the radius f calculated by the equality between the kinetic energy Q out of the barrier and the Coulomb potential energy at the barrier way out, as you can see from figure 4. More precisely:

$$f = \frac{1}{4\pi\varepsilon_0} \frac{z\tilde{Z}e^2}{Q}$$

In our case, converting Q in joules, the α particle will leave the barrier at:

$$f = \frac{1}{4 \times 3.14 \times 8.85 \cdot 10^{-12}} \frac{2 \times 96 \times (1.602 \cdot 10^{-19})^2}{9.8 \cdot 10^{-13}} \frac{Nm^2}{C^2} \frac{C^2}{J} = 4.52 \cdot 10^{-14}\ m = 45.2\ fm$$

Now we can calculate the time the α particle takes to cross the barrier. First, let us note from figure 4 that the kinetic energy of the α particle under the barrier becomes less negative during its tachyonic flight from d (the radius at which the α particle enters the barrier) to f (the radius at which the α particle leaves the barrier).

Then, just to make a rough estimate, we suppose that the average tachyonic kinetic energy of the α particle under the barrier is ½ (Q − H) = −11.6 MeV. Hence, putting T = ½ (Q − H) in relation (13), the superluminal α particle will travel at the average speed:

$$v = 299792458 \left(1 + \sqrt{\frac{3}{2} \frac{3700}{-11.6 + 3700}}\right) \frac{m}{s} = 667538653\ \frac{m}{s}$$

and it will cross the barrier, assuming that inside it does not bounce back and forth for some time, in:

$$t = \frac{f - d}{v} = \frac{(45.2 - 9.4) \cdot 10^{-15}}{667538653} \text{ m} \frac{s}{m} = 5.4 \cdot 10^{-23} \text{ s}$$

The tunnelling of the α particle is almost an instant process.
Obviously inverted dynamics cannot say anything on the probability to pass through the barrier because it is a classical theory. In fact, inverted dynamics does not deal with matter waves not using a statistical approach. That will be left to quantum mechanics.
In any case inverted dynamics can explain classically the tunnelling of the α particles from an energy point of view and it can also predict their superluminal speed.
Moreover, in this classical framework, the positive kinetic energy Q outside the barrier is also in agreement with α particles in the tachyonic state. In other words, according to relation (13) of inverted dynamics, the superluminal α particle at the exit of the barrier could sharply decrease its speed without changing state into a bradyon.
If that happens for some α-unstable nuclei, we could have some natural sources of superluminal α particles. It will be the task of quantum mechanics to predict the probability of such event and, more in general, the probability of conversion from bradyon to tachyon and from tachyon to bradyon.
At the end of this section, remaining in a classical environment, we suppose to switch a starship at rest into a tachyonic object. Surely this is a very ideal case since in a quantum environment such event could have zero probability, but it is presented anyway because it shows another inverted behavior with respect to subluminal mechanics.
We consider, as depicted in figure 5, a starship initially at rest at the surface of the Earth that is converted into a tachyon with initial speed v_0 pointing upwards. Now let us see how the starship speed changes during the flight to the upper atmosphere.

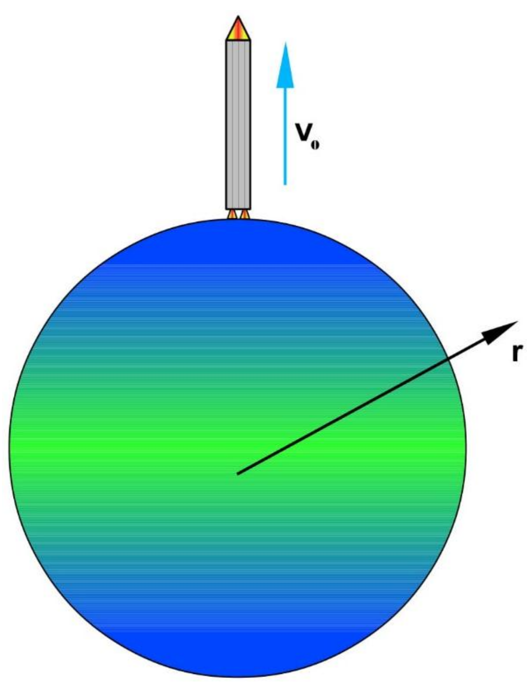

Figure 5 - *A tachyonic starship starting its flight from the surface of the Earth.*

For the first thing we begin considering Newton's Second Law F = dp/dt. Assuming that the gravitational force has the same form even when acting on superluminal particles, it follows:

$$-\gamma \frac{m_T m}{r^2} = \frac{d}{dt}\left(\frac{mc^2}{v - c}\right)$$

where r is the distance from the center of the Earth, γ is the gravitational constant, m_T is the Earth mass and m is the tachyonic starship inertial mass. Furthermore, being a superluminal starship, for the linear momentum p we have used relation (2) of inverted dynamics. Now we substitute expression (1) for the tachyonic inertial mass and then we calculate the derivative, that gives:

$$-\gamma \frac{m_T}{r^2} \frac{m_0 c^2}{(v - c)^2} = \frac{d}{dt}\left[\frac{m_0 c^4}{(v - c)^3}\right]$$

$$-\gamma \frac{m_T}{r^2} \frac{m_0 c^2}{(v - c)^2} = -\frac{3 m_0 c^4}{(v - c)^4} \frac{dv}{dt}$$

Deleting the common term $-m_0c^2/(v - c)^2$ from both sides we have:

$$\gamma \frac{m_T}{r^2} = \frac{3c^2}{(v - c)^2} \frac{dv}{dt}$$

At this point we multiply the two members for dr, where dr = v dt. Then our equation becomes:

$$\gamma \frac{m_T}{r^2}\, dr = \frac{3c^2}{(v - c)^2} \frac{dv}{dt}\, dr$$

$$\gamma \frac{m_T}{r^2}\, dr = 3c^2 \frac{v}{(v - c)^2}\, dv$$

$$\gamma \frac{m_T}{r^2}\, dr = \frac{3c^2}{(v - c)^2} \frac{dr}{dt}\, dv$$

Now to find the starship speed as a function of $r > r_T$ we have to carry out an integration on the two terms of the above differential equation:

$$\int_{r_T}^{r} \gamma \frac{m_T}{r^2}\, dr = \int_{v_0}^{v} 3c^2 \frac{v}{(v - c)^2}\, dv$$

where, according to relation (7), the initial velocity of the starship at rest, as soon as it is turned into a tachyon, is $v_0 = c\,(\sqrt{3/2} + 1)$ and r_T is the Earth radius. By noting that:

$$\frac{v}{(v - c)^2} = \frac{v - c + c}{(v - c)^2} = \frac{v - c}{(v - c)^2} + \frac{c}{(v - c)^2} = \frac{1}{v - c} + \frac{c}{(v - c)^2}$$

we can write:

$$\int_{r_T}^{r} \gamma \frac{m_T}{r^2} dr = \int_{v_0}^{v} 3c^2 \frac{1}{v-c} dv + \int_{v_0}^{v} 3c^3 \frac{1}{(v-c)^2} dv$$

$$\int_{r_T}^{r} \gamma \frac{m_T}{r^2} dr = \int_{v_0}^{v} 3c^2 \frac{1}{c} \frac{1}{\frac{v}{c}-1} dv + \int_{v_0}^{v} 3c^3 \frac{1}{(v-c)^2} dv$$

Solving the integrals, we have:

$$\left. \frac{-\gamma m_T}{r} \right|_{r_T}^{r} = \left. 3c^2 \ln\left(\frac{v}{c}-1\right) \right|_{v_0}^{v} - \left. \frac{3c^3}{v-c} \right|_{v_0}^{v}$$

For $r \to +\infty$ the above equation becomes:

$$3c^2 \ln\left(\frac{v_s}{c}-1\right) - \frac{3c^3}{v_s-c} = 3c^2 \ln\left(\frac{v_0}{c}-1\right) - \frac{3c^3}{v_0-c} + k_{gr} \tag{15}$$

where $k_{gr} = \gamma\, m_T / r_T$ and v_s is the starship speed in the empty space, outside the Earth.
Since $\gamma = 6.67 \cdot 10^{-11}$ m^3/kg s^2, $m_T = 5.98 \cdot 10^{24}$ kg, $r_T = 6.37 \cdot 10^6$ m, $c = 3 \cdot 10^8$ m/s, the initial speed $v_0 = 6.67 \cdot 10^8$ m/s and the planet constant $k_{gr} = 6.26 \cdot 10^7$ m^2/s^2, it follows that:

$$3c^2 \ln\left(\frac{v_s}{c}-1\right) - \frac{3c^3}{v_s-c} = -1.66 \cdot 10^{17} \tag{16}$$

For $v = v_0$ the function $f(v) = 3\, c^2 \ln(v/c - 1) - 3\, c^3/(v - c)$ at the first member of equation (16) is equal to $-1.66 \cdot 10^{17}$ m^2/s^2 and so, being equation (16) verified, it will be $v_s = v_0 = 6.67 \cdot 10^8$ m/s . We can see that the gravity does not affect the starship initial speed during its tachyonic flight to the upper atmosphere[g] and, in this way, it will arrive on Mars, which is on the average $78.34 \cdot 10^6$ km far from the Earth, in:

$$t = \frac{78.34 \cdot 10^9}{6.67 \cdot 10^8} \ \mathrm{m}\, \frac{\mathrm{s}}{\mathrm{m}} = 117 \ \mathrm{s} \cong 2 \ \mathrm{min}$$

Now we consider our tachyonic starship starting from a more massive object, a star for example. The Crab Pulsar is a neutron star, in the Crab Nebula, 6500 ly far from the Earth.
This neutron star is a remnant of the supernova SN 1054 with an estimated mass $m_P = 2.79 \cdot 10^{30}$ kg and radius $r_P = 10 \cdot 10^3$ m. In this case $k_{gr} = \gamma\, m_P / r_P = 1.86 \cdot 10^{16}$ m^2/s^2 that substituted in the second member

[g] More precisely, considering v_0 = 666961733 m/s and c = 299792458 m/s, the second member of equation (16) is equal to $-1.654870798 \cdot 10^{17}$. Then the equation is verified for v_s = 666961733.08 m/s, that is a value 0.00000001% higher than v_0. This is in agreement with the fact that a negative force increases the tachyon speed, as we have seen at the end of section 2, and so it should be clear that the initial velocity is unaffected by the gravity force only according
to the precision we are using in the example.

of equation (15), in place of the Earth k_{gr}, gives:

$$3c^2 \ln\left(\frac{v_S}{c}-1\right)-\frac{3c^3}{v_S-c}=-1.48\cdot 10^{17}$$

Now for $v = 6.81 \cdot 10^8$ m/s we have $f(v) = -1.48 \cdot 10^{17}\ m^2/s^2$ and so the above equation is satisfied for $v_s = 6.81 \cdot 10^8$ m/s. That gives a percentage speed variation with respect to the initial velocity v_0 equal to:

$$\frac{6.81\cdot 10^8 - 6.67\cdot 10^8}{6.67\cdot 10^8} \cong 2\ \%$$

Neither in this case the gravity affects to a significant extent the starship initial speed as it moves away from the neutron star.

