

P ≠ NP

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Abstract

A problem exists that's hard to solve but easy to verify a solution for.

$\exists V \in \{V : V \text{ is a deterministic polynomial time Turing machine}\}$
 $\forall M \in \{M : M \text{ is a deterministic Turing machine} \wedge$
 $\forall S \subseteq \Sigma$
 $\forall k \in \{k \in \mathbb{N} : k \leq |S|\}$
 $\forall w \in \{w \in \{0, 1\}^* : w \text{ is } S \text{ and } k \text{ encoded as a binary string} \wedge M \text{ accepts } w\} [$
 $M(w) = \exists b \in \{b \subseteq \{a \subseteq S : |a| = k\} : |b| = k\} [$

note: b is a k-subset of a k-subset of S

$V(b)$

note: V is a polynomial time verifier for M using b as a certificate

$]]$
 $\forall S \in \Sigma$
 $\forall k \in \{k \in \mathbb{N} : k \leq |S|\}$
 $\forall w \in \{w \in \{0, 1\}^* : w \text{ is } S \text{ and } k \text{ encoded as a binary string} \wedge M \text{ accepts } w\}$
 $\forall n \in \mathbb{N}$
 $\forall F \in \{F : F \text{ is a deterministic Turing machine} \wedge (F \text{ accepts } w \Leftrightarrow |w| = n)\} [$
 $F(w) = M(w)$

$] \Rightarrow$

$\forall S \in \Sigma$
 $\forall k \in \{k \in \mathbb{N} : k \leq |S|\}$
 $\forall w \in \{w \in \{0, 1\}^* : w \text{ is } S \text{ and } k \text{ encoded as a binary string} \wedge M \text{ accepts } w\}$
 $\forall n \in \mathbb{N}$
 $\forall F \in \{F : F \text{ is a deterministic Turing machine} \wedge (F \text{ accepts } w \Leftrightarrow |w| = n)\} [$
 $F \text{ has } O(n \wedge \log(n) \wedge \log(n)) \text{ certificates}$

$] \Rightarrow$

$\forall S \in \Sigma$
 $\forall k \in \{k \in \mathbb{N} : k \leq |S|\}$
 $\forall w \in \{w \in \{0, 1\}^* : w \text{ is } S \text{ and } k \text{ encoded as a binary string} \wedge M \text{ accepts } w\}$
 $\forall n \in \mathbb{N}$
 $\forall F \in \{F : F \text{ is a deterministic Turing machine} \wedge (F \text{ accepts } w \Leftrightarrow |w| = n)\} [$
 $F \text{ runs in } \geq O(2 \wedge \log(n)) \text{ steps}$

note: this is because each certificate is in a different node on the decision tree and a balanced decision tree requires the least amount of steps

$] \Rightarrow$

$\forall w \in \{w \in \{0, 1\}^* : w \text{ is } S \text{ and } k \text{ encoded as a binary string} \wedge M \text{ accepts } w\} [$
 $M \text{ runs in } \geq O(2 \wedge \log(|w|)) \text{ steps}$

$] \Rightarrow$ the decision problem of $M \notin P$

$] \Rightarrow P \neq NP$

note: the decision problem of M is in NP because YES solutions can be verified in polynomial time by V using b as a certificate