The Standard Model Architecture and Interactions

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Helmholtzian , Klein-Gordon equation , electromagnetism , preon , elementary particles , standard model , elementary particle , particle interactions , fermions , leptons , quarks , hadrons , mesons , baryons , weak interactions , strong interactions

ABSTRACT
Based on my 1984 linearization of the Klein-Gordon equations, potential functions generalizations of the electric and magnetic field strengths form a basis from which a compound model simply constructs the leptons; the simple differences between the quarks and leptons; how the quarks arose from the leptons; why there are these two types of fermions; and why there are precisely three generations for each of these types. The most elementary particle interactions classify the interactions between strong and weak, and further still between the W and Z type of weak interactions. Two simple conservation requirements give rise to all the fundamental particle interactions, and describe the structure of the weak intermediate envelopes. Further, a simple charge function determines the charge of every object. Further still, the only free assignable parameters for the entire model are four mass constants for each fermion generation.

This presentation is essentially a summary of my book: "a Mathematical Preon Foundation for the Standard Model"; but starting from the different standpoint of my Helmholtzian operator matrix product, rather than my constructive algebras (developed primarily in "reality is a Mathematical Model" and "The Weighted Matrix Product").

Recalling the Helmholtzian operator matrix product form:

$$\mathbf{j}^\mu \equiv (\Box - |m|^2)\mathbf{A}^\mu$$

$$= \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & D_1 \\ D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0 \end{pmatrix} \begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ A^0 \end{pmatrix}$$

So, it is natural to make the following definitions:

$$\begin{pmatrix} D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & D_1 \\ D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0 \end{pmatrix} \begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ A^0 \end{pmatrix} \equiv \begin{pmatrix} B_{\Updownarrow}^1 - E^1 \\ B_{\Updownarrow}^2 - E^2 \\ B_{\Updownarrow}^3 - E^3 \\ \overset{m}{\nabla}_{\Updownarrow} \cdot \mathbf{A}^* \end{pmatrix}$$

where:

$$B^0 \equiv 0 \equiv E^0 \ , \ E^i \equiv -D_0^{\Updownarrow} A^i - D_i A^0 \ , \ (i \neq 0) \ , \ \mathbf{A}_{\Updownarrow} \equiv \begin{pmatrix} A_- \\ A_+ \end{pmatrix}$$

Through some ingenious mathematical manipulations, it can be shown that in free space, the thus defined $\mathbf{E}$ and $\mathbf{B}$ (generalizations of the electric and magnetic field strengths) also satisfy the Klein-Gordon equations, so have a particle-nature.

Identifying a particle-nature member $\mathbf{R}$ as either an $\mathbf{E}$ or a $\mathbf{B}$, and $\mathbf{R}_+$ as either an $\mathbf{E}_+$ or a $\mathbf{B}_+$, then a notation consistent with common usage would denote it's particle-nature anti-member $\overline{\mathbf{R}_+}$ as the corresponding $\overline{\mathbf{E}_+}$ or a $\overline{\mathbf{B}_+}$ (and correspondingly for $\overline{\mathbf{R}_-}$, $\overline{\mathbf{E}_-}$, & $\overline{\mathbf{B}_-}$). And, of course, the particle-nature anti-member components correspond in the same way. Each of these members satisfies the Klein-Gordon equation, but only really do so as three-vectors with three components or triplets. And, each bag of triplets must be triplets or triplets of triplets or triplets of triplets of triplets, and so on (i.e.: 3^n of triplets).

The simplest , and thus, most fundamental members are triplets. The next most fundamental is triplets of triplets.

These will be considered, here.

Denoting a triplet of triplets by: $\mathbf{S_R} \equiv (\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3) = (\mathbf{R}_{1+}, \mathbf{R}_{2+}, \mathbf{R}_{3+}) + (\mathbf{R}_{1-}, \mathbf{R}_{2-}, \mathbf{R}_{3-})$, is a 3×3 matrix.