Finally, we consider the strongest gravity source, a black hole. A black hole, in the theoretical Schwarzschild picture, is a body where all the matter is concentrated in a mathematical point called singularity (see figure 6).

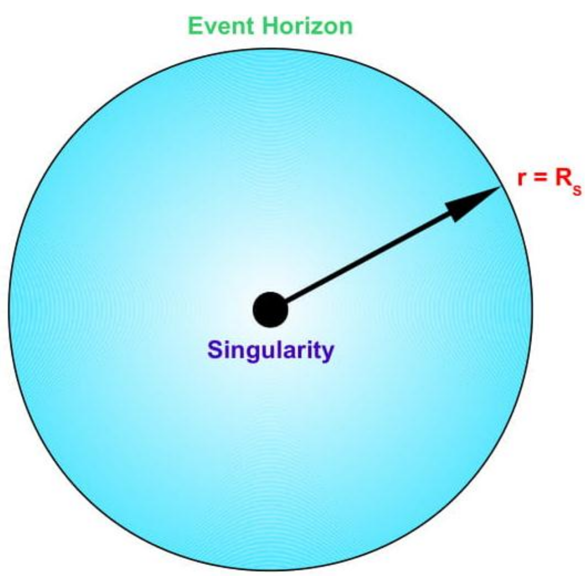

Figure 6 - *The simple structure of a black hole in the Schwarzschild picture.*

Anyway, its size is given by the Schwarzschild radius $R_S = 2\ \gamma\ m_B/c^2$ which defines the event horizon and distinguishes the inside of the black hole from the outside. Once inside, subluminal matter (matter in the bradyonic state) and light cannot get back out into the rest of the universe. Now we suppose that the tachyonic starship in figure 5 starts from the Schwarzschild radius of a supermassive black hole. Sagittarius A* is a bright and very compact astronomical radio source at the center of the Milky Way Galaxy, near the border of the constellations Sagittarius and Scorpius, at a distance of 29500 ly from the Earth. Its estimated mass is $m_B = 8.16 \cdot 10^{36}$ kg, its Schwarzschild radius is $R_S = 1.21 \cdot 10^{21}$ m and so the black hole constant will be $k_{gr} = \gamma\ m_B/R_S = 4.50 \cdot 10^{16}\ m^2/s^2$. Substituting this value in the second member of equation (15), in place of the Earth k_{gr}, it follows:

$$3c^2 \ln\left(\frac{v_s}{c} - 1\right) - \frac{3c^3}{v_s - c} = -1.21 \cdot 10^{17}$$

For $v = 7.03 \cdot 10^8$ m/s we have $f(v) = -1.21 \cdot 10^{17}$ m^2/s^2. So, from the equation written above, it will be $v_s = 7.03 \cdot 10^8$ m/s that gives a percentage speed variation with respect to the initial velocity v_0 equal to:

$$\frac{7.03 \cdot 10^8 - 6.67 \cdot 10^8}{6.67 \cdot 10^8} \cong 5\%$$

In this case we see that the gravity affects only slightly the initial motion of the starship. Tachyons, at this initial velocity, do not seem to feel the gravity very much.
Finally, let us suppose that the starship in figure 5 starts from the inside of the supermassive black hole Sagittarius A*, more precisely at $r = 2.095 \cdot 10^7$ m $< R_S$.
Now $k_{gr} = \gamma\, m_B/r = 2.60 \cdot 10^{19}$ m^2/s^2 that substituted in the second member of equation (15), in place of the Earth k_{gr}, gives:

$$3c^2 \ln\left(\frac{v_s}{c} - 1\right) - \frac{3c^3}{v_s - c} = 2.58 \cdot 10^{19}$$

For $v = 1.0 \cdot 10^{50}$ m/s we have $f(v) = 2.58 \cdot 10^{19}$ m^2/s^2. Hence, from the equation written above, we will have a superluminal velocity $v_s = 1.0 \cdot 10^{50}$ m/s. In this last case the tachyonic starship will be teleported outside Sagittarius A* at the astonishing speed of $1.0 \cdot 10^{34}$ ly/s. The supermassive black hole has become a very powerful gravity catapult.

3.3 Fermilab79

In this section we will consider the experimental results of Fermilab79 to calculate the muonic neutrino rest mass. The uncertainty associated to these data is greater than the one associated to MINOS and OPERA, but they can provide a further test of inverted dynamics using a very different experimental setup. Moreover, the observed neutrinos and antineutrinos are a mix of subluminal and superluminal particles and so we will able to compare, by the same experiment, the predictions of inverted dynamics and special relativity.
In Fermilab79 experiment, see ref. [8], muonic neutrinos and antineutrinos from pion decay were observed, that is $\pi^+ \to \mu^+ + \nu_\mu$ and $\pi^- \to \mu^- + \bar{\nu}_\mu$. At higher energies ($\geq 90$ GeV), muonic neutrinos and antineutrinos from kaon decay were also observed, that is $K^+ \to \mu^+ + \nu_\mu$ and $K^- \to \mu^- + \bar{\nu}_\mu$. Then a velocity comparison between neutrinos or antineutrinos and penetrating muons was obtained.
In TABLE II of ref. [8], the velocity differences are listed in the form $v_\nu/c - v_\mu/c$ and it is also reported the corresponding average total energy of neutrinos or antineutrinos. Furthermore, in the same reference, it is assumed that charged muons behave as expected from special relativity giving in this way a negligible velocity difference of the muons with respect to light. Hence the experimental data will be in the form $v_\nu/c - 1$ or, more simply, $v/c - 1$.

Now we consider, for example, the muonic neutrinos from pion decay ($^{\pi}\nu_{\mu}$) with an average total energy of 90 GeV. For them, see TABLE II of ref. [8], we have that $v/c - 1 = (6 \pm 3) \cdot 10^{-5}$, but this is not the final experimental result we have to use in our calculations. In fact, from ref. [8] again, the experimenters claim that the penetrating muons travel a slightly longer distance from target to detector, due to multiple scattering in the shield, than the straight-line path of the neutrinos or antineutrinos. The muon flight path along its trajectory in the shield is then some 0.03 to 0.06 meters longer than a straight-line path to the detector and corresponds to the estimated bias $(4 \div 7) \cdot 10^{-5}$.
This bias, as stated by the experimenters, has no obvious correlation with the energy setting of the beam and so all the data have to be shifted downwards for this interval. In other words, we consider the highest probable value for the quantity $v/c - 1$, that is $9 \cdot 10^{-5}$. Because all the muons associated to the quicker 90 GeV neutrinos have travelled from 0.03 to 0.06 meters longer, we will have that the correct value is between $(9 - 4) \cdot 10^{-5} = 5 \cdot 10^{-5}$ and $(9 - 7) \cdot 10^{-5} = 2 \cdot 10^{-5}$. Hence the highest probable value for the quantity $v/c - 1$ will be $5 \cdot 10^{-5}$. Now we consider the lowest probable value for the quantity $v/c - 1$, that is $3 \cdot 10^{-5}$. Even in this case, since all the muons associated to the slower 90-GeV neutrinos have travelled from 0.03 to 0.06 meters longer, we will have that the correct value is between $(3 - 4) \cdot 10^{-5} = -1 \cdot 10^{-5}$ and $(3 - 7) \cdot 10^{-5} = -4 \cdot 10^{-5}$. Hence the lowest probable value for the quantity $v/c - 1$ will be $-4 \cdot 10^{-5}$.
In conclusion the probable values for the speed of the observed 90-GeV neutrinos, given by the correlated quantity $v/c - 1$, will be in the range $(-4 \div 5) \cdot 10^{-5}$. Now let us note that this range is made up of two parts: one negative and one positive. The negative part, $(-4 \div 0) \cdot 10^{-5}$, means that some of the observed 90-GeV neutrinos travelled slower than light and so to calculate their rest mass we have to use special relativity. In particular, from the equation $E = mc^2$ it follows:

$$m_0c^2 = E\sqrt{1 - \frac{v^2}{c^2}}$$

where m_0c^2 is the muonic neutrino rest mass in energy units.
When $v/c - 1 = -4 \cdot 10^{-5}$, it is $v/c = 1 - 4 \cdot 10^{-5}$ that substituted in the previous relation gives:

$$m_0c^2 = 90 \cdot 10^9 \times \sqrt{1 - (1 - 4 \cdot 10^5)^2}\ \text{ eV} = 805 \cdot 10^6\ \text{ eV}$$

