Linear programming solves biclique problems, flaws in literature proof

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Abstract

The study of perebor *dates back to the Soviet-era mathematics, especially in the 1980s [1]. Post-Soviet mathematicians have been working on many problems in combinatorial optimization. One of them is Maximum Edge Biclique Problem (*MBP*). In [2], the author proves that MBP is NP-complete. In this note, we give a polynomial time algorithm for* MBP *by using linear programming (*LP*). Thus, some flaw needs to be found in Peeter's work. We leave this to the community.*

I. Linear programming formulation

In MBP, we are given a bipartite graph $G = (X \cup Y, E)$ and asked to find a maximum edge biclique. A biclique of G is one of its subgraphs that have an edge connecting every pair of vertices belonging to different parts.

Our algorithm works by trying every pair (k, l) to search for a (k, l) biclique, which has k vertices in X, l vertices in Y. After solving all these subproblems, we can easily obtain the result for MBP.

Our LP formulation is very simple. For k and l, we have an LP model. Every model is the same except for two constraints.

For each X-vertex, we create an x_i variable with domain [0, 1]. For each Y-vertex, we create a y_j variable with domain [0, 1]. For each non-edge (x_i, y_j), we have $constraint_{ij}$ that prevents solution having both x_i and y_j non-zero. It is equivalent to: $x_i \neq 0$ implies $y_j = 0$. To state this in LP, we use big-M method:

$M(x_i - 1) + y_j \leq 0$, where M is a big enough positive natural number

Two subproblem-dependent constraints are just $x_1 + ... + x_m = k$ and $y_1 + .. + y_n = l$. Our objective function is $x_1 + x_2 + ... + x_m + y_1 + ... + y_n$ to be maximized.

Note that in every solution returned by an LP solver, if $x_i \neq 0$ and $y_j \neq 0$, then (x_i, y_j) is an edge, otherwise this would violate $constraint_{ij}$.

So by taking, all the non-zero variables (vertices), we have a solution for our subproblems. If it is larger than needed, we only need to remove some vertices.

II. Conclusion and future vision

We can anticipate much work to be done on *perebor* in near future, after Soviet-era mathematics and [3]. For now, flaws are among the widespread literature proof. This calls for a large-scale scrutiny of all mathematics. Moreover, whether these colossal monuments can thrive in a (possibly) finite universe is of philosophical concerns.

References

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