Then we can write: $\mathbf{S_E} \equiv (\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3)$, and $\mathbf{S_B} \equiv (\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3)$.

The components of each vector written vertically:

$$\mathbf{S_R} \equiv \left(\begin{pmatrix} R^1 \\ R^2 \\ R^3 \end{pmatrix}_1 , \begin{pmatrix} R^1 \\ R^2 \\ R^3 \end{pmatrix}_2 , \begin{pmatrix} R^1 \\ R^2 \\ R^3 \end{pmatrix}_3 \right) \equiv \begin{pmatrix} R_1^1 & R_2^1 & R_3^1 \\ R_1^2 & R_2^2 & R_3^2 \\ R_1^3 & R_2^3 & R_3^3 \end{pmatrix}$$

Define the first fundamental objects as follows:

$L_i \equiv \mathbf{S}_{\mathbf{E}_i}$, $\Lambda_i \equiv \mathbf{S}_{\mathbf{B}_i}$

[where: $\mathbf{S}_{\mathbf{R}_1} \equiv (\mathbf{R}_1, \mathbf{0}, \mathbf{0})$, $\mathbf{S}_{\mathbf{R}_2} \equiv (\mathbf{0}, \mathbf{R}_2, \mathbf{0})$, $\mathbf{S}_{\mathbf{R}_3} \equiv (\mathbf{0}, \mathbf{0}, \mathbf{R}_3)$]

and define in-line notation: $\mathbf{S}_{\mathbf{R}_i} \equiv (\mathbf{R}^1, \mathbf{R}^2, \mathbf{R}^3)_i$

So, there are 3 pair of L, Λ :

$L_1 = (\mathbf{E}_1, \mathbf{0}, \mathbf{0}) = (E^1, E^2, E^3)_1$, $\Lambda_1 = (\mathbf{B}_1, \mathbf{0}, \mathbf{0}) = (B^1, B^2, B^3)_1$

$L_2 = (\mathbf{0}, \mathbf{E}_2, \mathbf{0}) = (E^1, E^2, E^3)_2$, $\Lambda_2 = (\mathbf{0}, \mathbf{B}_2, \mathbf{0}) = (B^1, B^2, B^3)_2$

$L_3 = (\mathbf{0}, \mathbf{0}, \mathbf{E}_3) = (E^1, E^2, E^3)_3$, $\Lambda_3 = (\mathbf{0}, \mathbf{0}, \mathbf{B}_3) = (B^1, B^2, B^3)_3$

And, let:

$$\eta_j(R_k^h) \equiv \begin{cases} R_k^h \ , \ j \neq 0 \\ E_k^h \ , \ j = 0 \ , \ \mathbf{R} = \mathbf{B} \\ B_k^h \ , \ j = 0 \ , \ \mathbf{R} = \mathbf{E} \end{cases} \ , \ \sigma_j(\mathbf{R}_k) \equiv \begin{pmatrix} \eta_{j-1}(R_k^1) \\ \eta_{j-2}(R_k^2) \\ \eta_{j-3}(R_k^3) \end{pmatrix}$$

$$\widehat{\mathbf{S}_{\mathbf{R}_{ij}}} \equiv (\delta_1^i \sigma_j(\mathbf{R}_1), \delta_2^i \sigma_j(\mathbf{R}_2), \delta_3^i \sigma_j(\mathbf{R}_3))$$

Corresponding to these fundamental objects, define these second order objects as 3 pair of triples, as follows:

$Q_{jh}^l = \widehat{\mathbf{S}_{\mathbf{E}_{hj}}}$, $Q_{jh}^\Lambda = \widehat{\mathbf{S}_{\mathbf{B}_{hj}}}$

with in-line notation: $\widehat{\mathbf{S}_{\mathbf{R}_{hj}}} \equiv (\eta_{j-1}(R^1), \eta_{j-2}(R^2), \eta_{j-3}(R^3))_h$