This is the largest probable value for the muonic neutrino rest mass.
On the other hand, when $v/c - 1 = 0$ (the muonic neutrinos travel at the speed of light), it is $v/c = 1$ that substituted in the previous relation gives:

$$m_0c^2 = 90 \cdot 10^9 \times \sqrt{1 - 1^2}\ \text{ eV} = 90 \cdot 10^9 \times 0\ \text{ eV} = 0\ \text{ eV}$$

This is the smallest probable value for the muonic neutrino rest mass.
In conclusion, using the experimental data from 90-GeV neutrino sample, the range in which the muonic neutrino rest mass probably lies, according to special relativity, is $0 \div 805$ MeV.
At this point we consider the positive part for the quantity $v/c - 1$, that is $(0 \div 5) \cdot 10^{-5}$. The positive part means that some of the observed 90-GeV neutrinos travelled faster than light and so to calculate their rest mass we have to use relation (9) of inverted dynamics as we have already done with the data of MINOS and OPERA. When $v/c - 1 = 5 \cdot 10^{-5}$, the largest probable value for the muonic neutrino rest

mass is:

$$m_0c^2 = \frac{2}{3} \times 90 \cdot 10^9 \times (5 \cdot 10^{-5})^2 \ \text{eV} = 150 \ \text{eV}$$

For v/c − 1 → 0 instead, it follows immediately from relation (9) that the greatest lower bound for the smallest probable value of the muonic neutrino rest mass is 0. In conclusion, using the experimental data from 90-GeV neutrino sample, the range in which the muonic neutrino rest mass probably lies, according to inverted dynamics, is 0 ÷ 150 eV. The value 0, in this case, means that the muonic neutrino rest mass is probably infinitesimal, but never null. In fact, see section 2, inverted dynamics only applies to massive particles which travel faster than light leaving the massless ones, which travel at the speed of light, to the well experimentally verified theoretical framework of special relativity.
Doing the same thing with the other neutrino and antineutrino experimental data reported in TABLE II of ref. [8], we get for the muonic neutrino rest mass the results shown in Table 6.

SAMPLE	E (GeV) Average	v/c – 1 (10^{-5}) Rough	v/c – 1 (10^{-5}) Corrected	m_0c^2 (MeV) Special Relativity	m_0c^2 (eV) Inverted Dynamics
${}^{\pi}\nu_{\mu}$	32	1 ÷ 5	−6 ÷ 1	0 ÷ 350	0 ÷ 2
${}^{\pi}\nu_{\mu}$	44	0 ÷ 14	−7 ÷ 10	0 ÷ 521	0 ÷ 293
${}^{\pi}\bar{\nu}_{\mu}$	45	2 ÷ 6	−5 ÷ 2	0 ÷ 450	0 ÷ 12
${}^{\pi}\bar{\nu}_{\mu}$	58	9 ÷ 17	2 ÷ 13	–	15 ÷ 653
${}^{\pi}\nu_{\mu}$	59	2 ÷ 6	−5 ÷ 2	0 ÷ 590	0 ÷ 16
${}^{\pi}\bar{\nu}_{\mu}$	64	5 ÷ 9	−2 ÷ 5	0 ÷ 405	0 ÷ 107
${}^{\pi}\nu_{\mu}$	69	2 ÷ 6	−5 ÷ 2	0 ÷ 690	0 ÷ 18
${}^{K}\nu_{\mu}$	90	3 ÷ 9	−4 ÷ 5	0 ÷ 805	0 ÷ 150
${}^{K}\nu_{\mu}$	120	−1 ÷ 13	−8 ÷ 9	0 ÷ 1518	0 ÷ 648
${}^{K}\bar{\nu}_{\mu}$	125	1 ÷ 7	−6 ÷ 3	0 ÷ 1369	0 ÷ 75
${}^{K}\bar{\nu}_{\mu}$	157	6 ÷ 14	−1 ÷ 10	0 ÷ 702	0 ÷ 1047
${}^{K}\nu_{\mu}$	170	4 ÷ 8	−3 ÷ 4	0 ÷ 1317	0 ÷ 181
${}^{K}\bar{\nu}_{\mu}$	183	7 ÷ 15	0 ÷ 11	–	0 ÷ 1476
${}^{K}\nu_{\mu}$	195	8 ÷ 14	1 ÷ 10	–	13 ÷ 1300

Table 6 – *Muonic neutrino rest mass according to special relativity and inverted dynamics calculated by using Fermilab79 experimental results.*

From the table above, we see that for each sample, where applicable, the predictions of special relativity and inverted dynamics agree since the two mass intervals overlap.

Furthermore, the two mass intervals overlap with the mass interval 0 ÷ 170 keV (90% C.L. determined by Assamagan et al. from pion decay (see ref. [6]).
In particular, the predictions of inverted dynamics for the muonic neutrino or antineutrino rest mass are well below the experimentally accepted highest probable value of 170 keV. Fermilab79 experimental setup, viewed as a balance, has more sensitivity when the neutrino is in the superluminal state and not when it is in the subluminal one. In fact, in this latter case, the mass interval amplitudes are at least six orders of magnitude greater than those relating to muonic neutrinos or antineutrinos in the teleported state.
Finally, considering only the results from tachyons, we see that all the mass intervals overlap, except for a very little number of acceptable slightly disagreements[h], giving the same result for the rest mass in the limit of the experimental uncertainties. Moreover, this mass intervals overlap, except for a very little number of acceptable slightly disagreements (see note h), with those deduced from the experimental data of MINOS and OPERA and so we can say that all these data are consistent. This provides a further confirmation of inverted dynamics laws and it also provides a further confirmation of the fact that particles and their antiparticles have the same rest mass. Anyway, in conclusion, judge by your own.

4 Conclusions

At the end of this work we want to say something on the superluminal kinematics. For example, we consider a photon and a teleported scintillator in front of it going in the same direction. This situation, as seen by us in the laboratory reference frame at the initial instant $t_0 = 0$, is depicted in figure 7 where the photon travels at the speed of light while the scintillator travels at the superluminal speed v > c.

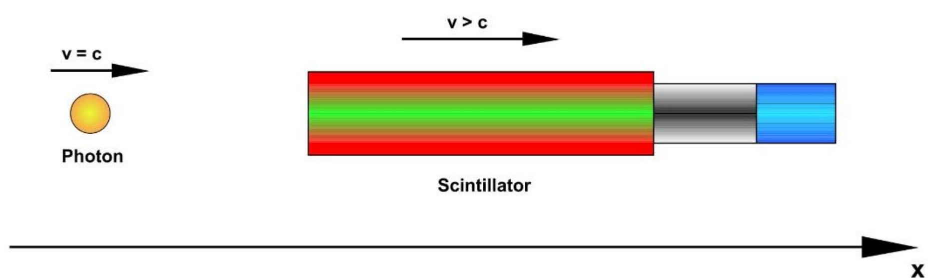

Figure 7 - *A photon and a superluminal scintillator travelling along the x axis of our laboratory reference frame. The picture is taken at t = 0.*

Then, in the laboratory frame, we will see the scintillator that leaves the photon behind because our detector travels faster than light for hypothesis. In other words, the photon will never reach the scintillator and so this latter one will never give a signal. Now we suppose there is another observer whose laboratory reference frame is fixed to the scintillator, that is we suppose of having a superluminal inertial reference frame in which our detector is at rest. What happens in this teleported laboratory reference frame? If the speed of light is invariant in every kind of inertial reference frame, see for

[h] This very little number of slightly disagreements is acceptable because, see ref. [4], the disagreements are small compared with the uncertainties in the several measurements involved.

example extended relativity in ref. [1], the superluminal observer will see the photon approaching and striking the scintillator. So, in a superluminal inertial reference frame the scintillator will give a signal. Since the scintillator is always the same, this is in contrast with the fact that in our laboratory frame, when $t > 0$, we will never have a signal from the counter unless the time for a superluminal observer flows in the opposite direction with respect to us.

In fact, see figure 8, in our laboratory frame for $t < t_1 < 0$ the scintillator is behind the photon and when $t = t_1 < 0$ the scintillator will reach the photon giving a signal, in our past. In other words, we have again an inverted behavior since our past will be the future for a superluminal observer, as his past will be our future. In this case the teleported state will be also a sort of time-machine state set to the past.

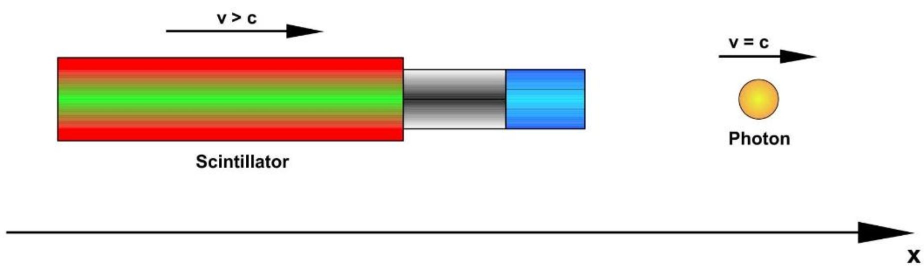

Figure 8 - *A photon and a superluminal scintillator travelling along the x axis of our laboratory reference frame. The picture is taken at $t < t_1 < 0$.*

But that is not the only possible case. It could happen that the time for a superluminal observer flows in our same direction. Since our future is also the future for a teleported observer, and in the future the scintillator does not give a signal for hypothesis, it follows that the speed of light is no longer invariant in a superluminal inertial reference frame.

Anyway, let us note that this is not in contrast with the well-known experimental results. In fact, we consider the decay of unstable sub-nuclear particles, for example the pions π^+ and π^-. In the decay, pions disappear and in their place other particles appear. If, at the initial instant, we have N_0 pions at rest (π^+ or π^-), at the time $t > 0$ their number will be:

$$N = N_0 e^{-t/\tau_0} \qquad (17)$$

where τ_0 is the proper lifetime measured in the rest frame of pions.

When the velocity of pions is subluminal and near the speed of light, it is observed that they travel along distances greater than those derived from their proper lifetime.