$Q_{11}^L = (B^1, E^2, E^3)_1$, $Q_{11}^\Lambda = (E^1, B^2, B^3)_1$

$Q^L_{21} = (E^1,B^2,E^3)_1$, $Q^\Lambda_{21} = (B^1,E^2,B^3)_1$
$Q^L_{31} = (E^1,E^2,B^3)_1$, $Q^\Lambda_{31} = (B^1,B^2,E^3)_1$
(which are, of course, merely a swapping of one component between the pair)
(Note: any number of swappings between the pair results in a member of these 9 matrices)
(- i.e. it is a group transformation, so it is sufficient to consider a single swapping)
(Note also that including the originals, there are 8 members)
The other two pair of triples are:
$Q^L_{12} = (B^1,E^2,E^3)_2$, $Q^\Lambda_{12} = (E^1,B^2,B^3)_2$
$Q^L_{22} = (E^1,B^2,E^3)_2$, $Q^\Lambda_{22} = (B^1,E^2,B^3)_2$
$Q^L_{32} = (E^1,E^2,B^3)_2$, $Q^\Lambda_{32} = (B^1,B^2,E^3)_2$

$Q^L_{13} = (B^1,E^2,E^3)_3$, $Q^\Lambda_{13} = (E^1,B^2,B^3)_3$
$Q^L_{23} = (E^1,B^2,E^3)_3$, $Q^\Lambda_{23} = (B^1,E^2,B^3)_3$
$Q^L_{33} = (E^1,E^2,B^3)_3$, $Q^\Lambda_{33} = (B^1,B^2,E^3)_3$
The following definitions:
$e^+ \equiv \overline{L_1}$, $\nu_e \equiv \Lambda_1$
$\mu^+ \equiv \overline{L_2}$, $\nu_\mu \equiv \Lambda_2$
$\tau^+ \equiv \overline{L_3}$, $\nu_\tau \equiv \Lambda_3$
correspond to the leptons.
And the following:
$u_R \equiv Q^l_{11}$, $d_R \equiv \overline{Q^\Lambda_{11}}$
$u_G \equiv Q^l_{21}$, $d_G \equiv \overline{Q^\Lambda_{21}}$
$u_B \equiv Q^l_{31}$, $d_B \equiv \overline{Q^\Lambda_{31}}$

$c_R \equiv Q^l_{12}$, $s_R \equiv \overline{Q^\Lambda_{12}}$
$c_G \equiv Q^l_{22}$, $s_G \equiv \overline{Q^\Lambda_{22}}$
$c_B \equiv Q^l_{32}$, $s_B \equiv \overline{Q^\Lambda_{32}}$

$t_R \equiv Q^l_{13}$, $b_R \equiv \overline{Q^\Lambda_{13}}$
$t_G \equiv Q^l_{23}$, $b_G \equiv \overline{Q^\Lambda_{23}}$
$t_B \equiv Q^l_{33}$, $b_B \equiv \overline{Q^\Lambda_{33}}$
correspond to all colors and flavors and generations of the quarks.
From this point on, represent the generations of the most fundamental objects by:

$e(i) \equiv \overline{L_i} = \overline{(E^1,E^2,E^3)_i}$, $u_j(i) \equiv Q^L_{ji} = (\eta_{j-1}(E^1),\eta_{j-2}(E^2),\eta_{j-3}(E^3))_i$
$\nu(i) \equiv \Lambda_i = (B^1,B^2,B^3)_i$, $d_j(i) \equiv \overline{Q^\Lambda_{ji}} = \overline{(\eta_{j-1}(B^1),\eta_{j-2}(B^1),\eta_{j-3}(B^1))_i}$
(i denoting column/generation, j denoting row/color)
So, in particular:

$e = e(1) = \overline{(E^1,E^2,E^3)_1}$	$\mu = e(2) = \overline{(E^1,E^2,E^3)_2}$	$\tau = e(3) = \overline{(E^1,E^2,E^3)_3}$
$\nu_e = \nu(1) = (B^1,B^2,B^3)_1$	$\nu_\mu = \nu(2) = (B^1,B^2,B^3)_2$	$\nu_\tau = \nu(3) = (B^1,B^2,B^3)_3$
$u_R = u_1(1) = (B^1,E^2,E^3)_1$	$c_R = u_1(2) = (B^1,E^2,E^3)_2$	$t_R = u_1(3) = (B^1,E^2,E^3)_3$
$u_G = u_2(1) = (E^1,B^2,E^3)_1$	$c_G = u_2(2) = (E^1,B^2,E^3)_2$	$t_G = u_2(3) = (E^1,B^2,E^3)_3$
$u_B = u_3(1) = (E^1,E^2,B^3)_1$	$c_B = u_3(2) = (E^1,E^2,B^3)_2$	$t_B = u_3(3) = (E^1,E^2,B^3)_3$
$d_R = d_1(1) = \overline{(E^1,B^2,B^3)_1}$	$s_R = d_1(2) = \overline{(E^1,B^2,B^3)_2}$	$b_R = d_1(3) = \overline{(E^1,B^2,B^3)_3}$
$d_G = d_2(1) = \overline{(B^1,E^2,B^3)_1}$	$s_G = d_2(2) = \overline{(B^1,E^2,B^3)_2}$	$b_G = d_2(3) = \overline{(B^1,E^2,B^3)_3}$
$d_B = d_3(1) = \overline{(B^1,B^2,E^3)_1}$	$s_B = d_3(2) = \overline{(B^1,B^2,E^3)_2}$	$b_B = d_3(3) = \overline{(B^1,B^2,E^3)_3}$

Examples of hadrons (second order compositions):
mesons:
$u_R : \overline{d_R} = (B^1,E^2,E^3)_1 : (E^1,B^2,B^3)_1 = \pi^+$.
$d_R : \overline{u_R} = \overline{(E^1,B^2,B^3)_1} : \overline{(B^1,E^2,E^3)_1} = \pi^-$.
$c_R : \overline{c_R} = (B^1,E^2,E^3)_2 : \overline{(B^1,E^2,E^3)_2} = \eta_c$.
$u_R : \overline{s_R} = (B^1,E^2,E^3)_1 : (E^1,B^2,B^3)_2 = K^+$.
$d_R : \overline{s_R} = \overline{(E^1,B^2,B^3)_1} : (E^1,B^2,B^3)_2 = K^0$.
$c_R : \overline{d_R} = (B^1,E^2,E^3)_2 : (E^1,B^2,B^3)_1 = D^+$.
$u_R : \overline{b_R} = (B^1,E^2,E^3)_1 : (E^1,B^2,B^3)_3 = B^+$.
$d_R : \overline{b_R} = \overline{(E^1,B^2,B^3)_1} : (E^1,B^2,B^3)_3 = B^0$.
These aren't all the mesons, but illustrates that they are of two families:
1) all the are matched: R^h_j with $\eta_0(R^h_m)$. (the charged ones)
2) all the are matched: R^h_j with $\overline{R^h_m}$. (the uncharged ones)
baryons:
$u_R : u_R : d_R = (B^1,E^2,E^3)_1 : (B^1,E^2,E^3)_1 : \overline{(E^1,B^2,B^3)_1} = p^+$.
$u_R : u_B : d_G = (B^1,E^2,E^3)_1 : (E^1,E^2,B^3)_1 : \overline{(E^1,B^2,B^3)_1} = p^+$.
$d_R : u_R : d_R = \overline{(E^1,B^2,B^3)_1} : (B^1,E^2,E^3)_1 : \overline{(E^1,B^2,B^3)_1} = n^0$.
$d_R : u_B : d_G = \overline{(E^1,B^2,B^3)_1} : (E^1,E^2,B^3)_1 : \overline{(E^1,B^2,B^3)_1} = n^0$.
As an $\mathbf{S_R}$ matrix, the proton and neutron incarnations are all the same. except for one swapped pair of elements.