In fact, when pions travel with speed $v < c$ along the x axis of our laboratory frame, the decay law (17) can be written in terms of distance $x = v\,t$ as:

$$N = N_0 e^{-x/v\tau} \qquad (18)$$

where, in the laboratory frame, the lifetime τ is not the same as the proper lifetime τ_0 measured in the rest frame of pions. Indeed, for time dilation predicted by special relativity, it is:

$$\tau = \frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{19}$$

In the experiment of Ayres et al. (see ref. [9]), the pion detector was moved through a distance of over 10 m and the change in rate due to the decay was observed. Using time-of-flight techniques, the beam momentum was also measured and from these two data the validity of time dilation equation (19) of special relativity has been experimentally verified. For more details you can see section 17.2 of ref. [7], subsection Decay Modes, as well.

Anyway Ayres et al. observed the decay of pions moving at $v/c = 0.913$, that is they observed subluminal pions. This is in agreement with the fact that in the subluminal world special relativity rules, as we have already said many times, but, unfortunately, we do not have any data on the decay rate of pions moving faster than light. Indeed, assuming that in the superluminal reference frame in which faster-than-light pions are at rest the decay law (17) holds again, the knowledge of the dependence of the lifetime τ from the superluminal speed of pions and their proper lifetime τ_0 should give the behavior of clocks in the teleported state.

For example, if $\tau < 0$, we have that the teleported clocks turn in the opposite direction (our past direction) with respect to the laboratory clocks and so we will see pions appearing along their path according to equation (18). That could be a sign of the speed of light invariance.

Conversely, if $\tau > 0$, the teleported clocks turn in our same direction and we will see pions disappearing along their path.

Hence the speed of light cannot be invariant for a teleported observer as we have seen previously.

Watching the topic from a more sophisticated and mathematical point of view, once we know the way the time flows in a superluminal inertial reference frame, we can try to shape the geometry of its spacetime.

Obviously, without any experimental data, any kind of geometry you can imagine could be the right one. In my opinion, even some weird behaviors for space, time, propagation of light and intrinsic properties of superluminal particles could be true; never say never[i].

Waiting for future research, inverted dynamics, being a theory based on the available experimental data at the moment, has not dealt with this kind of topics since MINOS, OPERA and Fermilab79 have only given information useful to shape the tachyon dynamics. In fact, we have worked in a single inertial reference frame or, in practice, in our common laboratory frame (see Appendix D for further considerations).

Surely inverted dynamics leaves many open and unsolved questions but, in the end, I hope it can be the first little step into the other half of the sky.

THANKS VERY MUCH TO ELENA BARETTA AND PAMELA CILIBERTI
FOR THEIR VALUABLE CONTRIBUTION TO THIS WORK.

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[i] Let us note that the example of the photon and the teleported scintillator given in this paragraph is only one of the possible views of reality at the moment.

APPENDIX A - Theoretical Introduction: Newton's Second Law in Dynamics

A fundamental observation is that the variation of the state of motion of a point is caused by the interaction of the point with its surrounding environment. This interaction is described by the concept of force.

In Newtonian dynamics the quantitative formulation of the connection between the force and the state of motion of the point is given by:

❂ **LAW (A1) - CLASSICAL NEWTON'S SECOND LAW**

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

where the linear momentum **p** is defined as $\mathbf{p} = m_0\mathbf{v}$.

The dynamical state of the point is determined by its linear momentum which depends on its rest mass m_0 and its velocity **v**.

The action of a force changes the linear momentum of the point and so it will change its velocity **v**. However, to describe the dynamical behavior of a point you need to know its inertial rest mass m_0 which usually can be measured by using a balance for example. The term inertial rest mass is due to the fact that this quantity provides a measure of the resistance of the point to change its state of motion or, in other words, its velocity. Given a force **F** the dynamical effect is greater when the inertial rest mass m_0 is smaller.

In relativistic dynamics the quantitative formulation of the connection between the force and the state of motion of the point is given by:

❆ **LAW (A2) - RELATIVISTIC NEWTON'S SECOND LAW**

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

where the linear momentum **p** is defined as

$$\mathbf{p} = m_0\gamma\mathbf{v} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\mathbf{v}$$

being

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

The main difference[j] between the laws (A1) and (A2) is the definition of the linear momentum **p**.

In fact, in the definition (A2) of the relativistic linear momentum, $\mathbf{p} = m_0\gamma\mathbf{v}$, you do no longer have a linear dependence from the velocity **v** like in the Newtonian definition (A1), $\mathbf{p} = m_0\mathbf{v}$, but there is also a

[j] Laws (A1) and (A2) are the same when $v \ll c$.

multiplicative factor γ (depending on v).
Relations (A1) and (A2) are experimental laws derived by the study of the motion of a point subject to a known force. It is needless to say, however, that the best confirmation of relations (A1) and (A2) is provided by the correctness of the predictions made by using them.
Now we consider inverted dynamics. In inverted dynamics the quantitative formulation of the connection between the force and the state of motion of the superluminal point is given by:

✹ **LAW (A3) - SUPERLUMINAL NEWTON'S SECOND LAW**

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

where the linear momentum **p** is defined as

$$\mathbf{p} = m_0\,\gamma_{sl}\,\frac{c^2}{v-c}\,\mathbf{i}_v = \frac{m_0 c^4}{(v-c)^3}\,\mathbf{i}_v$$

being

$$\gamma_{sl} = \frac{c^2}{(v-c)^2}$$

and $\mathbf{i}_v$ the versor in the direction of the velocity **v**.
The main difference between the laws (A2) and (A3) is the definition of the linear momentum **p**.
In fact, in the definition (A3) of the superluminal linear momentum **p** you do no longer have a linear dependence from the velocity **v**, through the non-constant coefficient $m_0\gamma$, like in the relativistic definition (A2), $\mathbf{p} = m_0\gamma\,\mathbf{v}$, but there is instead an inverse dependence from the velocity **v**, through the non-constant coefficient $m_0\gamma_{sl}$, expressed by the term $c^2/(v-c)\,\mathbf{i}_v$.
Furthermore, the multiplicative factors γ and γ_{sl} do not necessarily have to be equal, and indeed they are not ($\gamma \neq \gamma_{sl}$), because inverted dynamics and special relativity describe two different physical worlds: the superluminal one and the subluminal one respectively.
However, even relation (A3) is an experimental law because the best confirmation of it is provided by the correctness of the previsions made by using it.
In fact, as illustrated in section 2, following the same procedure used in classical dynamics we get that the total energy of a superluminal point is $E = 3/2\ m_0\gamma_{sl}c^2$ see equation (6), and because this relation has been verified by using real data, see the discussion in section 3, that provides an experimental confirmation of relation (A3).
Furthermore, it has to be noticed that the relation $E = 3/2\ m_0\gamma_{sl}c^2$ is very similar to the mass-energy equivalence in special relativity, $E = m_0\gamma c^2$. Since in special relativity the term $m_0\gamma$ is interpreted as the inertial mass of a point travelling at subluminal speed, in the same way we will interpret the term $m_0\gamma_{sl}$ as the inertial mass of a point travelling at superluminal speed, see equation (1).

APPENDIX B - Note on the muonic neutrino rest mass

In this note we will make a few considerations on the experimental value of the muonic neutrino rest mass determined in the theoretical framework of inverted dynamics.
We suppose, by assumption, that the inertial mass of a superluminal particle can be written as:

$$m = \frac{m_0 c^{5/2}}{(v - c)^{5/2}}$$

Now following the same considerations at the beginning of section 2 of the article, we have that:

$$P = \frac{d}{dt}\left(\frac{mc^2}{v - c}\right)(v - c) = \frac{d}{dt}\left[\frac{m_0 c^{9/2}}{(v - c)^{7/2}}\right](v - c) =$$

$$= -\frac{7}{2}\frac{m_0 c^{9/2}}{(v - c)^{9/2}}(v - c)\frac{dv}{dt} = -\frac{7}{2}\frac{m_0 c^{9/2}}{(v - c)^{7/2}}\frac{dv}{dt}$$

Multiplying the above equation for the interval of time dt, the infinitesimal work done by a force is:

$$dL = P\,dt = -\frac{7}{2}\frac{m_0 c^{9/2}}{(v - c)^{7/2}}\,dv$$

Since this last expression is equal to

$$d\left[\frac{7}{5}\frac{m_0 c^{9/2}}{(v - c)^{5/2}}\right] = -\frac{7}{5}\frac{5}{2}\frac{m_0 c^{9/2}}{(v - c)^{7/2}}\,dv = -\frac{7}{2}\frac{m_0 c^{9/2}}{(v - c)^{7/2}}\,dv$$

it follows that:

$$dL = d\left[\frac{7}{5}\frac{m_0 c^{9/2}}{(v - c)^{5/2}}\right]$$

As we have already seen $dL = dE$ and so the total energy of a superluminal particle can be written as:

$$E = \frac{7}{5}\frac{m_0 c^{9/2}}{(v - c)^{5/2}} \tag{B1}$$

or, in terms of mass-energy equivalence, $E = 7/5\ mc^2$.
At this point, using the data in table 3 of section 3.1 of the article and equation (B1), MINOS and OPERA experiments give for the muonic neutrino rest mass the results displayed in table B1.
From table B1 we see that the five experimental results overlap with one another and so these measurements are consistent with the total energy law in equation (B1). Furthermore, having all the intervals values well below 170 keV, the muonic neutrino rest mass obtained in this way is in agreement with the experimentally known one.