Charge is a function $c()$ with the followinh characteristics:
$c((R^1,R^2,R^3)_h) = c(R^1_h) + c(R^2_h) + c(R^3_h)$,
$c\left(\overline{R^i_h}\right) = -c(R^i_h)$,
$c(E^i_h) = x$,
$c(B^i_h) = y$,.

then the objects are:
$c(e(i)) = 3x, c(\nu(i)) = 3y, c(u_j(i)) = 2x + y, c(d_j(i)) = -(x + 2y)$
Calibrating this with: $-1 = c(e(1)) = 3x, 0 = c(\nu(1)) = 3y \Rightarrow x = -\frac{1}{3}, y = 0$
Operating this linear function on the objects, yields:
$c(e(i)) = -1, c(\nu(i)) = 0$
$c(u_j(i)) = \frac{2}{3}, c(d_j(i)) = -\frac{1}{3}$
These are precisely the charge characteristics of all the fermions.

The function for mass is a little more complicated, but for the first and second order objects works out to be:

$$[M(R_h^1) + M(R_h^2) + M(R_h^3)]e^{f\text{–}type(\theta(h))k(p\text{–}type(\theta(h)))}$$

where:

$$M(E_h^i) \equiv \frac{1}{3}m_{0h}^2 + m_{ih}^2$$

$$M(B_h^i) \equiv \frac{1}{2}(m_{jh}^2 + m_{kh}^2) \quad , \quad \big(j \neq i, k \neq i, j \neq k\big)$$

with:

$$\boxed{\begin{aligned} k(2) &= \ln 3 - \ln\left(2\frac{m_{e((h))}^2}{m_{u_j((h))}^2} + \frac{m_{\nu((h))}^2}{m_{u_j((h))}^2}\right) \\ k(1) &= \ln 3 - \ln\left(\frac{m_{e((h))}^2}{m_{u_j((h))}^2} + 2\frac{m_{\nu((h))}^2}{m_{u_j((h))}^2}\right) \end{aligned}}$$

$$\boxed{\begin{aligned} m_{0h}^2 &= m_{e((h))}^2 - m_{\nu((h))}^2 \\ m_{1h}^2 &= m_{2h}^2 = m_{3h}^2 = \frac{1}{3}m_{\nu((h))}^2 \end{aligned}}$$

An $\mathbf{S_R}$ matrix with more elements than a first or second order object is termed an **intermediate envelope**.
Note that the columns of a solitary $\mathbf{S_R}$ matrix determine an object generation, and the row configuration of the members of a second order object for a given column determine the color of the second order object.

The Fermion Interaction:

An interaction with entry ingredients A, B and exit ingredients C, D is denoted:

$$A + B \longrightarrow C + D$$

thus: $A + B \longrightarrow C + D \Rightarrow \overline{A} + \overline{B} \longrightarrow \overline{C} + \overline{D}$.

A fermion interaction is an interaction between first and second order objects, i.e.: between solitary $\mathbf{S_R}$ matrices.
The initial step in a fermion interaction proceess is fusion of two solitary $\mathbf{S_R}$ matrices, enrty-wise, into a single $\mathbf{S_R}$ matrix.
Next, the $\mathbf{S_R}$ matrix entries may flip between columns/generations with equal likelihood. The $\mathbf{S_R}$ matrix entries may flip between rows/colors. Entries in the same column of the form $R_m^\alpha : \overline{R_n^\alpha}$ may flip to $\eta_{\alpha-\alpha}(R_m^\alpha) : \eta_{\alpha-\alpha}(\overline{R_n^\alpha})$, or those of the form $R_m^\alpha : \eta_{\alpha-\alpha}(R_n^\alpha)$ may to $\eta_{\alpha-\alpha}(R_m^\alpha) : R_n^\alpha$; or not with equal likelihood. These are the only random flip events conserving charge, so are the only ones allowed. Conservation of row/color indices also restricts the possible flip events.
Finally, the $\mathbf{S_R}$ matrix separates into the two solitary matrices column resultants from the flip events occurring during the fusion.