EXPERIMENT	E (GeV)	m_0c^2 (eV)	Range (eV)
MINOS	3	0.040	0.005 ÷ 0.123
OPERA	13.9	0.022	0.004 ÷ 0.060
OPERA	17.0	0.038	0.019 ÷ 0.063
OPERA	28.1	0.061	0.021 ÷ 0.126
OPERA	42.9	0.121	0.036 ÷ 0.273

Table B1 - *Values of the muonic neutrino rest mass derived by using MINOS and OPERA results and equation (B1).*

Then, following the procedure at the end of section 3.1 of the article, a rough single estimate of the muonic neutrino rest mass and its associated uncertainty is:

$$m_0c^2 = 0.037 \pm 0.016 \text{ eV}$$

Moreover, we note that even the results of Fermilab79 experiment are in agreement with the MINOS and OPERA ones, as you can see in table B2 obtained using the data of table 6 in section 3.3 and the equation (B1) written above.

E (GeV)	Range (eV)	E (GeV)	Range (eV)
32	0 ÷ 0.007	90	0 ÷ 1.1
44	0 ÷ 3.1	120	0 ÷ 6.6
45	0 ÷ 0.057	125	0 ÷ 0.440
58	0.074 ÷ 8.0	157	0 ÷ 11.2
59	0 ÷ 0.075	170	0 ÷ 1.2
64	0 ÷ 0.808	183	0 ÷ 16.6
69	0 ÷ 0.088	195	0.044 ÷ 14.0

Table B2 – *Muonic neutrino rest mass calculated by using Fermilab79 experimental results and equation (B1).*

Due to the uncertainty in the known experimental value of the muonic neutrino rest mass, 0 ÷ 170 keV, even the expression illustrated in equation (B1) fits very well with MINOS, OPERA, Fermilab79 and Assamagan results. In the same manner, you could find other values for the exponent of the term $v - c$ that give an expression for the total energy in agreement with the experimental data. We will be able to remove this kind of indeterminacy in the total energy expression only when the muonic neutrino rest mass will be known more precisely or data taken with superluminal particles of well-defined rest mass will be available. Anyway, whatever the right exponent is, the theoretical framework of inverted dynamics remains the same since only the numerical results change.

For example, in the – 5/2 case, the α particle under $^{252}_{98}\mathrm{Cf}$ barrier will travel at a mean superluminal speed of (see section 3.2 of the article):

$$v = 299792458 \left[1 + \left(\frac{7}{5}\,\frac{3700}{-11.6+3700}\right)^{2/5}\right] \frac{m}{s} = 643205988 \;\frac{m}{s}$$

and it will cross the barrier, assuming that inside it does not bounce back and forth for some time, in:

$$t = \frac{f - d}{v} = \frac{(45.2 - 9.4)\cdot 10^{-15}}{643205988}\; m\,\frac{s}{m} = 5.6 \cdot 10^{-23}\; s$$

Hence the tunnelling of the α particle is almost an instant process again. In conclusion, the physics did not change.

APPENDIX C – Correct Operation of the OPERA Detector During the 2009-2011 Neutrino Data Taking

On the 22nd of September 2011 the first OPERA data was released (arXiv:1109.4897v1 [hep-ex]) creating a great outcry worldwide, but after some time the experimenters retracted their original results after further analysis. Now let us take a look at this further analysis.
In the latest version of the article "Measurement of the neutrino velocity with the OPERA detector in the CNGS beam" (arXiv:1109.4897v4 [hep-ex] 12 July 2012) it is reported that, starting from the beginning of December 2011, measurements of the time delay in the 8.3 km optical fiber between the ESAT GPS 1PPS output and the OPERA Master Clock output were made. A value 73.2 ns larger than the one determined in 2006 and 2007 was found. Further investigations revealed that the difference originated from an optical cable not properly connected and, when proper connections were restored, the value of the delay agreed with what was measured in 2006 and 2007 again.
According to my opinion, this is not properly a great scientific evidence since it seems to me like the following situation. A Saturday night at the end of 2007, not knowing what to do, I decided to go for a walk along the main street of my city; it was all floodlit and colorful. Then a Saturday night four years later, at the end of 2011, not knowing what to do I decided to go for a walk in the main street of my city again. This time there was darkness everywhere. Not having other information, I argued that the blackout happened on the Sunday night at the end of 2007, namely four years before.
This may be true, but it can also be true that the blackout happened on the Friday night at the end of 2011, namely a day before.
What reported in the article is a very weak experimental evidence supporting the need to retract the data included in its original version (22 September 2011). Indeed, it was searched for an additional independent information about the value of the LNGS fiber delay during the 2009-2011 neutrino data taking by the study of cosmic muon events in delayed coincidence in the OPERA and LVD detectors. Now let us take a careful look at this independent information reported in the article *Determination of a time-shift in the OPERA set-up using high energy horizontal muons in the LVD and OPERA detectors* (arXiv:1206.2488v1 [hep-ex] 12 June 2012).
Briefly, very high energy horizontal muons going first through the OPERA detector and then through the LVD detector were measured in delayed coincidence.
Both detectors are located in the same laboratory at Gran Sasso and they are separated by an average distance of 160 m, as you can see in figure C1.

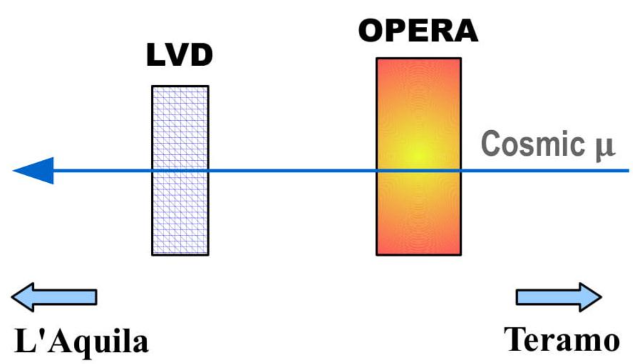

Figure C1 – *Location of the LVD and OPERA experiments (see arXiv:1206.2488v1).*

The data was collected from mid-2007 until 2012. The total number of cosmic horizontal muons detected by LVD and OPERA in delayed coincidence was 306.
The time difference between the UTC time of the LVD experiment (t_{LVD}) and the corrected UTC time of the OPERA experiment ($t^*_{OPERA} = t_{OPERA} - 74.4$ where t_{OPERA} is the UTC time of the OPERA experiment, while 74.4 ns is the correction due to the drift of the OPERA time-stamp within the DAQ cycle) for each cosmic horizontal muon event is considered as a function of the calendar date. The results are shown in figure C2 where the time difference is defined as $\delta t = t_{LVD} - t^*_{OPERA}$.

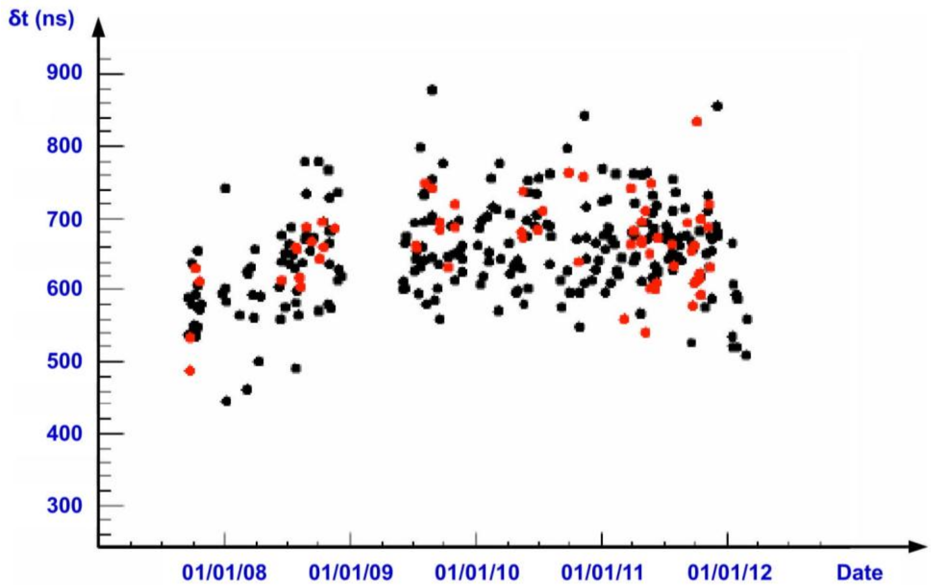

Figure C2 – *Distribution of the* $\delta t = t_{LVD} - t^*_{OPERA}$ *for corrected events. The black (red) dots represent events originated in the TT (RPC) OPERA sub-system (see arXiv:1206.2488v1).*

In order to study the stability of the time difference, $\delta t = t_{LVD} - t^*_{OPERA}$, versus calendar time, the data has been subdivided in different periods of the various solar years. Then, the mean and the associated uncertainty have been computed as summarized in the following table where the total number of events, 306, is distributed into eight samples, each one covering a given calendar time period.

Class	Year	From	To	No of Events	<δt> (ns)
A	2007	August	December	18	577 ± 10
A	2008-1	January	April	14	584 ± 20
A	2008-2	May	August	23	628 ± 11
B	2008-3	September	December	25	669 ± 11
B	2009	June	November	47	669 ± 9
B	2010	January	December	63	670 ± 8
B	2011	January	December	107	667 ± 5
A	2012	January	March	9	567 ± 16

Table C1 – *Summary of the* δt *distribution in the various calendar time periods (see arXiv:1206.2488v1).*

The results have been grouped in two classes:

- class A: between August 2007 to August 2008 and from January to March 2012;
- class B: from August 2008 to December 2011.

At this point, using the means in table C1, the mean value of δt for the events of class A and class B has been calculated giving a time difference $\Delta_{AB} = \langle\delta t_A\rangle - \langle\delta t_B\rangle$ between the two classes different from zero. From this result has been deduced, supposing the stability in time of LVD, that the OPERA detector had a negative time shift in the calendar time period from September 2008 to December 2011 of the order of $\Delta_{AB} = (-73 \pm 9)$ ns compared with the calendar time period from August 2007 to August 2008 and from January to March 2012 taken together.

However, according to my opinion, the means in table 1 used to calculate the mean values of δt in the two classes are not physically significant.