The first order objects (leptons) fermion-interactions occur as follows:

(1)

$$\begin{aligned}
&\nu(x) + \overline{e(y)} = (B^1, B^2, B^3)_x + (E^1, E^2, E^3)_y \\
&\Rightarrow (B_x^1 : E_y^1, B_x^2 : E_y^2, B_x^3 : E_y^3) && : \langle F\rangle \\
&\Rightarrow \Big(\eta_{h-1}(B_x^1) : \eta_{h-1}(E_y^1), \eta_{h-2}(B_x^2) : \eta_{h-2}(E_y^2), \eta_{h-3}(B_x^3) : \eta_{h-3}(E_y^3)\Big) && : \langle S\rangle \\
&\Rightarrow \Big(\eta_{h-1}(B_m^1) : \eta_{h-1}(E_n^1), \eta_{h-2}(B_m^2) : \eta_{h-2}(E_n^2), \eta_{h-3}(B_m^3)\overline{\eta_{h-3}(E_n^3)}\Big) && : \langle W\rangle \\
&\Rightarrow (\eta_{h-1}(B^1), \eta_{h-2}(B^2), \eta_{h-3}(B^3))_m + (\eta_{h-1}(E^1), \eta_{h-2}(E^2), \eta_{h-3}(E^3))_n \\
&= \begin{cases}
(E^1, B^2, B^3)_m + (B^1, E^2, E^3)_n = \overline{d_R(m)} + u_R(n) \;;\; h = 1, m \neq 3 \\
(B^1, E^2, B^3)_m + (E^1, B^2, B^3)_n = \overline{d_G(m)} + u_G(n) \;;\; h = 2, m \neq 3 \\
(B^1, B^2, E^3)_m + (E^1, E^2, B^3)_n = \overline{d_B(m)} + u_B(n) \;;\; h = 3, m \neq 3 \\
(B^1, B^2, B^3)_m + (E^1, E^2, E^3)_n = \nu(m) + \overline{e(n)} \;;\; h = 0 : \text{(none flipped)}
\end{cases}
\end{aligned}$$

The first order objects (leptons) fermion-interactions occur as follows:

(2)

$$\begin{aligned}
&e(x) + \overline{e(y)} = \overline{(E^1, E^2, E^3)_x} + (E^1, E^2, E^3)_y \\
&\Rightarrow \Big(\overline{E_x^1} : E_y^1, \overline{E_x^2} : E_y^2, \overline{E_x^3} : E_y^3\Big) && : \langle F\rangle \\
&\Rightarrow \Big(\overline{\eta_{h-1}(E_x^1)} : \eta_{h-1}(E_y^1), \overline{\eta_{h-2}(E_x^2)} : \eta_{j-2}(E_y^2), \overline{\eta_{i-3}(E_x^3)} : \eta_{j-3}(E_y^3)\Big) && : \langle S\rangle \\
&\Rightarrow \Big(\overline{\eta_{h-1}(E_m^1)} : \eta_{h-1}(E_n^1), \overline{\eta_{h-2}(E_m^2)} : \eta_{j-2}(E_n^2), \overline{\eta_{i-3}(E_m^3)} : \eta_{j-3}(E_n^3)\Big) && : \langle W\rangle \\
&\Rightarrow \overline{(\eta_{h-1}(E^1), \eta_{h-2}(E^2), \eta_{h-3}(E^3))_m} + (\eta_{h-1}(E^1), \eta_{h-2}(E^2), \eta_{h-3}(E^3))_n \\
&= \begin{cases}
\overline{(B^1, E^2, E^3)_m} + (B^1, E^2, E^3)_n = \overline{u_R(m)} + u_R(n) \;;\; h = 1 \\
\overline{(E^1, B^2, E^3)_m} + (E^1, B^2, E^3)_n = \overline{u_G(m)} + u_G(n) \;;\; h = 2 \\
\overline{(E^1, E^2, B^3)_m} + (E^1, E^2, B^3)_n = \overline{u_B(m)} + u_B(n) \;;\; h = 3 \\
\overline{(E^1, E^2, E^3)_m} + (E^1, E^2, E^3)_n = e(m) + \overline{e(n)} \;;\; h = 0 : \text{(none flipped)}
\end{cases}
\end{aligned}$$

(3)

$$\begin{aligned}
&\overline{\nu(x)} + \nu(y) = \overline{(B^1, B^2, B^3)_x} + (B^1, B^2, B^3)_y \\
&\Rightarrow \Big(\overline{B_x^1} : B_y^1, \overline{B_x^2} : B_y^2, \overline{B_x^3} : B_y^3\Big) && : \langle F\rangle \\
&\Rightarrow \Big(\overline{\eta_{h-1}(B_x^1)} : \eta_{h-1}(B_y^1), \overline{\eta_{h-2}(B_x^2)} : \eta_{j-2}(B_y^2), \overline{\eta_{i-3}(B_x^3)} : \eta_{j-3}(B_y^3)\Big) && : \langle S\rangle \\
&\Rightarrow \Big(\overline{\eta_{h-1}(B_m^1)} : \eta_{h-1}(B_n^1), \overline{\eta_{h-2}(B_m^2)} : \eta_{j-2}(B_n^2), \overline{\eta_{i-3}(B_m^3)} : \eta_{j-3}(B_n^3)\Big) && : \langle W\rangle
\end{aligned}$$

$$\Rightarrow \overline{(\eta_{h-1}(B^1),\eta_{h-2}(B^2),\eta_{h-3}(B^3))_m} + (\eta_{h-1}(B^1),\eta_{h-2}(B^2),\eta_{h-3}(B^3))_n$$

$$= \begin{cases} \overline{(E^1,B^2,B^3)_m} + (E^1,B^2,B^3)_n = d_R(m) + \overline{d_R(n)} \ ; \ h = 1 \\ \overline{(B^1,E^2,B^3)_m} + (B^1,E^2,B^3)_n = d_G(m) + \overline{d_G(n)} \ ; \ h = 2 \\ \overline{(B^1,B^2,E^3)_m} + (B^1,B^2,E^3)_n = d_B(m) + \overline{d_B(n)} \ ; \ h = 3 \\ \overline{(B^1,B^2,B^3)_m} + (B^1,B^2,B^3)_n = \overline{\nu(m)} + \nu(n) \quad ; \ h = 0 : \text{(none flipped)} \end{cases}$$

This is how the second order objects (quarks) are/were created from the most fundamental first order objects (leptons).

Weak type interactions may be further classified according to whether they are of the form (1), or of the form of (2)/(3).
Interactions of the form (1) (i.e.: $d + \nu \longrightarrow u + e$) are of the W type.
Interactions of the form (2)/(3) (i.e.: $f_1 + \overline{f_1} \longrightarrow f_2 + \overline{f_2}$) are of the Z type.