As explained in the article (arXiv:1206.2488v1), a cosmic muon can hit the TT subsystem or the RPC subsystem of the OPERA detector, and then it can hit the first, second or third tower (1T, 2T, 3T) of the LVD detector. Due to different delays in cables, the UTC time of each cosmic event in the detector depends not only on the measurement of its slave clock, $t_{clockLVD}$ or $t_{clockOPERA}$, but it also depends on where the cosmic horizontal muon hits the detector. The related data are summarized table C2.

Detector	**Detector Subsystem**	**UTC Time (ns)**
OPERA	TT	$t_{OPERA} = t_{clockOPERA} + 45230.3$
OPERA	RPC	$t_{OPERA} = t_{clockOPERA} + 45081.0$
LVD	Tower 1 (1T)	$t_{LVD} = t_{clockLVD} + 42166$
LVD	Tower 2 (2T)	$t_{LVD} = t_{clockLVD} + 42114$
LVD	Tower 3 (3T)	$t_{LVD} = t_{clockLVD} + 42091$

Table C2 – *UTC time definition for each subsystem of the two detectors (see arXiv:1206.2488v1).*

Hence we find that the time difference, $\delta t = t_{LVD} - t^*_{OPERA}$, has to be considered according to six different channels.

In fact, we suppose that a cosmic horizontal muon hits the TT subsystem of the OPERA detector first and the third tower of the LVD detector (3T) then. In this case, according to table C2, the time difference in nanoseconds is:

$$\delta t_{TT\text{-}3T} = t_{LVD} - t^*_{OPERA} = t_{LVD} - (t_{OPERA} - 74.4) = t_{clockLVD} + 42091 - (t_{clockOPERA} + 45230.3 - 74.4) =$$
$$= t_{clockLVD} - t_{clockOPERA} - 3064.9$$

where 74.4 ns, as previously noticed, is the correction due to the drift of the OPERA time-stamp within the DAQ cycle.

Now we suppose that a cosmic horizontal muon hits the RPC subsystem of the OPERA detector first and the first tower of the LVD detector (1T) then. In this case, according to table C2, the time difference in nanoseconds is:

$$\delta t_{RPC\text{-}1T} = t_{LVD} - {}_{clockOPERA}t^*_{OPERA} = t_{LVD} - (t_{OPERA} - 74.4) = t_{clockLVD} + 42166 - (t_{clockOPERA} + 45081.0 - 74.4) =$$
$$= t_{clockLVD} - t_{clockOPERA} - 2840.6$$

Finally, we suppose that $t_{clockLVD} - t_{clockOPERA}$ is the same both in the TT-3T event and in the RPC-1T event; we have:

$$\delta t_{RPC\text{-}1T} - \delta t_{TT\text{-}3T} = t_{clockLVD} - t_{clockOPERA} - 2840.6 - t_{clockLVD} + t_{clockOPERA} + 3064.9 = 224.3 \text{ ns}$$

At this point it should be clear that the positive time shift of time difference $\delta t_{RPC\text{-}1T}$ compared to the time difference $\delta t_{TT\text{-}3T}$ is due to a different timing of the RPC-1T subsystems combined together in a cosmic horizontal muon event with respect to the timing of the TT-3T subsystems combined together in a similar cosmic horizontal muon event, namely an event for which $t_{clockLVD} - t_{clockOPERA}$ is always the same.
So, during the analysis, the data should be divided into six channels (TT-1T, TT-2T, TT-3T, RPC-1T, RPC-2T, RPC-3T) according to where the cosmic horizontal muon hits the OPERA detector and subsequently the LVD detector to avoid the introduction of spurious time shifts caused by a different timing of the subsystems constituting the two detectors.
For example, we suppose that in a given period of time, which we will call Period 1, we have detected 9 events in the TT-3T channel, while in another given period of time, which we will call Period 2, we have detected 9 events in the RPC-1T channel. Assuming that all the horizontal muon events (18 events) are similar, namely $t_{clockLVD} - t_{clockOPERA}$ is always the same, it follows that the mean of the time differences δt over Period 2 will be greater than the mean of the time differences δt over Period 1 of 224.3 ns. This negative time shift, $\Delta_{12} = <\delta t_1> - <\delta t_2> = -224.3$ ns, is not caused by a different timing of the OPERA detector (or the LVD detector) during the Period 2, but it is due to the different detector subsystems involved.
Because the means of the time differences δt in table C1 are calculated using events from different channels, the inferred negative time shift of the OPERA setup during the 2009-2011 neutrino data taking, $\Delta_{AB} = (-73 \pm 9)$ ns, may not be physically significant.
A proper analysis should be to consider the events in the six channels separately and then studying the behavior of the time difference $\delta t = t_{LVD} - t^*_{OPERA}$ over the time. In this case if there is a systematic error caused by an improper connection of the 8.3 km optical fiber, it will be seen in all the six channels and it will have the same value in the limit of the experimental uncertainties. However, that cannot be done since we do not have enough cosmic horizontal muon events. In fact, let us consider for example the calendar time period from January 2012 to March 2012. In this case, as you can see in figure C3, we do not have any event in the RPC channels (there is not any red dot in the figure to the right of the blue line). Without any event, we cannot do any comparison.
And the same holds if we consider the calendar time period from January 2008 to April 2008: even in this case, as you can see in figure C3, we do not have any event in the RPC channels (there is not any red dot in the figure between the two green lines). At this point, we can note that in figure C2, where the time difference distribution ($\delta t = t_{LVD} - t^*_{OPERA}$) over the years of the 306 cosmic horizontal muon events is illustrated, at least a distinction between the TT events (black dots) and the RPC events (red dots) is made.

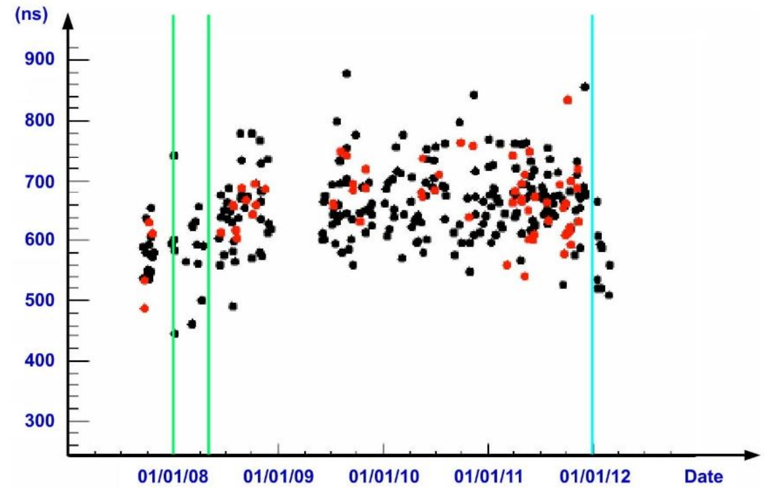

Figure C3 – *Calendar time periods from January 2008 to April 2008 and from January 2012 to March 2012.*

Now we consider the TT events in the time period from August 2007 to August 2008, when the 8.3 km optical fiber was properly connected, along with the TT events in the time period from January 2012 to March 2012, when the proper connection of the 8.3 km optical fiber was restored (class A events). Then we consider the RPC events in the time period from September 2008 to December 2011 when an improper connection of the 8.3 km optical fiber was supposed to happen (class B events). In order to make a correct comparison between the two sets of data, we remember that the time difference in the RPC channels has a positive time shift compared to the time difference in the TT channels when considering similar cosmic horizontal muon events ($t_{clockLVD} - t_{clockOPERA}$ is always the same). Since for each cosmic horizontal muon event no information is given about the stricken tower, we will proceed in the following way. First, we will suppose that all the RPC events of class B are in the RPC-3T channel, while all the TT events of class A are in the TT-1T channel.

Now the minimum measured value of δt for the TT events of class A was about 420 ns (value of δt corresponding to the black dot intersecting the lower horizontal green line in figure C4). Hence, according to table C2, we have that:

$$\delta t_{TT\text{-}1Tmin} = 420 = t_{clockLVD} + 42166 - (t_{clockOPERA} + 45230.3 - 74.4) = t_{clockLVD} - t_{clockOPERA} - 2989.9$$

At this point we can derive the minimum value of $t_{clockLVD} - t_{clockOPERA}$ for the TT events of class A. Indeed, from the previous relation it follows that:

$$(t_{clockLVD} - t_{clockOPERA})_{TT\text{-}1Tmin} = (442 + 2989.9)\ \text{ns} \simeq 3432\ \text{ns}$$

Instead, the maximum measured value of δt for the TT events of class A was about 780 ns (value of δt corresponding to the black dot intersecting the upper horizontal green line in figure C4). Hence, according to table C2, we have that:

$$\delta t_{TT\text{-}1Tmax} = 780 = t_{clockLVD} + 42166 - (t_{clockOPERA} + 45230.3 - 74.4) = t_{clockLVD} - t_{clockOPERA} - 2989.9$$

Then the maximum value of $t_{clockLVD} - t_{clockOPERA}$ for the TT events of class A is:

$$(t_{clockLVD} - t_{clockOPERA})_{TT\text{-}1Tmax} = (780 + 2989.9)\ \text{ns} \simeq 3770\ \text{ns}$$

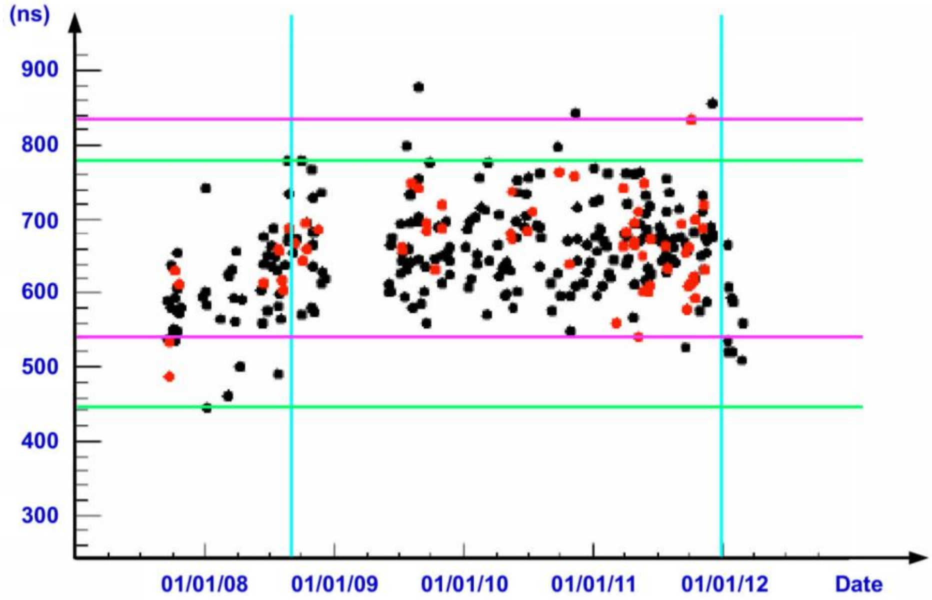

Figure C4 – *Class A events located to the left of the first vertical blue line and to the right of the second vertical blue line and class B events located between the two vertical blue lines.*

In conclusion, we see that the values of $t_{clockLVD} - t_{clockOPERA}$ for cosmic horizontal muon events detected in the TT subsystem of the OPERA detector when the timing system of this latter one is supposed to work properly (class A events) are in the range[k] $[3410 \text{ ns}, 3770 \text{ ns}]_{TT\text{-}1T}$.
Now the minimum measured value of δt for the RPC events of class B was about 540 ns (value of δt corresponding to the red dot intersecting the lower horizontal magenta line in figure C4). Hence, according to table C2, we have that:

$$\delta t_{RPC\text{-}3Tmin} = 540 = t_{clockLVD} + 42091 - (t_{clockOPERA} + 45081.0 - 74.4) = t_{clockLVD} - t_{clockOPERA} - 2915.6$$

At this point we can derive the minimum value of $t_{clockLVD} - t_{clockOPERA}$ for the RPC events of class B. Indeed, from the previous relation it follows that:

$$(t_{clockLVD} - t_{clockOPERA})_{RPC\text{-}3Tmin} = (540 + 2915.6) \text{ ns} \simeq 3456 \text{ ns}$$

Instead the maximum measured value of δt for the RPC events of class B was about 840 ns (value of δt corresponding to the red dot intersecting the upper horizontal magenta line in figure C4). Hence, according to table C2, we have that:

$$\delta t_{RPC\text{-}3Tmax} = 840 = t_{clockLVD} + 42091 - (t_{clockOPERA} + 45081.0 - 74.4) = t_{clockLVD} - t_{clockOPERA} - 2915.6$$

Then the maximum value of $t_{clockLVD} - t_{clockOPERA}$ for the RPC events of class B is:

$$(t_{clockLVD} - t_{clockOPERA})_{RPC\text{-}3Tmax} = (840 + 2915.6) \text{ ns} \simeq 3756 \text{ ns}$$

In conclusion, we see that the values of $t_{clockLVD} - t_{clockOPERA}$ for cosmic horizontal muon events detected in

[k] You take into account that the time difference δt for similar events will not have a single value, but it will be represented by a range of most probable values since the cosmic horizontal muons enter the detectors along different trajectories at different energies.

the RPC subsystem of the OPERA detector when the timing system of this latter one is supposed to work improperly (class B events) are in the range $[3456 \text{ ns}, 3756 \text{ ns}]_{RPC\text{-}3T}$.
So it follows that $[3456 \text{ ns}, 3756 \text{ ns}]_{RPC\text{-}3T} \subset [3410 \text{ ns}, 3770 \text{ ns}]_{TT\text{-}1T}$.
In other words, all the values of class B in the RPC-3T channel are similar to the values of class A in the TT-1T channel not showing in this way any difference between the two periods of time. More simply, we can write the previous relation as $(t_{clockLVD} - t_{clockOPERA})_{RPC\text{-}3T\,(B)} = (t_{clockLVD} - t_{clockOPERA})_{TT\text{-}1T\,(A)}$.
Now we will suppose that all the RPC events of class B are in RPC-3T channel again, while all the TT events of class A are in the TT-2T channel.
Following the same procedure illustrated above, and using the data of table C2 and the one in figure C4, we see that the measured values of $t_{clockLVD} - t_{clockOPERA}$ for the events of class A in the TT-2T channel are in the range $[3462 \text{ ns}, 3822 \text{ ns}]_{TT\text{-}2T}$, while the measured values of $t_{clockLVD} - t_{clockOPERA}$ for the events of class B in the RPC-3T channel are in the range $[3456 \text{ ns}, 3756 \text{ ns}]_{RPC\text{-}3T}$ as seen previously. In this case we can write $(t_{clockLVD} - t_{clockOPERA})_{RPC\text{-}3T\,(B)} \leq (t_{clockLVD} - t_{clockOPERA})_{TT\text{-}2T\,(A)}$. In the following table C3, using the data in table C2 and figure C4, are summarized the measured intervals of $t_{clockLVD} - t_{clockOPERA}$ for the events of class A in the TT channels and the measured intervals of $t_{clockLVD} - t_{clockOPERA}$ for the events of class B in the RPC channels according to the stricken tower.

RPC Channels	**$t_{clockLVD} - t_{clockOPERA}$ (ns) Class B**	**TT Channels**	**$t_{clockLVD} - t_{clockOPERA}$ (ns) Class A**
RPC-1T	[3381, 3681]	TT-1T	[3410, 3770]
RPC-2T	[3433, 3733]	TT-2T	[3462, 3822]
RPC-3T	[3456, 3756]	TT-3T	[3485, 3845]

Table C3 – *Measured* $t_{clockLVD} - t_{clockOPERA}$ *intervals for class A events in the TT channels and class B events in the RPC channels according to the stricken tower.*

Then, comparing the intervals in table C3, you can see that $(t_{clockLVD} - t_{clockOPERA})_{RPC(B)} \leq (t_{clockLVD} - t_{clockOPERA})_{TT(A)}$ whatever the stricken tower is. Hence, even if we do not know the stricken towers, we are sure that the range of $t_{clockLVD} - t_{clockOPERA}$ for the RPC events in the calendar time period from September 2008 to December 2011 (class B events) has values similar or smaller than the values in the calendar time period from August 2007 to August 2008 along with the calendar time period from January 2012 to March 2012 (class A events).
In other words, considering all the RPC events in the calendar time period from September 2008 to December 2011, you cannot see any negative time shift of the order of –73 ns caused by an improper connection of the 8.3 km optical fiber in the OPERA experimental setup. In fact, a negative time shift due to the OPERA setup would result in the following relation:

$$\Delta_{AB} = -73 = <(t_{clockLVD} - t_{clockOPERA})_A> - <(t_{clockLVD} - t_{clockOPERA})_B>$$

which can be written as:

$$<(t_{clockLVD} - t_{clockOPERA})_B> = <(t_{clockLVD} - t_{clockOPERA})_A> + 73$$

Hence, in the presence of a negative time shift of the order of –73 ns due to the optical fiber not properly

connected, it has to be $(t_{clockLVD} - t_{clockOPERA})_B \geq (t_{clockLVD} - t_{clockOPERA})_A$.
More precisely, the $t_{clockLVD} - t_{clockOPERA}$ intervals for the events in the time period from September 2008 to December 2011 (class B events) should have values similar or greater than the values in the time period from August 2007 to August 2008 along with the time period from January 2012 to March 2012 (events of class A). Obviously, this has to be true even when we consider the data according to its channel of detection. In the presence of a negative time shift it should be $(t_{clockLVD} - t_{clockOPERA})_{RPC(B)} \geq (t_{clockLVD} - t_{clockOPERA})_{TT(A)}$, but this was not the case. As we have just shown using the experimental data in figure C2, it has been observed $(t_{clockLVD} - t_{clockOPERA})_{RPC(B)} \leq (t_{clockLVD} - t_{clockOPERA})_{TT(A)}$ and so it is very unlikely that the OPERA setup had a negative time shift of the order of –73 ns in the calendar time period from September 2008 to December 2011.
Conversely, the data in figure C2 shows that the OPERA setup seems to have worked properly in this period of time.
A further confirmation, even if a bit qualitative, is obtained comparing the values of δt associated to the TT events of class A and the values of δt associated to the TT events of class B. As you can see in the following figure C5, the values of δt associated to the TT events of class A (black dots to the left of the first vertical blue line and black dots to the right of the second vertical blue line) are between the two horizontal green lines.