It will be seen that all the interactions to follow fall into these catagories.
The second order objects (quarks) fermion-interactions are far more computationally long and intensive, but all the fermion-interactions may be summarized, as follows: :

No interaction has deterministic exit ingredients, but every interaction has a deterministic family of exit ingredients with probability outcome.
All the interactions allow change of generation.
The interactions are summarized as follows:
(in the following, unless otherwise stated: $(i,j,h,k,x,y,m,n \in \{1,2,3\})$)

(1)

$$\nu(x) + \overline{e(y)} \longrightarrow \begin{cases} \overline{\nu(m)} + e(n) \\ \overline{d_h(m)} + u_h(n) \quad , \quad (m \in \{1,2\}) \end{cases}$$

(2)

$$e(x) + \overline{e(y)} \longrightarrow \begin{cases} e(m) + \overline{e(n)} \\ \overline{u_h(m)} + u_h(n) \end{cases}$$

(3)

$$\overline{\nu(x)} + \nu(y) \longrightarrow \begin{cases} \overline{\nu(m)} + \nu(n) \\ d_h(m) + \overline{d_h(n)} \quad , \quad (m,n \in \{1,2\}) \end{cases}$$

(4)

$$\overline{u_i(x)} + u_j(y) \longrightarrow \begin{cases} \overline{u_i(m)} + u_j(n) \\ e(m) + \overline{e(n)} \\ d_h(m) + \overline{d_k(n)} \ , \ k = 1 + (h - 1 + |i-j|)(\text{mod}\,3) \ , \ (m,n \in \{1,2\}) \end{cases}$$

(5)

$$d_i(x) + \overline{d_j(y)} \longrightarrow \begin{cases} d_i(m) + \overline{d_j(n)} \quad , \quad (m,n \in \{1,2\}) \\ \overline{u_h(m)} + u_k(n) \ , \ k = 1 + (h - 1 + |i-j|)(\text{mod}\,3) \\ \overline{\nu(m)} + \nu(n) \quad , \quad (i = j) \\ \qquad\qquad , \ (x,y \in \{1,2\}) \end{cases}$$

(6)

$$d_i(x) + \overline{u_j(y)} \longrightarrow \begin{cases} \overline{\nu(m)} + e(n) \quad , \ i = j \\ d_h(m) + \overline{u_k(n)} \quad , \ k = 1 + (h - 1 + |i-j|)(\text{mod}\,3) \\ \qquad\qquad , \ (x,m \in \{1,2\}) \end{cases}$$

(7)

$$d_i(x) + d_j(y) \longrightarrow d_h(m) + d_k(n) \quad , \ k = 1 + (h - 1 + |i-j|)(\text{mod}\,3) \ , \ (x,y,m,n \in \{1,2\})$$

(8)

$$d_i(x) + u_j(y) \longrightarrow d_i(m) + u_j(n) \ , \ (x,m \in \{1,2\})$$

(9)

$$u_i(x) + u_j(y) \longrightarrow u_i(m) + u_j(n)$$

(10)

$$e(x) + \overline{u_j(y)} \longrightarrow \begin{cases} e(m) + \overline{u_j(n)} \\ \overline{u_j(m)} + e(n) \end{cases}$$

(11)

$$e(x) + u_j(y) \longrightarrow e(m) + u_j(n)$$

(12)

$$e(x) + \overline{d_j(y)} \rightarrow \begin{cases} e(m) + \overline{d_j(n)} \\ \overline{u_j(m)} + \nu(n) \end{cases}$$

(13)
$$e(x) + d_j(y) \rightarrow e(m) + d_j(n)$$

(14)
$$\nu(x) + \overline{u_j(y)} \rightarrow \begin{cases} \nu(m) + \overline{u_j(n)} \\ \overline{d_j(m)} + e(n) \end{cases}$$

(15)
$$\nu(x) + u_j(y) \rightarrow \nu(m) + u_j(n)$$

(16)
$$\nu(x) + \overline{d_j(y)} \rightarrow \nu(m) + \overline{d_j(n)}$$

(17)
$$\nu(x) + d_j(y) \rightarrow \nu(m) + d_j(n)$$

It is clear, that:
$$A + \overline{B} \rightarrow C + \overline{D} \Rightarrow A + D \rightarrow C + B$$

Thus, the above simple mathematical construction has just been clearly and concisely shown to define, analyze, and determine a foundation for the standard model with a simple interaction process consistent with experiment.

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