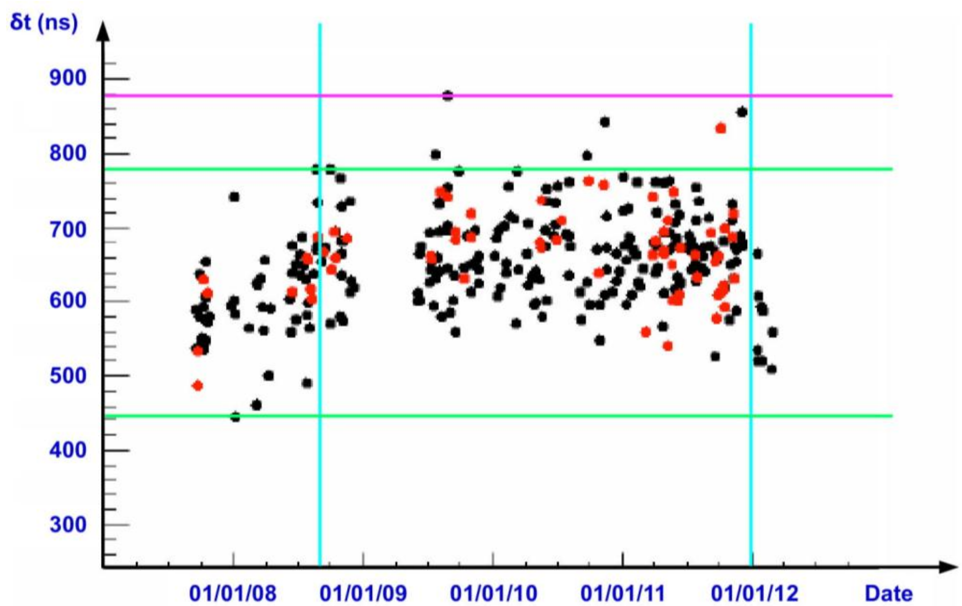

Figure C5 – *Comparison between TT events of class A and TT events of class B (see text).*

And the same is true for most of the values of δt associated to the TT events of class B (black dots between the two vertical blue lines). Hence these latter ones are compatible with the TT events of class A not showing, in this way, any difference in the measurements between the two periods of time.
Furthermore, we note that the global time accuracy of each event detected by the LVD detector is about 50 ns (see arXiv:1206.2488v1) and so the time accuracy of $\delta t = t_{LVD} - t^*_{OPERA}$ has to be at least equal to this value. For this reason, the maximum measured value of δt associated to the TT events of class A, 780 ns, will be written as 780 ± 50 ns.

At this point, a measured value of δt equal to 880 ns is compatible with 780 ns because the corresponding uncertainty intervals, namely 780 ± 50 ns and 880 ± 50 ns, overlap. Then it follows that all the values of δt such that 780 ns < δt ≤ 880 ns (values between the upper horizontal green line and the horizontal magenta line) are compatible with the maximum measured value of δt associated to the TT events of class A.
Hence, as clearly shown in figure C5, the remaining TT events of class B not included between the two horizontal green lines are compatible with the maximum measured value of δt associated to the TT events of class A since the corresponding black dots are included between the upper horizontal green line and the horizontal magenta line.
In conclusion, the values of δt taken during the time period from September 2008 to December 2011 (TT events of class B) are in agreement with the ones taken when the 8.3 km optical fiber was properly connected not showing, in this way, any negative time shift.
The same approach has been used in the comparison between the RPC events of class A and the RPC events of class B. As you can see in figure C6, the values of δt associated to the RPC events of class A (red dots to the left of the first vertical blue line) are between the two horizontal green lines.

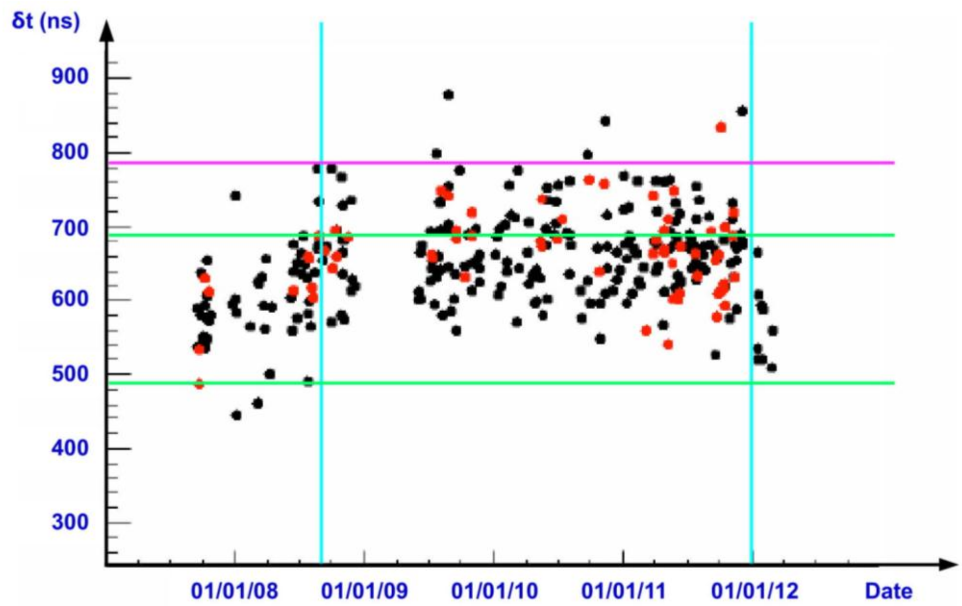

Figure C6 – *Comparison between RPC events of class A and RPC events of class B.*

And the same is true for more than half of the values of δt associated to the RPC events of class B (red dots between the two vertical blue lines). Hence these latter ones are compatible with the RPC events of class A not showing, in this way, any difference in the measurements between the two periods of time.
Furthermore, since the time accuracy of $\delta t = t_{LVD} - t^*_{OPERA}$ is at least equal to 50 ns, the maximum measured value of δt associated to the RPC events of class A, 690 ns, will be written as 690 ± 50 ns. At this point, a measured value of δt equal to 790 ns is compatible with 690 ns because the corresponding uncertainty intervals, namely 690 ± 50 ns and 790 ± 50 ns, overlap. Then it follows that all the values of δt such that 690 ns < δt ≤ 790 ns (values between the upper horizontal green line and the horizontal magenta line) are compatible with the maximum measured value of δt associated to the RPC events of class A. So, as

you can see in figure C6, the remaining RPC events of class B not included between the two horizontal green lines, except for one event, are compatible with the maximum measured value of δt associated to the RPC events of class A since the corresponding red dots are included between the upper horizontal green line and the horizontal magenta line. Even in this case the values of δt taken during the calendar time period from September 2008 to December 2011 (RPC events of class B) are in agreement with the ones taken when the 8.3 km optical fiber was properly connected not showing, in this way, any negative time shift.

In conclusion, the timing system of the OPERA setup seems to have always worked properly.

APPENDIX D – Space and Time

Inverted dynamics is not based on a space-time. In fact, inverted dynamics is defined in the laboratory reference frame.

Usually, the room corner and the three edges outgoing from it, which determine the origin O and the three orthogonal axes respectively, represent the simplest laboratory reference frame. Furthermore, if the fictitious forces are negligible in it, this can be viewed as an inertial reference frame.

Now we take a proper instrument to measure the distance from the room edges of a moving object, a rule for example, and a clock to correlate the object position to the time. As everyone knows, the time measured by the clock at rest in the room is unaffected by its particular location. Indeed, if I put the clock on the table or if I put the clock on the windowsill, the motion of its hands will be unchanged and the instrument will always give the same result whatever its position in the room is. Hence the time flow is independent from the clock location in the laboratory. In other words, there is not any connection between space and time in a given inertial reference frame. This is not surprising since, according to our experience in real life, space and time are two qualitatively very different physical quantities.

At this point, why do we talk about space-time? This concept comes into play when we have to relate the measurements taken in two different inertial reference frames, R and R′, in relative motion.

More precisely, the observations in the two inertial reference frames are connected by the Lorentz Transformations. In their simplest form, we have that $t' = (1 - v^2/c^2)^{-1/2} \cdot (t - vx/c^2)$ where v is the relative speed of R′ with respect to R. Then, unlike Galilean relativity, the time of an event in R′ can be reconstructed knowing its position and time in R together. It is clear that in this case space and time are strictly bound and so it makes sense to talk about space-time.

However, because inverted dynamics is defined in a fixed inertial reference frame, namely the laboratory frame, the concept of space-time is not applicable to this theory. In a fixed inertial reference frame, we just have space and time.

Moreover, if I may, I would like to make a comment. If you take a careful look at special relativity equations (mass, momentum, total energy) you can see that the related physical quantities go to infinity for $v \to c^-$, as experimentally well verified in particle accelerators, and furthermore they become imaginary physical quantities when v is greater than c. According to the mainstream interpretation, this means that a particle cannot go faster than light as already said at the beginning of the article. In my opinion instead, the one thing you can deduce is that special relativity does not have any sense for superluminal velocities because, having well defined and tight boundaries, it just describes moving objects with speed between [0, c]. Obviously, since a particle cannot go from subluminal velocities to superluminal velocities in a continuous way, these two worlds are separated. For this reason, the theory of superluminal motions not necessarily has to be an attempt to apply the principles and the mathematical framework of special relativity also in this case. In fact, nobody has found the so-called Superluminal Lorentz Transformations in the three space coordinates until now.

These transformations of coordinates should keep the speed of light invariant between two reference frames moving at relative superluminal velocity and they should turn the pseudo-length of a subluminal event in the Minkowski space-time, $ds^2 = (c\,dt)^2 - (dx)^2$, into the new pseudo-length $d\check{s}^2 = (dx)^2 - (c\,dt)^2$ corresponding to a superluminal event.

Furthermore, we do not even know if the speed of light is really invariant when we consider inertial reference frames moving at superluminal relative speed. Indeed, this is a fact, all the experiments on the invariance of the speed of light and its consequences deal with inertial reference frames moving at subluminal relative speed and not at superluminal relative speed, see Michelson-Morley (the Earth, on which the experimental setup rests, moves at the subluminal speed of 30 km/s around the Sun), Fizeau or the Doppler effect for example. Hence, as explained in section 4, the extension of the invariance of the speed of light to inertial reference frames moving at superluminal relative speed is just one of the possible pictures of a reality we do not know except for some information obtained by the experiments MINOS, Opera and Fermilab79.

At the moment, the pseudo-length $d\check{s}^2 = (dx)^2 - (c\,dt)^2$ in the Minkowski space-time may not describe our world since there is a lack of experimental evidence on that.

In conclusion, the special relativity rules forbid the space trip from one star to another in reasonable times; someone calls that the “Einstein’s cage”. However, in my opinion, special relativity is viewed by the most part of this generation of scientists as the theory of everything instead of giving to this theory its right place. The result was to think that because special relativity does not have any sense for superluminal motions, faster-than-light particles cannot exist even when the observations have shown the opposite; it is always an instrument failure.

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